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Analytical solution for free vibration analysis of Laminated Composite plate

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ABSTRACT-In this paper a variationally consistent polynomial shear deformation theory is presented for the free vibration of thick isotropic square and rectangular plate. In this displacement based theory, the in-plane displacement field use parabolic function in terms of thickness coordinate to include the shear deformation effect. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. Results of frequency are obtained from free vibration of simply supported isotropic square and rectangular plates and compared with those of other refined theories and frequencies from exact theory.

Keywords: Shear deformation; Laminated plate; Shear correction factor; Free vibration.

I. INTRODUCTION

Plates are the basic structural components that are widely used in various engineering disciplines such as aerospace, civil, marine and mechanical engineering. The transverse shear and transverse normal deformation effects are more pronounced in shear flexible plates which may be made up of isotropic, orthotropic, anisotropic or laminated composite materials. In order to address the correct structural behavior of structural elements made up of these materials; development of refined theories, which take into account refined effects in static and dynamic analysis of structural elements, becomes necessary. The study of plate vibration dates back to the early eighteen century, with the German physicist, who observed the nodal patterns for a flat square plate. Since then there has been a tremendous research interest in the subject of plate vibrations. Several thin plate vibration solutions based on Kirchhoff's plate theory are available in the literature. The classical plate theory based on Kirchhoff's hypothesis [1] is not adequate for the analysis of shear flexible plates due to the neglect of transverse shear deformation and the rotary inertia in the theory; as a consequence, it under predicts deflections and over predicts all the vibration frequencies for thick plates, and the higher frequencies for the thin plates. The most suitable starting point for the analysis of both thin and thick plates seems to be a theory in which the classical hypothesis of zero transverse shear strains is relaxed.

At first, Reissner proposed that the rotations of the normal to the plate mid-surface in the transverse plane could be introduced as independent variables in the plate theory. Reissner has developed a stress based theory which incorporates the effect of shear. Mindlin [2] simplified Reissner's assumption that normal to the plate mid-surface before deformation remains straight but not necessarily normal to the plate mid-surface after deformation and the stress normal to the mid-surface is disregarded as in the case of classical plate theory of Kirchhoff. Mindlin employed displacement based approach. In Mindlin's theory, transverse shear stress is assumed to be constant through the thickness of the plate, but this assumption violates the shear stress free surface conditions. The theory includes both the shear deformation and rotary inertia effects. Both effects decrease the frequencies. There are still other effects not accounted for by the Mindlin are stretching in the thickness direction and the warping of the normal to the mid-plane, which are more important in case of thick plates.

Mindlin's theory satisfies constitutive relations for transverse shear stresses and shear strains by using shear correction factor. The value of this factor is not unique but depends on the material, geometry, loading and boundary condition parameters. Wang discussed these theories in detail and developed the relationships between bending solutions of Reissner and Mindlin plate theory. Usually, in two dimensional plate theories, displacement components are considered power series expressions in thickness coordinate (z). Depending on the number of terms retained in the power series expressions, various higher order theories for homogeneous and laminated plates can be developed Reddy [3,4] utilize some simplification of the generalized displacement function.The simplified higher order theories, generally third order shear deformation theories give parabolic variation of transverse shear stress through the thickness of the plate satisfying the shear stress free boundary conditions on the top and bottom surfaces of

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the plate. Thus, these theories do not require shear correction factors. Levinson formulated a theory based on displacement approach which does not require shear correction factor. However, Levinson's theory is variationally inconsistent since the field equations and boundary conditions are not derived using principle of virtual work. Srinivas et al. [5] developed exact elasticity solutions for the flexure and free vibration of simply supported homogeneous, isotropic, thick rectangular plates. The exact elasticity solutions play important role in validation of results of two dimensional thick plate theories. surveyed plate theories particularly applied to thick plate vibration problems. In the development of such theories use of polynomials, trigonometric functions, hyperbolic functions and exponential functions in terms of thickness coordinate is widely and wisely made by

Ghugal and Sayyad [6,7] have used trigonometric shear deformation theory for the free vibration analysis of orthotropic plates and a variationally consistent trigonometric shear deformation theory for free vibration of homogenous, isotropic plate is developed. It has four variables and includes effects of transverse shear and transverse normal strain. The theory satisfies the tangential traction free boundary conditions (zero shear stress conditions) on the top and bottom surfaces of the plate. The primary objective of this investigation is to present the frequencies of flexural mode, thickness shear and thickness stretch modes of free vibration of thick plates.

