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Modelling Multiechelon Inventory Systems for Repairable Items using Spreadsheets

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Abstract - Multiechelon inventory models are key to optimizing overall inventory management costs in organizations that have multiple operating sites and associated maintenance policies. However, these models are extremely complex and difficult to understand due to the level of mathematics involved. Practitioners, especially those without any background of probability theory and integral calculus, often find it difficult to implement this technique in their organizations. Scholars in operations management have advocated using spreadsheets to help practitioners as well as students to not only easily comprehend these complex models but also to assist in building tools that can be used in different scenarios. An added advantage is that spreadsheets have inbuilt functions for probability distributions. The primary objective of this paper is to demonstrate the use of spreadsheets in translating a complex multiechelon inventory model into an effective tool that can help students learn the concepts faster and practitioners can use the tool to manage their inventory better. While the technique can be used for managing any item, in this paper we focus on managing repairable items with multiple non-identical bases serviced by a central repair depot. Towards the end of this paper we have also presented the results of a survey conducted to study the effectiveness of teaching multiechelon theory to students with minimal background in probability and statistics. More than 82% of the respondents felt this method of teaching multiechelon theory would help them learn and solve problems faster. Possible extensions to make this model more flexible is also discussed in this paper.

Keywords: Multiechelon inventory, Repairable items, Base-depot model, Non-identical bases, Spreadsheet modeling.

I. INTRODUCTION

The main objective is to implementation of data mining to solve a problem related to searching a research paper .The other objectives are as follows.

Traditional inventory control models, though severely constrained by assumptions, have been successfully applied in practice to address single stocking level decisions [1][2]. Multiechelon inventory management models deal with the problem of stocking items at multiple levels, or echelons. Multiechelon inventory models have been in existence for over three decades, and is a topic of great interest to practitioners involved in supply chain management for a simple reason – decisions made at an echelon in the supply chain has some impact on another. Due to their closeness to current reality these models have found applications in several domains including manufacturing, transportation & distribution [3][4]. However, these models have been adopted more in management of repairable items for military equipment. Repairable items are those that are not disposed-off on failure, but are sent to a repair facility where the failed parts are restored to their original, working condition. The relatively higher investments in military equipment, the need for higher degree of control of management of parts and need for higher cost savings have made the topic of multiechelon inventory models for repairable items more attractive to inventory managers [5].

In its basic form, a multiechelon system for repairable items consists of several operating sites, with each operating site hosting several working items. Operating sites are supported by:

- a base, that maintains a stock of items in a working condition, and
- a central repair facility, or a depot. The repair facility also has a stock of working items and a repair queue for items awaiting/undergoing repair.

When a failure occurs at an operating site the failed item is sent to the depot for repair where it enters the repair queue. A spare item is shipped from the inventory maintained at the depot to replenish the base stock. If a spare item is unavailable a backorder is placed at the depot. A spare item from the stock maintained at the base is shipped to the operating site. If a spare item is unavailable at the base a backorder is placed on the base. Items that are repaired replenish the stock maintained at the depot.

The multiechelon technique for recoverable item control, or METRIC [6] and its variants - MOD-METRIC, VARI-METRIC etc. - have been very popular amongst researchers to manage repairable items. A detailed review of multiechelon inventory management for repairable items

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can be found in [7]. Application of these models can be found in maintenance of engines for aircraft/transport vehicles [8], high-technology manufacturing companies [9], process industries such as chemicals / fertilizer plants, steel plants and power generation besides several others including military organizations.

While multiechelon inventory models are key to optimizing inventory costs they are extremely complex and difficult to understand. Practitioners, especially those with little knowledge of probability theory and integral calculus, find it difficult to implement it in their organizations. Inventory managers and practitioners have to acquire/hire programming skills, or rely on expensive software and plugins to model and make decisions related to inventory policies. Spreadsheets offer a good alternative.

The objective of this paper is to describe the steps that practitioners may execute to model the multiechelon theory using commonly available spreadsheets, without using any special plugins or macros. By using commonly available spreadsheets practitioners can make the process of managing inventory more visible and repeatable. They can extend the spreadsheet models to perform what-if analysis to identify and resolve issues in their inventory system. Further, modelling using spreadsheets also eliminates the need to purchase expensive software since spreadsheets are available in all organizations.

