Modal Analysis of Mono Leaf Spring

Using Analytical, F.E.A (Pro-e Mechanica) and Experimental (Vibration Shaker) Approach

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Abstract:In this research work, study is carried out on mono steel leaf springs used by commercial vehicles. The chief thought behind this work is to examine the vibrational behavior of two leaf springs one having Upturned eye and one having Berlin eye, both is having almost similar parameters i.e. width, thickness & load carrying capacity. The material used for the deuce is steel with almost similar properties. The modal analysis is accomplished theoretically for determining the natural frequency of both the springs for first five mode shapes. To validate the theoretical modal analysis, modal analysis is conducted with aid of simulation by FEA and also through experimentation on vibration shaker. Modeling is finished using Pro-E 4.0 and analysis performed on Pro-E Mechanica platform. The result of all deuce-ace analyzed operations almost coincides and compared. From the outcomes comparative graphs are plotted for acceleration v/s frequency to obtain the resonance frequency thus to incur the value for unhazardous design of system & to further optimize the system by modifying either the material properties and eye conditions or both.

Index Terms– *Modal analysis, Resonance, finite element analysis, berlin eye, upturned eyeand mode shapes.*

I. INTRODUCTION

Leaf springs are commonly used in the vehicle suspension system and are subjected to millions of varying stress cycles leading to fatigue failure. A lot of research has been done for improving the performance of leaf spring. The automobile industry has shown interest in the replacement of steel spring with composite leaf spring. But composite materials are costly so manufacturers are also looking for alternative solution for the said problem. At the same time automobile manufacturers are trying different eye condition for leaf spring while keeping the material same to improve vibrational behavior of the same. The automobile manufacturers can reduce product development cost and time while improving the safety, comfort, and durability of the vehicles they produce with this analysis. Also during the analysis, the leaf spring is subjected to frequency with which the leaf spring has to operate most of the times in the vehicles. Modal analysis is a worldwide used methodology that allows fast and reliable identification of system dynamics in complex structures. Modal analysis refers to measuring and predicting the mode shapes and frequencies of a structure. Through modal analysis it is possible to predict the vibration behavior of leaf spring.

II. SELECTION OF SPECIMEN

The specimen selected foe testing are produced and used by leading automobile manufacturer Mahindra and Mahindra. Two mono leaf spring were selected One with upturned eye and other with Berlin eye. Following table will give an overview of properties of mono leaf spring selected

Table 1. Properties of the Selected Specimen

Eye Condition	Dimensions l×w×t (mm)	Material	Mass Density (kg/m³)	Young's Modulus (N/mm²)	Poisson's Ratio	Moment of Inertia (m⁴)
Upturned	1150×70×7	Steel	7850	2.1×10^5	0.266	2×10^{-9}
Berlin	1139×60×7	Steel	7860	2.1×10^5	0.3	2.56×10^{-9}

III. RESEARCH METHODOLOGY

A. Theoretical Euler Bernoulli Beam Theory

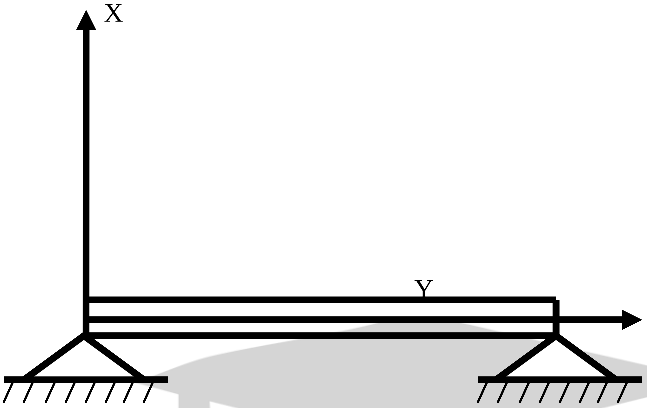
Beam theory is a simplification of the linear theory of elasticity which provides a means of calculating the load carrying and deflection characteristics of beams. It covers the case for small deflections of a beam that are subjected to lateral loads only. The Euler's equation for natural frequency is given as

$$\omega_n = \frac{n^2\pi^2}{L^2}\left[\sqrt{\frac{EI}{\rho A}}\right]$$

Where,

n = Mode Shape

I = Moment of inertia of system
ρ = Density of material
A = Area of cross section
L = Length of spring
E = Modulus of Elasticity
For simply supported beam