II. THEORETICAL FORMULATION

2.1 Laminated plate under consideration

Consider a rectangular laminated plate composed of orthotropic layers as shown in figure 1.The plate is assumed in Cartesian coordinate (x,y,z) system with origin o .it is convenient to take the y-plane of the coordinate system to the undeformed middle taken to be positive in a downword direction from the middle plane.

2.2 Displacement field.

For the bending analysis, the displacement field at a point in the laminated plate is expressed as:-

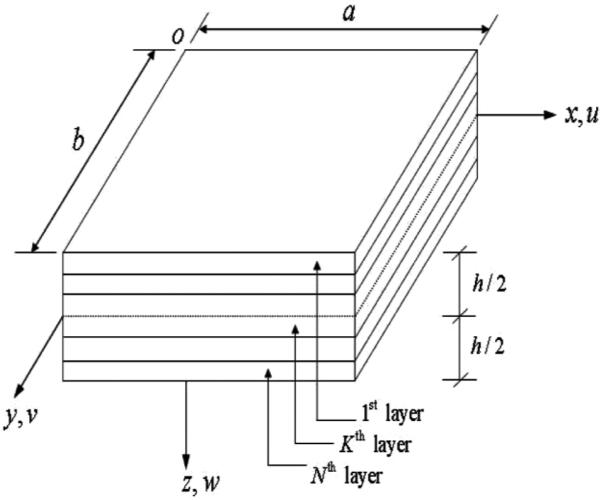

Figure1: plate geometry and coordinate System

$$u = u_0 - z\frac{\partial w_0}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_1}{\partial x} - \frac{16z^5}{5h^2}\frac{\partial w_2}{\partial x}$$

$$v = v_0 - z\frac{\partial w_0}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_1}{\partial x} - \frac{16z^5}{5h^2}\frac{\partial w_2}{\partial x} \qquad (1)$$

$$w = w_0 + w_1 + w_0$$

Where,

$u_0(x,y), v_0(x,y), w_0(x,y), w_1(x,y), w_2(x,y)$

Where u, v, w are the in-plane displacement of the mid-plane in x,y and z direction respectively w_0, w_1, w_2 are the shear rotations

2.3 Strain-Displacement Relationship

For the small plate deformation the six strain component are plane of the laminate.The z-axis is

$(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz})$ and three displacement component (u, v, w) are related according to the well-known liner kinematic relation.

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial w_0}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_1}{\partial x} - \frac{16z^5}{5h^2}\frac{\partial w_2}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial x} = \frac{\partial v_0}{\partial y} - z\frac{\partial w_0}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_1}{\partial x} - \frac{16z^5}{5h^2}\frac{\partial w_2}{\partial x}$$

$$\varepsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left\{\left[\frac{\partial u_0}{\partial y} - z\frac{\partial w_0}{\partial x \partial y} - \frac{4z^3}{3h^2}\frac{\partial w_1}{\partial x \partial y} - \frac{16z^5}{5h^2}\frac{\partial w_2}{\partial x \partial y}\right] + \left[\frac{\partial v_0}{\partial x} - z\frac{\partial w_0}{\partial x \partial y} - \frac{4z^3}{3h^2}\frac{\partial w_1}{\partial x \partial y} - \frac{16z^5}{5h^2}\frac{\partial w_2}{\partial x \partial y}\right]\right\} \qquad (2)$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = (1 - \frac{4z^2}{h^2})\frac{\partial w_1}{\partial x} + (1 - \frac{16z^4}{h^4})\frac{\partial w_2}{\partial x}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = (1 - \frac{4z^2}{h^2})\frac{\partial w_1}{\partial x} + (1 - \frac{16z^4}{h^4})\frac{\partial w_2}{\partial x}$$

2.4 Stress – Strain Relationship

The stress component associated with strain is given component by eq. (3) considering transverse shear deformation in the plate coordinate can be expressed as follows:-