This paper has been organized as follows: previous work done by researchers in this area is discussed briefly in section 2 of this paper. Section 3 describes the detailed steps involved in the actual implementation of the theory using spreadsheets. Section 4 contains discussions on results while section 5 presents concluding remarks, including possible extension to this work.

II. LITERATURE REVIEW

The use of spreadsheet modeling to teach industrial engineering students and practitioners has been advocated by several operations management scholars. Academics have extensively used inbuilt MS Excel[1] functions in their works to help practitioners comprehend inventory management and supply chain management principles [3][10][11].

Axsater [12] has described the steps involved in directly computing the inventory costs in a multiechelon system. However, implementation of the algorithm, which is recursive, will require knowledge of computer programming. Silver et al. [3] have used a spreadsheet to model a multiechelon system for the warehouse-retailer model. They modelled a system for repairable items, based on Graves algorithm [3][9], involving one depot and two identical bases, and have described the computation of decision variables in detail. Barlow [13] has modelled deterministic as well as probabilistic inventory control models using spreadsheets. He has demonstrated how spreadsheets can be used to easily theory into practice, and argues that by using spreadsheets students would be better placed to transfer decision-making and problem-solving skills into the workplace. Ragsdale [14] has shown how a business person can use spreadsheet model to analyze a business model before he decides to choose a specific plan for implementation. His work has extended beyond simple inventory theory to economic and investment decisions. Liu et al. [15] have presented a reliable spreadsheet-based inventory simulation model and applied it to a pharmacy chain. Results of the model showed a significant reduction of out-of-stock situation, increased annual revenues, and overall reduction in inventory for the pharmacy stores. They also demonstrated that spreadsheet models are able to educate students and practitioners better. Cobb [16] presented an alternative method of computing reorder point and order interval using spreadsheets that can help practitioners understand and implement the concept of inventory management easily. Strakos [17] successfully conducted an exercise with a group of graduate students that used spreadsheets to bridge the gap between learning exchange curves in inventory management theory, and applying the learning to practice.

Multiechelon systems are characterized by locations that are well spread out with each base operating under different economic and environmental conditions. Also, the number of items at the operating sites is not always the same, and, therefore, the failures (demand) would not be the same at each of those locations.

As can be seen from the review, researchers have strongly encouraged as well as demonstrated the use of spreadsheets in modelling and obtaining solutions to operations management problems. This paper attempts to model one of the most complex problems in inventory and supply chain management., and present ideas to develop a simple reusable tool that practitioners may use at their workplace The work presented in this paper is, however, different from the previous works in the domain in the following ways:

- We have modelled a multiechelon system for repairable items with one central depot and three bases, all of which have different characteristics in terms of costs, demand and lead times. While there are several algorithms available in the domain, we specifically use Graves algorithm [3] [9] because it not only approximates the mean number of backorders but also computes the variance of the number of repairable items in the repair process.
- We have comprehensively described the steps involved in the computation of parameters necessary to enable practitioners make decisions involved in setting up a multiechelon system for repairable items.
- We have described the model building using a spreadsheet that can be used by practitioners quite

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easily, and can be extended to different scenarios with minimal changes. The implementation of the Graves algorithm [3][9] using spreadsheet does not require any knowledge of programming / macros. Since the modelling allows for different characteristics for each of the bases, practitioners would find the tool very handy and realistic as well.

In the next section, we describe the steps involved in modelling the multiechelon system for repairable items using spreadsheets.

III. MODELLING USING SPREADSHEET

As described earlier, we use the Graves algorithm to model a repairable item system with one central depot and three non-identical bases. The spreadsheet may be modified slightly to use it in other scenarios such as more number of bases. The algorithm assumes a one-for-one $(S-1, S)$ replenishment policy since it is most appropriate for slow-moving, expensive spares.

Every system must have some indicators that measures its performance. The indicators, or performance measures, in the multiechelon system for repairable items are:

- Number of backorders, at the depot as well as each of the bases
- Variance of the number of backorders, at the depot as well as each of the bases
- Expected inventory on-hand, at the depot as well as each of the bases
- Total inventory cost for the system including carrying cost and shortage costs

The objective of the model is to determine the target (order-up-to) inventory at each of the bases and depot such that the total inventory cost is minimized using a one-for-one ordering policy. We use the same notations used in Silver et al. (1998) for the modelling exercise. Fig. 1 summarizes the steps involved in the modelling process.