EI, m = Constant with x

$$EI\frac{d^4W}{dx^4} + \frac{d^2w}{dt^2} = P_z$$

Boundary Conditions:

At X=0 , W=0

At X=l , $M = EI\frac{d^2w}{dx^2} = 0$

With:

$$W(x) = C_1 \sin\lambda x + C_2 \cosh\lambda x + C_3 \sin\lambda x + C_4 \cos\lambda x$$

Put the resulting four equations in matrix form

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ \sin\lambda l & \cos\lambda l & \sin\lambda l & \cos\lambda l \\ \sin\lambda l & \cos\lambda l & -\sin\lambda l & -\cos\lambda l \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

From above matrix

$$w(0) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{d^2w}{dx^2}(0) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$w(l) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\frac{d^2w}{dx^2}(l) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Solution of determinant matrix generally yields values of λ which then yield frequencies and associated modes (as was done for multiple mass systems in a somewhat similarfashion)
In this case, the determinant of the matrix yields:

$$C\sin\lambda l = 03$$

Note: Equations (1 & 2) give $c_2 = c_4 = 0$

Equations (3 & 4) give 2 $c_3 \sin\lambda l = 0$

⇒Nontrivial: $\lambda l = n\pi$

The nontrivial solution is:

$\lambda l = n\pi$ (Eigenvalue problem) Recalling that:

$$\lambda = \left[\frac{\mathrm{m}\omega^2}{\mathrm{EI}}\right]^{\frac{1}{4}}$$

$\frac{m\omega^2}{EI} = \frac{n^4\pi^4}{l^4}$ (Change n to r to be consistent with previous notation)

$\omega_n = \frac{n^2\pi^2}{L^2}\left[\sqrt{\frac{EI}{\rho A}}\right]$(Natural frequency)

B. FEM Analysis

The idea of finite element originated when the advances in aircraft structural analysis started. This method is used to solve physical problem governed by a differential equation/energy theorem. Finite element analysis is numerical technique to handle complex geometry, any material properties, any boundary conditions, and any loading conditions. Mathematical model of any geometry describes its behavior by differential equations and boundary conditions. The mathematical model is dividing the object of interest into finite number of elements the term degree of freedom is commonly used for physical objects; if the number of DOF of the model is finite the model is called discreet and continuous. When the physical object is divided into discreet parts then the infinite DOF is converted into finite DOF. A finite element mesh is obtained after assemblage of all elements at nodes is represented by a system of algebraic equations and is solved for unknowns by integral formulations. This method provides an approximate solution but it has receive importance in engineering field due to its diversity and flexibility as an analysis tool.

Modeling is done using Pro-E (Wild Fire) 4.0 and analysis is carried out using Pro-E Mechanica software. Pro-E is a 3D CAD, CAM, CAE and associative solid modeling app. It is one of a suite of 10 collaborative applications that provide solid modeling, assembly modeling, 2D orthographic views, finite element analysis, direct and parametric modeling, sub divisional and NURBS surface modeling, and NC and tooling functionality for mechanical designers.

Procedure for designing and analysis are as follows:

1. Select the dimension of leaf spring.

Parameters	Upturned Eye(mm)	Berlin Eye(mm)
Outer diameter of eye	50	44
Thickness	7	7
Full length	1150.5	1139
Half length	575.2	569.5
Width	70	60
Radius of curvature(R)	968	875
Angle	27	25