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \qquad (3)$$

$$\delta u_0 = -\frac{\partial Nx}{\partial x} - \frac{\partial Nxy}{\partial y}$$

$$\delta v_0 = -\frac{\partial Ny}{\partial y} - \frac{\partial Nxy}{\partial x}$$

$$\delta w_0 = -\frac{\partial^2 M^{b}{}_x}{\partial x^2} - \frac{\partial^2 M^{b}{}_y}{\partial y^2} - 2\frac{\partial^2 M^{b}{}_y}{\partial x \partial y}$$

$$\delta w_1 = -\frac{\partial^2 M^{S_1}{}_x}{\partial x^2} - \frac{\partial^2 M^{S_1}{}_y}{\partial y^2} - 2\frac{\partial^2 M^{S_1}{}_{xy}}{\partial x \partial y} - \frac{\partial Q^{S_1}{}_x}{\partial x} - \frac{\partial Q^{S_1}{}_y}{\partial y}$$

$$\delta w_2 = -\frac{\partial^2 M^{S_2}{}_x}{\partial x^2} - \frac{\partial^2 M^{S_2}{}_y}{\partial y^2} - 2\frac{\partial^2 M^{S_2}{}_{xy}}{\partial x \partial y} - \frac{\partial Q^{S_2}{}_x}{\partial x} - \frac{\partial Q^{S_2}{}_y}{\partial y}$$

Where Qij are the transformed elastic coefficient,

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}, \quad Q_{12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}},$$

$$Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}, \quad Q_{66} = G_{12}, \qquad (4)$$

$$Q_{55} = G_{23}, \quad Q_{44} = G_{13}$$

Where E_1, E_2 are the elastic moduli, μ_{12} and μ_{21} are Poisson's ratios and G_{12}, G_{23}, G_{13} are the shear moduli of the material.

2.5 Governing equation and boundary conditions.

Governing equation and boundary conditions are obtained using principal of virtual work.

$$\begin{aligned}&\int_0^a \int_0^b \int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} \\ &+ \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz})\,dxdydz \\ &+ \rho \int_0^a \int_0^b \int_{-h/2}^{h/2} \left(\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v \right. \\ &\left. + \frac{\partial^2 w}{\partial t^2}\delta w\right) dxdydz = 0\end{aligned} \qquad (5)$$

inserting strains from Eq.(2) and stress from Eq.(3) into Eq.(5). Integrating by parts and collecting coefficient of $\partial u_0, \partial v_0, \partial w_0, \partial w_1, \partial w_2$ the following governing equation are obtain inserting stress resultant in terms of unknown variables

$$\begin{aligned}\delta u_0 = &-A_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{11}\frac{\partial^3 w_0}{\partial x^3} + C_{11}\frac{\partial^3 w_1}{\partial x^3} + D_{11}\frac{\partial^3 w_2}{\partial x^3} \\ &-A_{12}\frac{\partial^2 v_0}{\partial x \partial y} + B_{12}\frac{\partial^3 w_0}{\partial x \partial y^2} + C_{12}\frac{\partial^3 w_1}{\partial x \partial y^2} + D_{12}\frac{\partial^3 w_2}{\partial x \partial y^2} \\ &-A_{66}\frac{\partial^2 u_0}{\partial x \partial y} + B_{66}\frac{\partial^3 w_0}{\partial x \partial y^2} + C_{66}\frac{\partial^3 w_1}{\partial x \partial y^2} + D_{66}\frac{\partial^3 w_2}{\partial x \partial y^2} \\ &-A_{66}\frac{\partial^2 v_0}{\partial x \partial y} + B_{66}\frac{\partial^3 w_0}{\partial x \partial y^2} + C_{66}\frac{\partial^3 w_1}{\partial x \partial y^2} + D_{66}\frac{\partial^3 w_2}{\partial x \partial y^2}\end{aligned}$$

$$\begin{aligned}\delta v_0 = &-A_{12}\frac{\partial^2 u_0}{\partial x \partial y} + B_{12}\frac{\partial^3 w_0}{\partial x \partial y^2} + C_{12}\frac{\partial^3 w_1}{\partial x \partial y^2} + D_{12}\frac{\partial^3 w_2}{\partial x \partial y^2} \\ &-A_{22}\frac{\partial^2 v_0}{\partial y^2} + B_{22}\frac{\partial^3 w_0}{\partial y^3} + C_{22}\frac{\partial^3 w_1}{\partial y^3} + D_{22}\frac{\partial^3 w_2}{\partial y^3} \\ &-A_{66}\frac{\partial^2 u_0}{\partial x \partial y} + B_{66}\frac{\partial^3 w_0}{\partial x^2 \partial y} + C_{66}\frac{\partial^3 w_1}{\partial x^2 \partial y} + D_{66}\frac{\partial^3 w_2}{\partial x^2 \partial y} \\ &-A_{66}\frac{\partial^2 v_0}{\partial x^2} + B_{66}\frac{\partial^3 w_0}{\partial x^2 \partial y} + C_{66}\frac{\partial^3 w_1}{\partial x^2 \partial y} + D_{66}\frac{\partial^3 w_2}{\partial x^2 \partial y}\end{aligned}$$