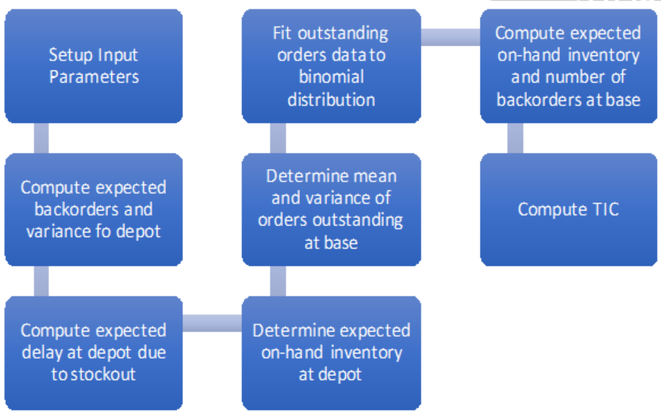

Figure 1: Summary of steps involved in the modelling process

Following are the steps one need to follow to model the multiechelon system using a spreadsheet[2]:

STEP 1: Setup input parameters

The first step in the modelling process in a spreadsheet is to setup a workbook and create cell references for input parameters. The key input parameters for this multiechelon problem are:

- N is the number of bases in operation.
- x_i is the demand for a repairable item at base i.
- L_i is the transportation time from central depot to base i, and L_c is the lead time to the depot.
- S_i is the order-up-to inventory at base i and S_c is the order-up-to inventory at the depot

Table 1: Setup Input Parameters

Cell	Description	Formula
D2	Number of bases in operation	
D6…F6	Demand for a repairable item at each base	
G6	Expected demand at depot	=SUM(D6:F6)
D7…G7	Transportation time	
D8…G8	Order-up-to inventory at each of the locations	
D9…G9	Carrying cost per unit per period (week)	
D10…G10	Shortage cost per unit per period (week)	
C12	Mean demand	=G6*G7

Fig. 1 shows the input parameters – expected demand at each of the identical bases, transportation time, target inventory, carrying costs and shortage costs - for the problem in consideration. Note that the order-up-to inventory at the depot, $S_c = 2$ unit. The total demand at the depot is the sum of demand generated by all the bases. In other words, the total demand at the depot is

$$x_c = \sum_{i=1}^{N} x_i \#(1)$$

Since the lead time at the depot is L_c, the expected demand during the lead time at the depot would be

$$= L_c \sum_{i=1}^{N} x_i \#(2)$$

As can be seen from Fig. 2, for the given input parameters, the expected demand for repairable items at the depot is 6.0 units, and is stored in cell C12.

	B	C	D	E	F	G
1	MULTIECHELON INVENTORY SYSTEM - INPUT PARAMETERS					
2	Number of bases	N	3			
3						
4			Base (i)			
5	Description	Variable	1	2	3	Depot
6	Expected Demand	x_i	0.4	0.8	1.2	2.4
7	Transportation Time (weeks)	L_i	2	1	1.5	2.5
8	Order-up-to Inventory	S_i	2	3	5	2
9	Carrying cost per unit per period (week)	C_o	10	12.5	12	8.5
10	Shortage cost per unit per period (week)	C_s	8	8	8	5
11						
12	Mean Demand	6				

Figure 2: Input parameters

[2] We use Microsoft's MS Excel v 2017 for Mac OS for this exercise

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STEP 2: Compute expected number of backorders its variance (at depot)

The next step is to compute the expected number and the variance of the number of backorders at the depot. The expected number of backorders is given by:

$$E(B_c) = \sum_{j=S_c+1}^{\infty} (j - S_c)\, p(j|x_c L_c) \#(3)$$

and the variance of the number of backorders is given by:

$$var\,(B_c) = \sum_{j=S_c+1}^{\infty} (j - S_c)^2\, p(j|x_c L_c) - \left(E(B_c)\right)^2 \#(4)$$

