

Buckling Analysis of Cantilever Twisted FGM Plate for Buckling Load and Non-Buckling Dimensional Buckling Load

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ABSTRACT - The demand and application of composites are increasing nowadays. Composite materials in the form of plate or plate-like structures are widely used in wind turbine blades and ship building due to its high specific strength and stiffness. For high thermal applications, Functionally Graded Materials (FGM) are used in preference to laminated composites because of its good performance in the thermal field. The pre-twisted cantilever plates have major use in turbine blades, fan blades, compressor blades, chopper blades, marine propellers and chiefly in gas turbines. These structures are often subjected to thermal environments, and hence FGMs are a good alternative to metal plates. The present work deals with the study of buckling analysis of cantilever twisted functionally graded material plates. The analysis is done by using ANSYS, and the results are validated using ABAQUS. A SHELL-281 element having six degrees of freedom per node is employed in ANSYS. The functionally graded material plate with a uniform variation of the material property through the thickness is estimated as a laminated section containing number of layers, and each layer is taken as isotropic. The power law is used to determine material properties in each layer. From convergence studies, ten by ten mesh and twelve number of layers are found to give good accuracy. Buckling behavior of cantilever twisted FGM plate for the various parameters like twist angle, side to thickness ratio, aspect ratio and gradient index are studied.

Key words: Buckling, Cantilever, Ansys, FGM, Property

I. INTRODUCTION

A composite material is a structural material made from two or more constituent materials with significantly distinct physical or chemical properties, which when fused produce a material with characteristics unlike that of the individual components. The main advantage of a composite material is that they are light as well as strong. Functionally Graded Materials (FGM) are a set of composites that exhibit a uniform change of material properties from one face to another and hence eliminate the stress concentration, normally encountered in laminated composites. The characteristics of these FGM's are the ability to yield a new composite material with uniform composition variation from thermal resistant ceramics to fracture resistant metals. The FGM concept originated in the year 1984 in Japan during a space research program. This program envisaged the manufacture of a temperature resistant material to resist a temperature of 2000 Kelvin and a temperature gradient of 1000 Kelvin having a thickness below 10mm. The structural component of an FGM can be characterized by the material constituents. It shows the rate of change of material properties. The gradient index governs the

chemical configuration, geometric configuration and physical state of FGM. Primarily FGM involves two material mixtures in which material configuration changes from one surface to another. Variation of porosity from one face to another face also yields functionally graded material. A steady rise in porosity builds impact resistance, thermal resistance, and low density. These FGM's have significant applications in civil and mechanical structures including Thermal structures like Rocket heat shield, heat exchanger tubes, wear resistance linings, thermos-elastic generators, diesel and turbine engines etc. The major applications of pre-twisted cantilever panels are in turbine blades, fan blades, compressor blades, chopper blades, marine propellers and chiefly in gas turbines. Nowadays, in research field the twisted plates have become key structural units. Because of the use of twisted panels in turbomachinery, aerospace and aeronautical industries, it is necessary to understand both vibration and buckling characteristics of the pre-twisted panels.

1.2 Importance of Present study

Composite materials in the form of plate or plate-like structures are widely used in wind turbine blades and a