$$\begin{aligned}\delta w_0 = &-B_{11}\frac{\partial^3 u_0}{\partial x^3} + E_{11}\frac{\partial^4 w_0}{\partial x^4} + F_{11}\frac{\partial^4 w_1}{\partial x^4} + H_{11}\frac{\partial^4 w_2}{\partial x^4} \\ &-B_{12}\frac{\partial^3 v_0}{\partial x^2 \partial y} + E_{12}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + F_{12}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + H_{12}\frac{\partial^4 w_2}{\partial x^2 \partial y^2} \\ &-B_{12}\frac{\partial^3 u_0}{\partial x \partial y^2} + E_{12}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + F_{12}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + H_{12}\frac{\partial^4 w_2}{\partial x^2 \partial y^2} \\ &-B_{22}\frac{\partial^3 v_0}{\partial y^3} + E_{22}\frac{\partial^4 w_0}{\partial y^4} + F_{22}\frac{\partial^4 w_1}{\partial y^4} + H_{22}\frac{\partial^4 w_2}{\partial y^4} \\ &-2B_{66}\frac{\partial^3 u_0}{\partial x \partial y^2} + 2E_{66}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + 2F_{66}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + 2H_{66}\frac{\partial^4 w_2}{\partial x^2 \partial y^2} \\ &-2B_{66}\frac{\partial^3 v_0}{\partial x^2 \partial y} + 2E_{66}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + 2F_{66}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + 2H_{66}\frac{\partial^4 w_2}{\partial x^2 \partial y^2}\end{aligned}$$

$$\begin{aligned}\delta w_1 = &-C_{11}\frac{\partial^3 u_0}{\partial x^3} + F_{11}\frac{\partial^4 w_0}{\partial x^4} + I_{11}\frac{\partial^4 w_1}{\partial x^4} + J_{11}\frac{\partial^4 w_2}{\partial x^4} \\ &-C_{12}\frac{\partial^3 v_0}{\partial x^2 \partial y} + F_{12}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + I_{12}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + J_{12}\frac{\partial^4 w_2}{\partial x^2 \partial y^2} \\ &-C_{12}\frac{\partial^3 u_0}{\partial x \partial y^2} + F_{12}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + I_{12}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + J_{12}\frac{\partial^4 w_2}{\partial x^2 \partial y^2} \\ &-C_{22}\frac{\partial^3 v_0}{\partial y^3} + F_{22}\frac{\partial^4 w_0}{\partial y^4} + I_{22}\frac{\partial^4 w_1}{\partial y^4} + J_{22}\frac{\partial^4 w_2}{\partial y^4} \\ &-2C_{66}\frac{\partial^3 u_0}{\partial x \partial y^2} + 2F_{66}\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + 2I_{66}\frac{\partial^4 w_1}{\partial x^2 \partial y^2} \\ &+2J_{66}\frac{\partial^4 w_2}{\partial x^2 \partial y^2}\end{aligned}$$