Dimension of leaf springs

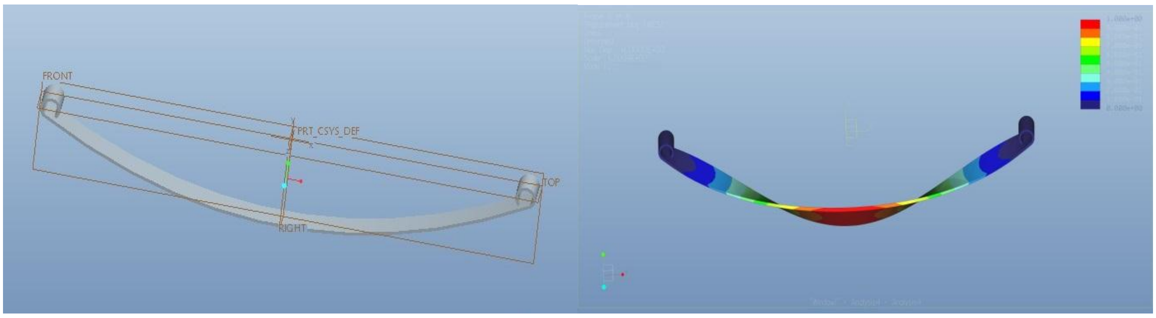
2. Open PRO-E software in which we have to design leaf spring from given dimensions.
3. Select the front plane for drawing the leaf spring
4. Draw the centreline at both the axis
5. Draw a circle from given dimension of inner diameter of eye at the right hand side and mirror this circle along its Y-axis
6. Make the arc of leaf spring considering the angle between tangent to eye and the base
7. Trim the extra part of circle
8. Add the thickness of spring by offset command. Extrude this sketch by given width.
9. Import this model in PRO-E MECHANICA work bench.
10. Apply boundary condition for the simply supported beam and cantilever beam
11. Enter the material properties which are obtained from research papers

Parameter	Upturned Eye Material(65Si7)	Berlin Eye Material(SiMn)
Young's modulus(E)	2.1×10^5 N/mm^2	2.1×10^5 N/mm^2
Poisson's ratio	0.266	0.3
Yield stress	250 Mpa	1680 N/mm^2
Density	7850 Kg/m^3	7.86×10^{-6} Kg/mm^3

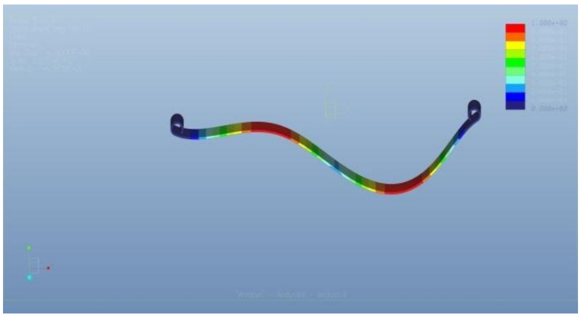
Material properties of leaf spring

12. Click on the mechanica result and edit the frequency range then run the analysis part which shows results
13. Natural frequency and the mode shapes are obtained after analysis in PRO-E MECHANICA software.

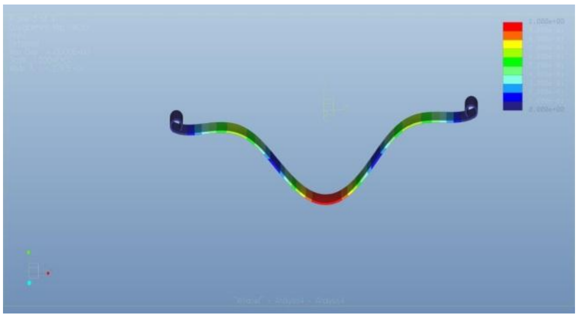
3-D MODEL AND MODE SHAPES OF UPTURNED EYE LEAF SPRING:



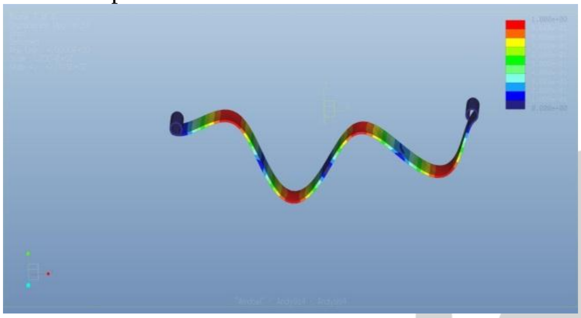
3-D Model of Upturned Eye Leaf spring



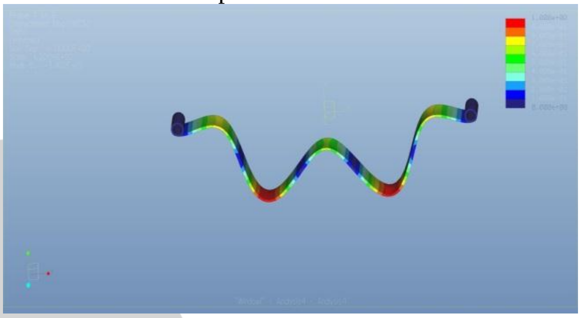
Mode Shape 1



Mode Shape 2



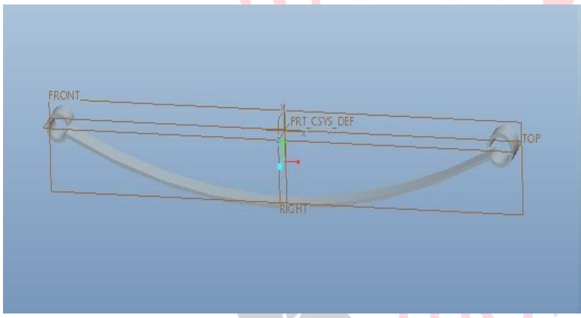
Mode Shape 3



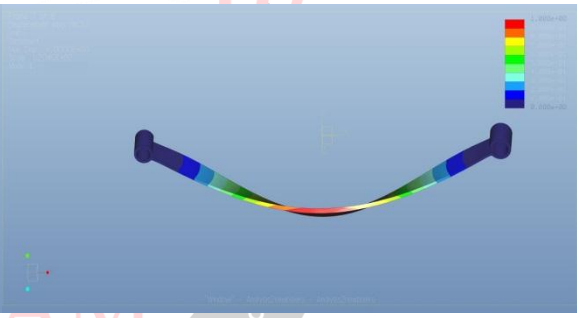
Mode Shape 4

Mode Shape 5

3-D MODEL AND MODE SHAPES OF BERLIN EYE LEAF SPRING:

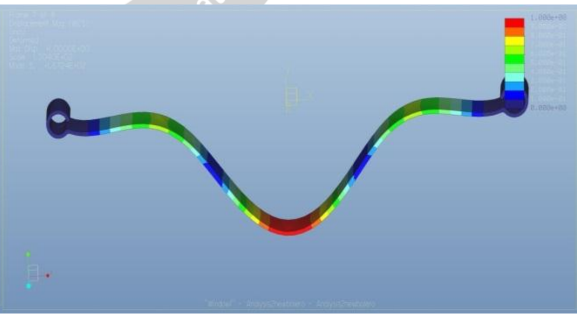


3-D Model of Berlin Eye Leaf Spring

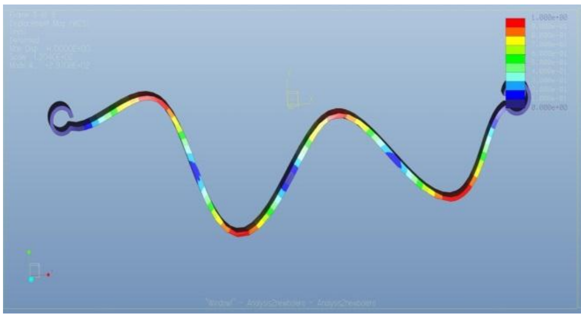


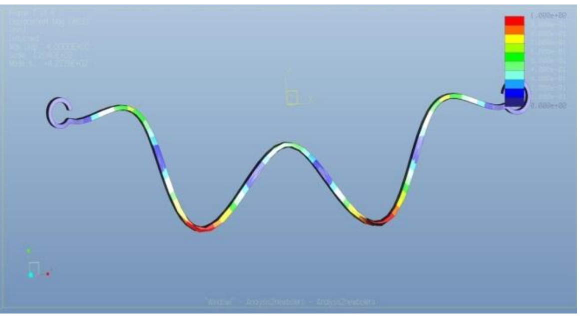
Mode Shape 1

Mode Shape 2



Mode Shape 3

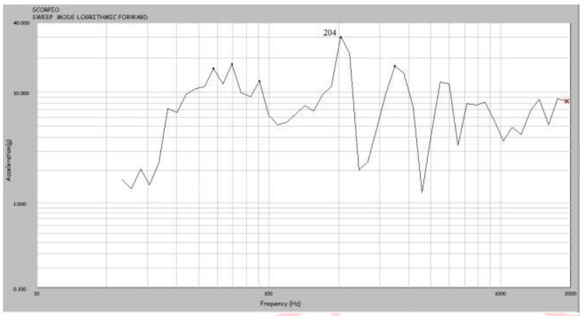




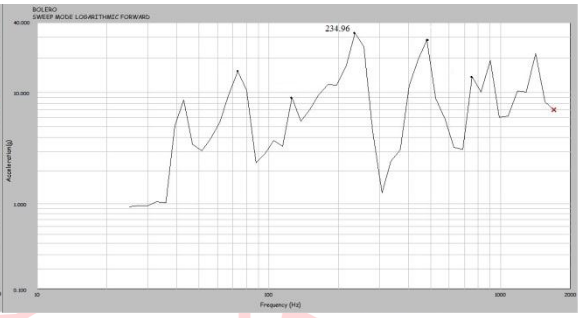
Mode Shape 4 Mode Shape 5

C. Experimental Setup

In an experimental setup to find out the vibration characteristic of leaf spring,we have employed vibration shaker along with the arrangement of spring clamping in metal fixture and vibration is given to spring by means of a metal rod used as a probe. Two different methods can be used to clamp the leaf spring to obtain the vibration characteristics at different end conditions. But we have applied the case for simply supported beam only for the study purpose. The leaf spring is kept exactly above the center of shaker, so that the probe can pass through center hole of leaf spring in order to transmit the vibration from shaker to leaf spring. Two accelerometers are attached one on the leaf spring and another one on vibratory part of the exciter to sense the vibration.The force excitation is given in the range of 10 Hz to 2 KHz, which can be provided either with control panel or the computer, since both are connected via interfacing wire. The excitation is generated by vibration controller and amplified byexciter. Three types of test can be performed on shaker i.e. random test, sine test and shock test. We are using sine test on springs, a sweep sine test generates only one frequency at a time and sweeps this frequency through a present frequency range and amplitude. The plot of displacement v/s frequency and acceleration v/s frequency is obtained from sine software as prescribed by the user manual.



Sweep Mode Logarithmic Forward of Upturned Eye Leaf Spring



Sweep Mode Logarithmic Forward of Berlin Eye Leaf Spring

IV. RESULTS AND DISCUSSION

As leaf spring is a Continuous System hence Euler- Bernoulli Beam theory was applied for the theoretical calculation of Mode shape and natural frequencies of both mono leaf springs. Leaf Spring was analyzed using Simply Supported Condition in Analytical, F.E.A and Experimental setup. As leaf spring is a continuous system hence it can have infinite mode shapes, but we have considered only first 5 mode shapes.

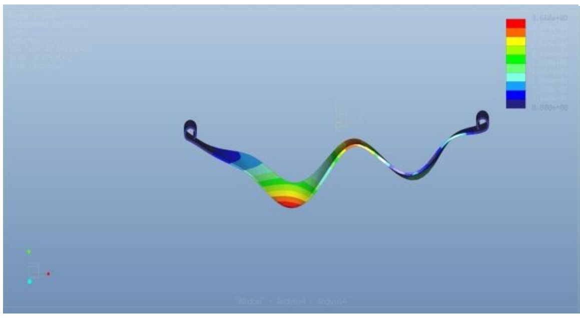
In table 2 results obtained from Shaker Testing are based on resonance frequency. Due to contact type accelerometer sensor only the bending modes were identified. Hence only the Modal Parameter within the excited frequency range of 10 Hz to 2000 Hz was captured. Results obtained from all the three approaches are tabulated in the following table

Table 2. Results

Mode Shape	Leaf Spring with Upturned Eye			Leaf Spring with Berlin Eye		
	Analytical	F.E.A	Experimental	Analytical	F.E.A	Experimental
1	12.3897	48	57.84	14.4534	39	42.87
2	49.5588	65	69.3	57.8135	89	73.33
3	111.5573	123	91.16	130.0806	167	125.32
4	198.2352	217	204	213.2544	297	234.96
5	446.0293	317	349.99	361.3350	427	480.32

Thus from Table 2 it is clear that the result obtained from all the three approaches are more or less within the good agreement with each other. This shows us that the measured values and calculated values are nearly the same.

On combining the first five mode shape and plotting the result in F.E.A in Fig no 1for Upturned Eye and Fig no 2 for Berlin Eye it shows us that the largest deflection is seen in 4th Mode Shape. It means that the frequency in 4th mode shape is the resonant frequency.