certain type of ships, particularly naval ships. Functionally graded material plates are finding increasing application in many structures, especially where the temperature is high. The plates are also subjected to loads due to fluid or hydrodynamic loading. Thus, understanding and proper application of composite materials have helped to control the lifetime and stability of these constructions. Hence, the buckling analysis plays a crucial role in the design context. From the literature review, it shows that there is plenty of work done in the area of flat FGM plates. However, no work has been done on the buckling behavior of twisted FGM material panels and hence the present study. Reddy (2000) presented the study of FG plates using third-order shear deformation theory. The material distribution and modulus of elasticity along thickness were assumed to vary based on power-law distribution. The results showed the influence of volume fraction and modular ratio on deflections and transverse shear stresses. Javaheri and Eslami (2002) derived equilibrium equations and stability equations of rectangular FG plate using higher-order shear deformation theory (HSDT) subjected to thermal load. The derived equation was found to be identical to the stability and equilibrium equations of laminated composite plates. Buckling behaviour of FG plates with geometrical imperfections under in-plane compressive loading was studied by Shariat et al. (2005). The Classical Plate Theory was used for the derivation of equations of equilibrium, stability, and compatibility. From their study, it was concluded that the imperfect FG plate has greater buckling load than that of the perfect plate. As the imperfection increases, Yang the critical buckling load also increases which can be reduced by increasing power law index et al. (2006) presented the sensitivity of post-buckling behaviour of FG material plates to initial geometric imperfections such as local type, global type, and sine type imperfections in general modes. The formulations used were based on Reddy's Higher order Shear Deformation Theory and von Karman type geometric non-linearity. The results showed that the post-buckling strength was comparatively insensitive to sine mode and global imperfections. However, it was highly sensitive to local imperfections that were situated at the center of the plate. They also concluded that the post-buckling curves were lowered by an increase in the side to thickness ratio, gradient index and aspect ratio. They observed that these curves were less sensitive to imperfection sensitivity of the post-buckling reaction of the plate. Shariat and Eslami (2007) used third-order shear deformation theory for the analysis of buckling of thick rectangular FG plates under various mechanical and thermal loads. The mechanical loadings were uniaxial compression, biaxial compression and biaxial compression with tension. The thermal loads were a uniform rise in temperature and non-linear rise in temperature. It was concluded that for the thick plates, the critical buckling load was over- predicted by the classical plate theory and in order to have precise buckling load values it was

recommended that the third-order shear deformation theory was necessary. Prakash et al. (2008) presented post-buckling behavior of FGM skew plates based on shear deformable finite element approach under thermal loads. The temperature field was assumed to vary along the thickness direction only and to be constant over the plate surface. The thermal load carrying capacity increased with increasing skew angle. Mahadavian (2009) 7 considered simply supported rectangular plates under non-uniform compression loads for the analysis of buckling of FG plate and derived equations of equilibrium and stability for the same and also achieved results for FG sample.

II. CHARACTERISTICS OF TWISTED PLATE

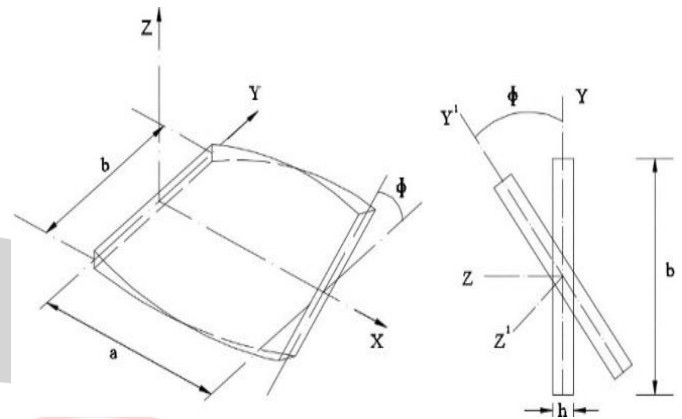


Figure 1: Laminated twisted plate

The Figure illustrates a twisted FGM plate. Here

Φ = Twist angle

a and b = length and width of the plate respectively.

h = Thickness of the plate.

Governing Differential Equations

Consider an element of Pretwisted panel with radius of curvatures R_x in x-direction and R_y in y-direction shown in figure 3.2. the internal forces acting on elements are membrane forces (N_x, N_y and N_{xy}), shearing forces (Q_x and Q_y) and the moment resultants (M_x, M_y and M_{xy}).

$$\begin{aligned}
 \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2} \left(\frac{1}{R_x} - \frac{1}{R_y} \right) \frac{\partial M_{xy}}{\partial y} + \frac{Q_x}{R_x} + \frac{Q_y}{R_{xy}} &= P_1 \frac{\partial^2 u}{\partial t^2} + P_2 \frac{\partial^2 \theta_x}{\partial t^2} \\
 \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} - \frac{1}{2} \left(\frac{1}{R_y} - \frac{1}{R_x} \right) \frac{\partial M_{xy}}{\partial x} + \frac{Q_y}{R_y} + \frac{Q_x}{R_{xy}} &= P_1 \frac{\partial^2 v}{\partial t^2} + P_2 \frac{\partial^2 \theta_y}{\partial t^2} \\
 \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} - 2 \frac{N_{xy}}{R_{xy}} + N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} &= P_1 \frac{\partial^2 w}{\partial t^2} \\
 \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x &= P_1 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2} \\
 \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y &= P_1 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2}
 \end{aligned} \tag{3.01}$$

Where,

N_x^0 - External loading in x-direction and N_y^0 - External loading in y-direction.