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$$-C_{22}\frac{\partial^3 v_0}{\partial y^3}+F_{22}\frac{\partial^4 w_0}{\partial y^4}+I_{22}\frac{\partial^4 w_1}{\partial y^4}+J_{22}\frac{\partial^4 w_2}{\partial y^4}$$
$$-2C_{66}\frac{\partial^3 u_0}{\partial x\partial y^2}+2F_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+2I_{66}\frac{\partial^4 w_1}{\partial x^2\partial y^2}$$
$$+2J_{66}\frac{\partial^4 w_2}{\partial x^2\partial y^2}$$
$$-2C_{66}\frac{\partial^3 v_0}{\partial x^2\partial y}+2F_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2}$$
$$+2I_{66}\frac{\partial^4 w_1}{\partial x^2\partial y^2}+2J_{66}\frac{\partial^4 w_2}{\partial x^2\partial y^2}$$
$$-M_{55}\frac{\partial^2 w_1}{\partial x^2}-N_{55}\frac{\partial^2 w_1}{\partial x^2}$$
$$-M_{44}\frac{\partial^2 w_1}{\partial y^2}-N_{44}\frac{\partial^2 w_1}{\partial y^2}$$
$$\delta w_2=-D_{11}\frac{\partial^2 u_0}{\partial x^3}+H_{11}\frac{\partial^4 w_0}{\partial x^4}+J_{11}\frac{\partial^4 w_1}{\partial x^4}+L_{11}\frac{\partial^4 w_2}{\partial x^4}$$
$$-D_{12}\frac{\partial^3 v_0}{\partial x^2\partial y}+H_{12}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+J_{12}\frac{\partial^4 w_1}{\partial x^2\partial y^2}+L_{12}\frac{\partial^4 w_2}{\partial x^2\partial y^2}$$
$$-D_{12}\frac{\partial^2 u_0}{\partial x\partial y^2}+H_{12}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+J_{12}\frac{\partial^4 w_1}{\partial x^2\partial y^2}+L_{12}\frac{\partial^4 w_2}{\partial x^2\partial y^2}$$
$$-D_{22}\frac{\partial^3 v_0}{\partial y^3}+H_{22}\frac{\partial^4 w_0}{\partial y^4}+J_{22}\frac{\partial^4 w_1}{\partial y^4}+L_{22}\frac{\partial^4 w_2}{\partial y^4}$$
$$-2D_{66}\frac{\partial^3 u_0}{\partial x\partial y^2}+2H_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+2J_{66}\frac{\partial^4 w_1}{\partial x^2\partial y^2}+2L_{66}\frac{\partial^4 w_2}{\partial x^2\partial y^2}$$
$$-2D_{66}\frac{\partial^3 v_0}{\partial x^2\partial y}+2H_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+2J_{66}\frac{\partial^4 w_1}{\partial x^2\partial y^2}+2L_{66}\frac{\partial^4 w_2}{\partial x^2\partial y^2} \quad (5)$$
$$-N_{55}\frac{\partial^2 w_1}{\partial x^2}-P_{55}\frac{\partial^2 w_1}{\partial x^2}-N_{44}\frac{\partial^2 w_1}{\partial y^2}-P_{44}\frac{\partial^2 w_1}{\partial y^2}$$

Similarly for density of mass component are as follows:-

$$\delta u_0=-IA\frac{\partial^2 u_0}{\partial t^2}+IB\frac{\partial^3 w_0}{\partial x\partial t^2}+IC\frac{\partial^3 w_1}{\partial x\partial t^2}+ID\frac{\partial^3 w_2}{\partial x\partial t^2}$$
$$\delta v_0=-IA\frac{\partial^2 v_0}{\partial t^2}+IB\frac{\partial^3 w_0}{\partial y\partial t^2}+IC\frac{\partial^3 w_1}{\partial y\partial t^2}+ID\frac{\partial^3 w_2}{\partial y\partial t^2}$$
$$\delta w_0=IB\frac{\partial^3 u_0}{\partial x\partial t^2}-IE\frac{\partial^4 w_0}{\partial x^2\partial t^2}-IF\frac{\partial^4 w_1}{\partial x^2\partial t^2}-IH\frac{\partial^4 w_2}{\partial x^2\partial t^2}$$
$$\delta w_1=IC\frac{\partial^3 u_0}{\partial y\partial t^2}-IF\frac{\partial^4 w_0}{\partial y^2\partial t^2}-II\frac{\partial^4 w_1}{\partial y^2\partial t^2}-IJ\frac{\partial^4 w_2}{\partial x^2\partial t^2}$$
$$+IC\frac{\partial^3 v_0}{\partial y\partial t^2}-IF\frac{\partial^4 w_0}{\partial y^2\partial t^2}-II\frac{\partial^4 w_1}{\partial y^2\partial t^2}-IJ\frac{\partial^4 w_2}{\partial x^2\partial t^2}+$$
$$IA\frac{\partial^2 w_0}{\partial t^2}+IA\frac{\partial^2 w_1}{\partial t^2}+IA\frac{\partial^2 w_2}{\partial t^2}$$
$$\delta w_2=ID\frac{\partial^3 u_0}{\partial x\partial t^2}-IH\frac{\partial^4 w_0}{\partial x^2\partial t^2}-IJ\frac{\partial^4 w_1}{\partial x^2\partial t^2}-IL\frac{\partial^4 w_2}{\partial x^2\partial t^2} \quad (6)$$
$$+ID\frac{\partial^3 v_0}{\partial y\partial t^2}-IH\frac{\partial^4 w_0}{\partial y^2\partial t^2}-IJ\frac{\partial^4 w_1}{\partial y^2\partial t^2}-IL\frac{\partial^4 w_2}{\partial x^2\partial t^2}+$$
$$IA\frac{\partial^2 w_0}{\partial t^2}+IA\frac{\partial^2 w_1}{\partial t^2}+IA\frac{\partial^2 w_2}{\partial t^2}$$