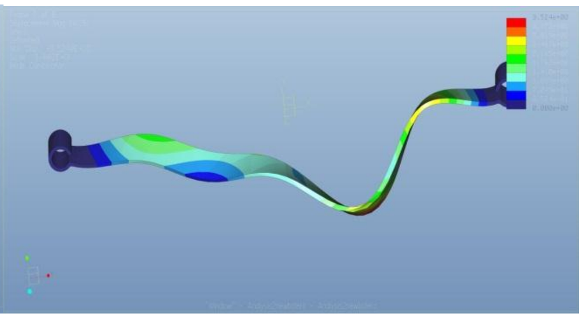
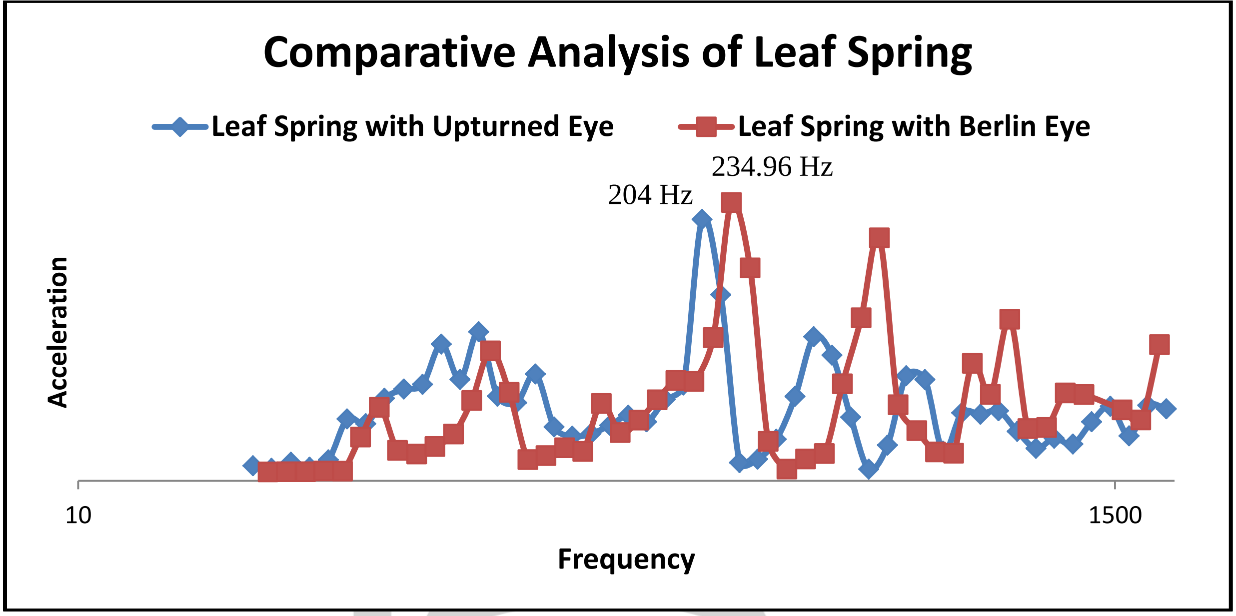


Fig No 1, Upturned Eye Fig No 2, Berlin Eye

On plotting the graph of Frequency V/s Acceleration from the data available after Shaker Test it can be seen that the highest peak of frequency in Leaf Spring with upturned eye is at 204 Hz and for leaf spring with berlin eye is at 234.96 Hz. This Frequency corresponds to the frequency of 4th mode shape in Analytical and F.E.A. results. Hence it can be seen that in Leaf Spring with Upturned Eye resonant occurs at 204 Hz and in Leaf Spring with Berlin Eye resonant occurs at 234.96 Hz.



From the comparative analysis it is clear that resonant frequency of Leaf Spring with Berlin Eye is 1.1518 times more as compared to resonant frequency of Leaf Spring with upturned eye. It is clear that to provide ride comfort to passenger, leaf spring has to be designed in such a way that its natural frequency is maintained to avoid resonant condition. Therefore leaf spring must be designed in such a way to have natural frequency higher than the frequency generated due to irregularities from the road surface to avoid resonance. From the data above it is be observed that Leaf Spring with berlin eye will provide better ride comfort.

V. ACKNOWLEDGMENT

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