R_x - Radius of curvature in x-direction, R_y - Radius of curvature in y-direction and

R_{xy} - Radius of twist.

$$(P_1, P_2, P_3) = \sum_{k=1}^n \int_{z_{k-1}}^{z_k} (\rho)_k (1, z, z^2) dz \tag{3.02}$$

The linear constitutive relations are given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{11} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{3.03}$$

Where, $Q_{11} = \frac{E}{(1-\nu^2)}$ 3.04

$Q_{12} = \frac{\nu E}{(1-\nu^2)}$ 3.05

$Q_{44} = \frac{E}{2(1+\nu)} = Q_{55} = Q_{66}$ 3.06

For the FGM plates, the constitutive relations are expressed as:

$\{F\} = [D]\{\epsilon\}$ 3.07

Where, $[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{21} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & I_{16} & I_{26} & I_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{66} \end{bmatrix}, \{F\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix}, \{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ K_x \\ K_y \\ K_{xy} \\ \phi_x \\ \phi_y \end{Bmatrix}$

Coefficients of stiffness are expressed as:

$$(A_{i,j}, B_{i,j}, D_{i,j}) = \sum_{k=1}^n [Q_{ij}]_k (1, z, z^2) dz \quad \text{For } (i, j=1, 2, 6) \quad 3.08$$

$$S_{ij} = k \sum_{k=1}^n \int_{z_{k-1}}^{z_k} [Q_{ij}]_k dz$$

The forces and moment resultants can be obtained by integrating stresses over thickness.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_x z \\ \sigma_y z \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz \quad 3.09$$

Where,

σ_x - Normal stress in x-direction, σ_y - normal stress in y-direction.

τ_{xy} , τ_{yz} and τ_{xz} are shear stresses in xy, yz and xz planes respectively.

3.3 Strain Displacement Relations

The total strain is considered in two parts namely linear strain and non-linear strain. The element stiffness matrix is derived using linear strain part and the geometric stiffness part is derived by using nonlinear strain part. The total strain is expressed as

$$\begin{aligned} \epsilon_{y1} &= \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y \\ \gamma_{xy1} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \\ \gamma_{yz1} &= \frac{\partial w}{\partial y} + \theta_y - \frac{v}{R_y} - \frac{u}{R_{xy}} \\ \gamma_{xz1} &= \frac{\partial w}{\partial x} + \theta_x - \frac{u}{R_x} - \frac{v}{R_{xy}} \end{aligned} \quad 3.10$$

Bending components are given by,

$$\begin{aligned} k_x &= \frac{\partial \theta_x}{\partial x} & k_y &= \frac{\partial \theta_y}{\partial y} \\ k_{xy} &= \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2} \left(\frac{1}{R_y} - \frac{1}{R_x} \right) \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \end{aligned} \quad 3.11$$

Finite element formulations

For complex boundary and geometrical conditions where analytical approach is not so easily feasible, the finite element approach will be opted. Here, in this work the plate is assumed to be a layered panel having number of layers, in which each layer is assumed as homogenous and isotropic. The first-order shear deformation theory is used for the present formulation to analyze the FG material twisted panel. An isoparametric quadratic shell element with eight nodes at its mid-surface shown in Figure 3.3 is considered for the analysis. In this shell element u , v , w , θ_x and θ_y are the five degrees of freedom each node. The Jacobian matrix J is used to transform the isoparametric element from the natural coordinate to the Cartesian

coordinate system

Derivation of element matrices

The linear strains expressed in terms of displacements is given by,

$$\{\varepsilon\} = [B]\{d_e\} \quad 3.18$$

Here, $\{d_e\} = \{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \dots, u_8, v_8, w_8, \theta_{x8}, \theta_{y8}\}$ 3.19

$$[B] = [[B_1], [B_2], \dots, [B_8]] \quad 3.20$$

$$[B_i] = \begin{bmatrix} N_{i,x} & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,y} & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,y} & N_{i,x} & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix} \quad 3.21$$

In natural coordinate system, element matrices are derived as:

Element plane elastic stiffness matrix

$$[k_p] = \int_{-1}^1 \int_{-1}^1 [B_p]^T [D_p] [B_p] |J| d\xi d\eta \quad 3.22$$

Element elastic stiffness matrix

$$[k_e] = \int_{-1}^1 \int_{-1}^1 [B]^T [D] [B] |J| d\xi d\eta \quad 3.23$$

Geometric stiffness matrix

The nonlinear strains with curvature component are used to derive the element geometric stiffness matrix for the twisted plate by employing the technique described by Cook, Malkus and Plesha [3]. Due to applied edge loading, the geometric stiffness matrix depends on in-plane stress distribution in the element. Finite element method is employed in carrying out plane stress analysis to determine the stresses.