3. Analysis of Laminated Plates.

The following middle surface displacement function are assumed which satisfies the boundary condition and the governing equation of simply supported laminated composite plates;

$$\begin{aligned}
u_0(x,y)&=u_{mn}\cos\alpha x\sin\beta y,\\
v_0(x,y)&=v_{mn}\sin\alpha x\cos\beta y,\\
w_0(x,y)&=w_{mn}\sin\alpha x\sin\beta y,\\
w_1(x,y)&=w_{mn}\sin\alpha x\sin\beta y,\\
w_2(x,y)&=w_{mn}\sin\alpha x\sin\beta y,
\end{aligned} \quad (7)$$

Where $\alpha=\frac{m\Pi}{a}$ *and* $\beta=\frac{n\Pi}{b}$

Substitutions of solution from given by Eq. into governing equation (5)-(6) result into system of the algebraic equation which can be written into matrix form as follows:

$$\left\{\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15}\\ k_{12} & k_{22} & k_{23} & k_{24} & k_{25}\\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35}\\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45}\\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55}\end{bmatrix}-\omega^2\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15}\\ M_{12} & M_{22} & M_{23} & M_{24} & M_{25}\\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35}\\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45}\\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55}\end{bmatrix}\right\}\begin{Bmatrix} u_{mn}\\ v_{mn}\\ w_{mn}\\ w_{mn}\\ w_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\\0\end{Bmatrix}$$

In a compact equation can be written as follows $([k]-\omega^2[M])\{\Delta\}=\{0\}$ Where [k],[M], $\{\Delta\}$ and ω are the stiffness matrix, mass matrix, amplitude vector and natural frequencies, respectively. The element of stiffness matrix [k] are defined as follows;

$$k_{11}=A_{11}\alpha^2+A_{66}\beta^2$$
$$k_{12}=(A_{11}+A_{66})\alpha\beta$$
$$k_{13}=-B_{11}\alpha^3-(B_{12}+2B_{66})\alpha\beta^2$$
$$k_{14}=-C_{11}\alpha^3-(C_{12}+2C_{66})\alpha\beta^2$$
$$k_{15}=-D_{11}\alpha^3-(D_{12}+2D_{66})\alpha\beta^2$$
$$k_{21}=(A_{12}+A_{66})\alpha\beta$$
$$k_{22}=A_{22}\beta^2+A_{66}\alpha^2$$
$$k_{23}=-(B_{12}+2B_{66})\alpha^2\beta-B_{22}\beta^3$$
$$k_{24}=-(C_{12}+2C_{66})\alpha^2\beta-C_{22}\beta^3$$
$$k_{25}=-(D_{12}+2D_{66})\alpha^2\beta-D_{22}\beta^3$$

$$k_{31} = -B_{11}\alpha^3 - (B_{12} + 2B_{66})\alpha\beta^2$$
$$k_{32} = -(B_{12} + 2B_{66})\alpha^2\beta - B_{22}\beta^3$$
$$k_{33} = E_{11}\alpha^4 + (2E_{12} + 4E_{66})\alpha^2\beta^2 + E_{22}\beta^4$$
$$k_{34} = F_{11}\alpha^4 + (2F_{12} + 4F_{66})\alpha^2\beta^2 + F_{22}\beta^4$$
$$k_{35} = H_{11}\alpha^4 + (2H_{12} + 4H_{66})\alpha^2\beta^2 + H_{22}\beta^4$$
$$k_{41} = -C_{11}\alpha^3 - (C_{12} + 2C_{66})\alpha\beta^2$$
$$k_{42} = -(C_{12} + 2C_{66})\alpha^2\beta - C_{22}\beta^3$$
$$k_{43} = F_{11}\alpha^4 + (2F_{12} + 4F_{66})\alpha^2\beta^2 + F_{22}\beta^4$$
$$k_{44} = I_{11}\alpha^4 + (2I_{12} + 4I_{66})\alpha^2\beta^2 + I_{22}\beta^4 + M_{55}\alpha + M_{44}\beta$$
$$k_{45} = J_{11}\alpha^4 + (2J_{12} + 4J_{66})\alpha^2\beta^2 + J_{22}\beta^4 + N_{55}\alpha + N_{44}\beta$$
$$k_{51} = -D_{11}\alpha^3 - (D_{12} + 2D_{66})\alpha\beta^2$$
$$k_{52} = -(D_{12} + 2D_{66})\alpha^2\beta - D_{22}\beta^3$$
$$k_{53} = H_{11}\alpha^4 + (2H_{12} + 4H_{66})\alpha^2\beta^2 + H_{22}\beta^4$$
$$k_{54} = J_{11}\alpha^4 + (2J_{12} + 4J_{66})\alpha^2\beta^2 + J_{22}\beta^4 + N_{55}\alpha + N_{44}\beta \tag{8}$$
$$k_{55} = L_{11}\alpha^4 + (2L_{12} + 4L_{66})\alpha^2\beta^2 + L_{22}\beta^4 + P_{55}\alpha + P_{44}\beta$$