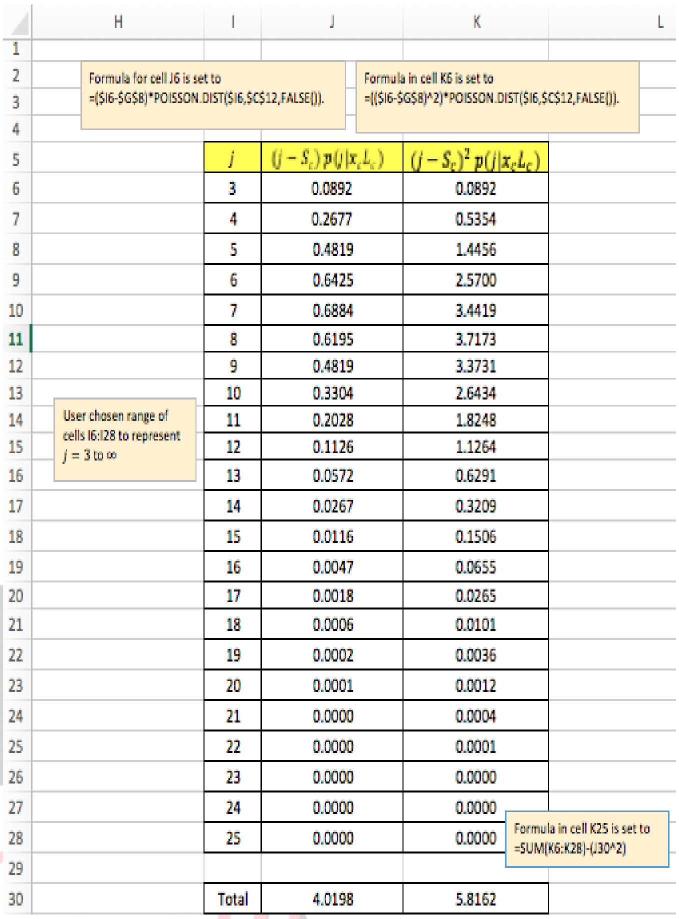
where

- S_c is the order-up-to level in the depot, and
- $p(j|x_c L_c)$ is the probability that a Poisson random variable may take on for a given j with a mean of 6.0 units.

Since $S_c = 2, j$ can take values from 3 to ∞. Table 2 shows the cell references and formula to be used in the computation process. In MS Excel, the expected number of backorders and the variance at the depot can be computed using the in-built Poisson distribution functions. Sample calculations are shown in Fig. 3. As can be seen from Fig. 3, $E(B_c) = 4.0198$ units and the variance, $var\,(B_c)$ is 5.8162 units^2.

Table 2: Cell references for computing expected number of backorders and variance

Cell	Description	Formula
I6…I28	Index j. Since $S_c = 2, j$ can take values from 3 to ∞. Note that the values in Col J and K tend to 0 as value of j increases. We, therefore, stop at $j = 25$.	
J6…J28	Expected number of backorders for a given value of j.	=($I6-$G$8)*POISSON.DIST ($I6,$C$10,FALSE()) Copy this formula in cell J6 and drag the cell handle down to J28. (See Eq. 3)
K6…K28	Required for calculating the variance of the number of backorders for a given value of j.	=(($I6-G$8)^2)*POISSON.DIST ($I6,$C$10,FALSE()) Copy this formula in cell K6 and drag the cell handle down to K28. (See Eq. 4)
J30	Expected number of backorders	=SUM(J6:J28)
K30	Variance of the number of backorders	=SUM(K6:K28)-(J30^2)



	j	$(j - S_c)\,p(j\|x_c L_c)$	$(j - S_c)^2\,p(j\|x_c L_c)$
6	3	0.0892	0.0892
7	4	0.2677	0.5354
8	5	0.4819	1.4456
9	6	0.6425	2.5700
10	7	0.6884	3.4419
11	8	0.6195	3.7173
12	9	0.4819	3.3731
13	10	0.3304	2.6434
14	11	0.2028	1.8248
15	12	0.1126	1.1264
16	13	0.0572	0.6291
17	14	0.0267	0.3209
18	15	0.0116	0.1506
19	16	0.0047	0.0655
20	17	0.0018	0.0265
21	18	0.0006	0.0101
22	19	0.0002	0.0036
23	20	0.0001	0.0012
24	21	0.0000	0.0004
25	22	0.0000	0.0001
26	23	0.0000	0.0000
27	24	0.0000	0.0000
28	25	0.0000	0.0000
29			
30	Total	4.0198	5.8162