The strain energy is given by,

$$U_2 = \int_v [\sigma^0]^T \{\varepsilon_{nl}\} dv \quad 3.25$$

The non-linear strain components are given by,

$$\varepsilon_{xnl} = \frac{1}{2} \left(\frac{\partial u}{\partial x} \right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial x} \right)^2 - \frac{1}{2} \left(\frac{\partial w}{\partial x} - \frac{u}{R_x} \right)^2 + \frac{1}{2} z^2 \left[\left(\frac{\partial \theta_x}{\partial x} \right)^2 + \left(\frac{\partial \theta_y}{\partial x} \right)^2 \right] \quad 3.26$$

$$\varepsilon_{ynl} = \frac{1}{2} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{1}{2} \left(\frac{\partial v}{\partial y} \right)^2 - \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{v}{R_y} \right)^2 + \frac{1}{2} z^2 \left[\left(\frac{\partial \theta_x}{\partial y} \right)^2 + \left(\frac{\partial \theta_y}{\partial y} \right)^2 \right]$$

$$\gamma_{xnl} = \frac{\partial u}{\partial x} \left(\frac{\partial u}{\partial y} \right) + \frac{\partial v}{\partial x} \left(\frac{\partial v}{\partial y} \right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x} \right) \left(\frac{\partial w}{\partial y} - \frac{v}{R_y} \right) + z^2 \left[\left(\frac{\partial \theta_x}{\partial x} \right) \left(\frac{\partial \theta_x}{\partial y} \right) + \left(\frac{\partial \theta_y}{\partial x} \right) \left(\frac{\partial \theta_y}{\partial y} \right) \right]$$

$$U_2 = \int_A \frac{h}{2} \left[\sigma_x^0 \left\{ \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x} \right)^2 \right\} + \sigma_y^0 \left\{ \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} - \frac{v}{R_y} \right)^2 \right\} + 2\tau_{xy}^0 \left\{ \left(\frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \right) + \left(\frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x} \right) \left(\frac{\partial w}{\partial y} - \frac{v}{R_y} \right) \right\} \right] dx dy + \int_A \frac{h^3}{24} \left[\sigma_x^0 \left\{ \left(\frac{\partial \theta_x}{\partial x} \right)^2 + \left(\frac{\partial \theta_y}{\partial x} \right)^2 \right\} + \sigma_y^0 \left\{ \left(\frac{\partial \theta_x}{\partial y} \right)^2 + \left(\frac{\partial \theta_y}{\partial y} \right)^2 \right\} + 2\tau_{xy}^0 \left\{ \left(\frac{\partial \theta_x}{\partial x} \frac{\partial \theta_x}{\partial y} \right) + \left(\frac{\partial \theta_x}{\partial x} \frac{\partial \theta_y}{\partial y} \right) \right\} \right] dx dy \tag{3.27}$$

This can also be written as

$$U_2 = \frac{1}{2} \int_V [f]^T [S] [f] dV \tag{3.28}$$

Where

$$\{f\} = \left[\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \left(\frac{\partial w}{\partial x} - \frac{u}{R_x} \right), \left(\frac{\partial w}{\partial y} - \frac{v}{R_y} \right), \frac{\partial \theta_x}{\partial x}, \frac{\partial \theta_x}{\partial y}, \frac{\partial \theta_y}{\partial x}, \frac{\partial \theta_y}{\partial y} \right]^T \tag{3.29}$$

And, $[S] = \begin{bmatrix} [S] & 0 & 0 & 0 & 0 \\ 0 & [S] & 0 & 0 & 0 \\ 0 & 0 & [S] & 0 & 0 \\ 0 & 0 & 0 & [S] & 0 \\ 0 & 0 & 0 & 0 & [S] \end{bmatrix}$ 3.30