The element of mass matrix [M] are given as follows;

$$M_{11} = IA$$
$$M_{12} = 0$$
$$M_{13} = -IB\alpha$$
$$M_{14} = -IC\alpha$$
$$M_{15} = -ID\alpha$$

$$M_{21} = 0$$
$$M_{22} = IA$$
$$M_{23} = -IB\beta$$
$$M_{24} = -IC\beta$$
$$M_{25} = -ID\beta$$

$$M_{31} = -IB\alpha$$
$$M_{32} = -IB\beta$$
$$M_{33} = (\alpha^2 + \beta^2)IE + IA$$
$$M_{34} = (\alpha^2 + \beta^2)IF + IA$$
$$M_{35} = (\alpha^2 + \beta^2)IH + IA$$

$$M_{41} = -IC\alpha$$
$$M_{42} = -IC\beta$$
$$M_{43} = (\alpha^2 + \beta^2)IF + IA \tag{9}$$
$$M_{44} = (\alpha^2 + \beta^2)II + IA$$
$$M_{45} = (\alpha^2 + \beta^2)IJ + IA$$
$$M_{51} = -ID\alpha$$
$$M_{52} = -ID\beta$$
$$M_{53} = (\alpha^2 + \beta^2)IH + IA$$
$$M_{54} = (\alpha^2 + \beta^2)II + IA$$
$$M_{55} = (\alpha^2 + \beta^2)IL + IA$$

3.1 Numerical Result.

In this paper, free vibration analysis of simply supported square and rectangular plates for aspect ratio (side to thickness ratio, *a*/*h*) 10 is attempted.

The simply supported plates considered are composed of isotropic material. The results obtained using trigonometric shear deformation theory are compared with exact results and with those of other refined theories available in literature. Following non-dimensional form is used for the purpose of presenting the results in this paper.

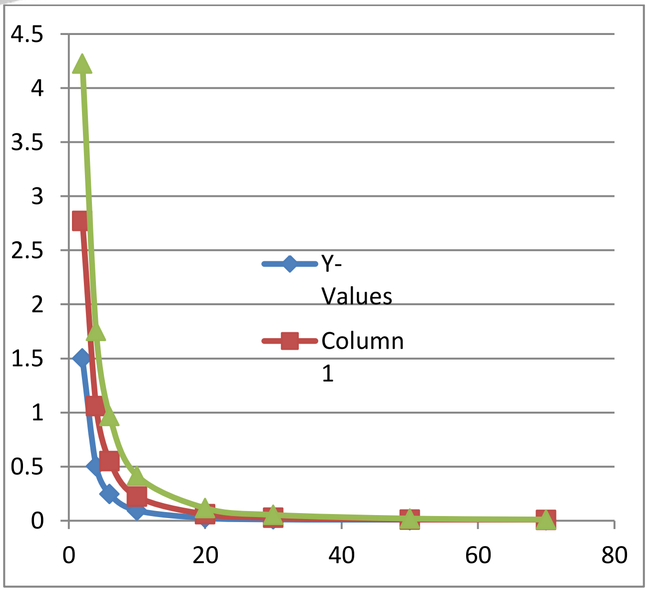

Figure-1.3 shows that natural frequencies of isotropic rectangular plate ($b/a = \sqrt{2}$) for aspect ratio1