Where, $[S] = \begin{bmatrix} \sigma_x^0 & \tau_{xy}^0 \\ \tau_{xy}^0 & \sigma_y^0 \end{bmatrix} = \frac{1}{h} \begin{bmatrix} N_{xx}^0 & N_{xy}^0 \\ N_{xy}^0 & N_{yy}^0 \end{bmatrix}$ 3.31

The strain energy becomes

$$U_2 = \frac{1}{2} \{q\}^T [G]^T [S] [G] \{q\} dV = \frac{1}{2} \{q_e\}^T [K_g]_e \{q_e\} \tag{3.32}$$

Where the element geometric stiffness matrix is expressed as:

$$[K_g]_e = \int_{-1}^1 \int_{-1}^1 [G]^T [S] [G] |J| d\xi d\eta \tag{3.33}$$

$$[G] = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ N_{i,y} & 0 & 0 & 0 & 0 \\ 0 & N_{i,x} & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ 0 & 0 & N_{i,x} & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & N_{i,y} & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & 0 & N_{i,y} \end{bmatrix} \tag{3.34}$$

III. METHODOLOGY

This work involves creating a finite element model of a functionally graded twisted plate subjected to in-plane uniform compressive load. The initial step is to build a model of a functionally graded plate using ANSYS. First a flat FGM plate will be modelled, and buckling behavior will be analyzed, and the results are compared with previous studies. Then a twisted functionally graded plate will be modelled and analyzed for its characteristics subjected to in-plane loads. Results will be analyzed and validated with the calculations using ABAQUS.

The three steps involved in modelling and analysis of plates are:

- I. Pre- Processor
- II. Solution
- III. General Postprocessor

Material modelling

FGMs consist of a mixture of metal and ceramic by gradually varying the volume fraction of the constituent materials. A simple rule of mixture based on power-law is

assumed to obtain the effective mechanical properties of FGM plate.

In which h is the thickness of the plate. n is the gradient index that is always positive, and z is the distance from the centre of layer under consideration to the centre of plate in which $-(h/2) \leq z \leq$

$+(h/2)$. Change of V_f over plate thickness is shown in Figure 2

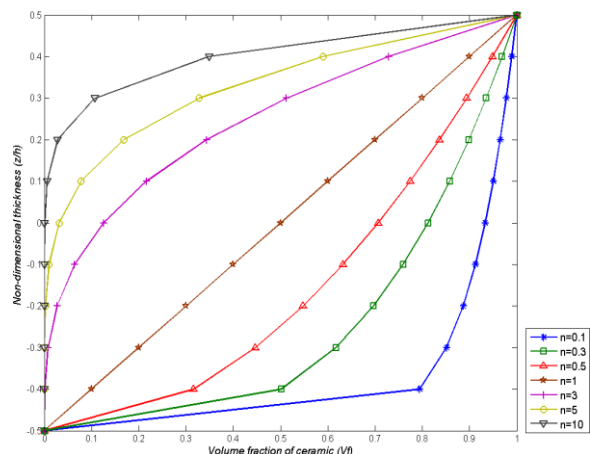


Figure 2 Change of Volume fraction (Vf) over plate thickness

Since the material constituents of the FG material varies over the thickness, the numerical model is made into divisions consisting of a number of layers as shown in Figure 3.5. Each layer is assumed to be isotropic.

IV. CONCLUSIONS

The present work enables to arrive at the following important conclusions:

- ❖ With the increase in angle of twist, the non-dimensional buckling load decreases.
- ❖ As the aspect ratio (a/b) increases, the non-dimensional buckling load of a twisted FGM plate decreases largely. This is because when the aspect ratio increases, the length of the plate in the direction of the in-plane compression load also increases resulting in the decreased stiffness. Hence, the amount of critical buckling load required to cause critical buckling decreases.
- ❖ The non-dimensional buckling load also increases with increase in the side to thickness ratio (b/d). Because, as the side to thickness ratio of the twisted plate increases, the stiffness of the plate decreases as well, and thus it increases critical buckling load.
- ❖ The non-dimensional buckling load decreases with increase in the material index of a pre-twisted functionally graded material plate. This is because, as the material gradient goes on increasing, the metal content in the plate also increases but the ceramic content decreases resulting in reduced stiffness and, therefore, the critical buckling load goes on decreasing with increase in gradient index.

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