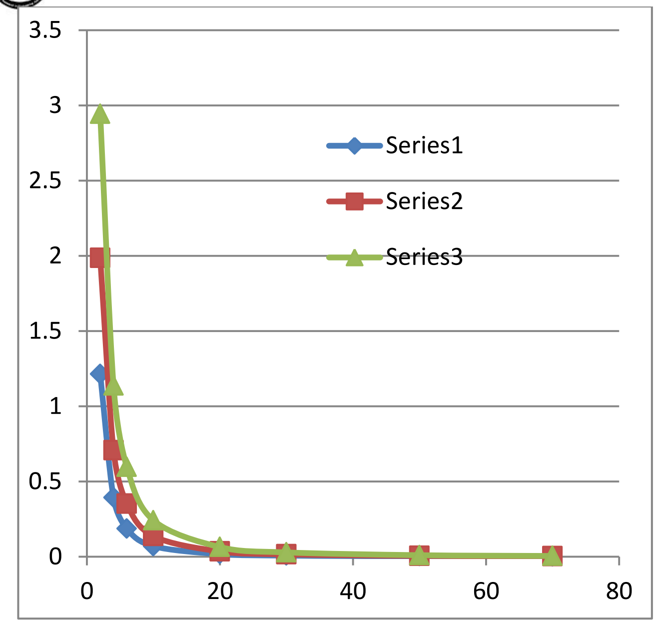

Figure-1.2 shows that natural frequencies of isotropic square plate (b/a = 1) for aspect ratio 10

Table 1 Comparison of non-dimensional natural frequencies of isotropic square plate (*b/a* = 1) for aspect ratio 10.

M,n	Present	Exact [5]	Ghugal and Sayyad [6]	Reddy [4]	Mindlin [2]	CPT [1]
(1,1)	0.0930	0.0932	0.0933	0.0931	0.0930	0.0955
(1,2)	0.2220	0.2226	0.2231	0.2219	0.2219	0.2360
(1,3)	0.4151	0.4171	0.4184	0.4150	0.4149	0.4629
(2,2)	0.3406	0.3421	0.3431	0.3406	0.3406	0.3732
(2,3)	0.5208	0.5239	0.5258	0.5208	0.5206	0.5951
(2,4)	0.7453	0.7511	0.7542	0.7453	0.7446	0.8926
(3,3)	0.6839	0.6889	0.6917	0.6839	0.6834	0.8090
(4,4)	1.0783	1.0889	1.0945	1.0785	1.0764	1.3716

Table 2 Comparison of non-dimensional natural frequencies of isotropic rectangular plate (b/a =) for aspect ratio 10

M,n	Present	Exact [5]	Ghugal and Sayyad [6]	Reddy [4]	Mindlin [2]	CPT [1]
(1,1)	0.0704	0.0704	0.0705	0.0704	0.0703	0.0718
(1,2)	0.1373	0.1376	0.1393	0.1374	0.1373	0.1427
(1,3)	0.2424	0.2431	0.2438	0.2426	0.2424	0.2591
(1,4)	0.3783	0.3800	0.3811	0.3789	0.3782	0.4182
(2,1)	0.2012	0.2018	0.2023	0.2041	0.2012	0.2128
(2,2)	0.2625	0.2634	0.2642	0.2628	0.2625	0.2821
(2,3)	0.3596	0.3612	0.3623	0.3601	0.3595	0.3958
(2,4)	0.4863	0.4890	0.4906	0.4874	0.4861	0.5513
(3,1)	0.3968	0.3987	0.3999	0.3975	0.3967	0.4406
(3,2)	0.4511	0.4535	0.4550	0.4520	0.4509	0.5073
(3,3)	0.5378	0.5411	0.5431	0.5392	0.5375	0.6168

IV. CONCLUSION

In this paper, a variationally consistent trigonometric shear deformation theory is applied to free vibration of isotropic square and rectangular plates. The effects of transverse shear and transverse normal deformation are both included in the present theory. The theory gives realistic variation of transverse shear stress through the thickness of plate and satisfies the shear stress free boundary conditions on the top and bottom planes of the plate. The theory requires no shear correction factor. The result of frequencies are compared with exact frequencies and those of other higher order theories. It is observed that the frequencies obtained by present theory are in excellent agreement with the frequencies of exact theory. The present theory is capable to produce frequencies of thickness of bending mode of vibration. The theory yields the exact dynamic shear correction factor from the thickness shear motion which is a most important factor in the dynamic analysis of plates.

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