Figure 3: Computing Expected Number and Variance of Backorders at Depot

STEP 3: Compute expected delay at depot due to stock-out

Next, we need to compute the expected delay caused due to stock-out at the depot. The waiting time at the depot, $E(W_c)$, can be determined using the following:

$$E(W_c) = \frac{E(B_c)}{\sum_{i=1}^{N} x_i} \#(5)$$

Since the expected number of backorders is 4.0198 units, and the total demand at the depot is 2.4 units, using Eq. 5, we can compute the expected delay at the depot due to stock-out, which is:

$$E(W_c) = \frac{4.0198}{2.4} = 1.6749 \text{ weeks}$$

STEP 4: Determine expected on-hand inventory at depot

The next step is to determine the inventory on-hand at the depot, $E(I_c)$, which is given by:

$$E(I_c) = \sum_{j=0}^{S_c-1} (S_c - j)\, p(j|x_c L_c) \#(6)$$

Recall that since $S_c = 2, j$ can take values only from 0 to 1. Table 3 shows the cell references and formulae required for computation of the expected inventory on hand at the depot. Fig. 4 shows the sample calculations for determining the on-hand inventory at the depot, which in this case is 0.0198 units.

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Table 3: Calculation of on-hand inventory at depot

Cell	Description	Formula
M6…M7	Value of index j	
N6…N7	On-hand inventory at depot for a given value of j.	=($G$8-M6)*POISSON.DIST ($M6,$C$12,FALSE()). Copy this formula to cell N6 and drag the cell handle down to N7. (See Eq. 6)
N14	Expected on-hand inventory at depot	=SUM(N6:N7)

Formula for cell N6 is set to =($G$8-M$6)*POISSON.DIST($M6,$C$12,FALSE()). Drag cell handle down through to N7.

	L	M	N
5		j	$(S_c - j)\,p(j\mid x_c L_c)$
6		0	0.0050
7		1	0.0149
8			
9			
10			
11			
12			
13			
14		Total	0.0198

Formula in cell N9 is set to =SUM(N6:N7)

Figure 4: Calculation of on-hand inventory at depot

Fig. 5 shows the summary of performance measures of the multiechelon system for the depot, given the order-up-to inventory at the bases and the depot, and a one-for-one ordering policy.

	A	B	C	D
14		**Summary - Depot**		
15		Description	Variable	Value
16		Expected no. of backorders	$E(B_c)$	4.0198
17		Variance of no. of backorders	$var(B_c)$	5.8162
18		Expected waiting time (weeks)	$E(W_c)$	1.6749
19		Expected inventory on hand	$E(I_c)$	0.0198

Figure 5: Solution summary (depot)

Now that we have the indicators for the depot we can proceed to find the same for each of the bases, given the order-up-to inventory at those locations.

STEP 5: Determine mean and variance of orders outstanding at base

In order to determine the mean and variance of outstanding orders at all the bases, we need to first compute the average lead time at the base, which is equal to the time to transport items between the depot and the base i, represented by L_i, and the expected delay in the depot due to stock-out, represented by $E(W_c)$. Thus, the average lead time $\bar{L}_i$ at base i is:

$$\bar{L}_i = L_i + E(W_c)\#(7)$$

In this case, we have $L_1 = 2, L_2 = 1$ and $L_3 = 1.5$ weeks, and $E(W_c) = 1.6749$ weeks. Substituting these values in Eq. 7, we get $\bar{L}_1 = 3.6749$ weeks, $\bar{L}_2 = 2.6749$ weeks and $\bar{L}_3 = 3.1749$ weeks respectively for each of the bases. Using the average lead time values we can now calculate the mean and variance of the number of orders outstanding at each of the bases. The mean number of orders outstanding is a function of the lead time and the demand. Specifically, the mean number of outstanding orders, at base i, is given by:

$$E(O_i) = x_i\bar{L}_i\#(8)$$

Since the mean demand at the base 1, $x_1, = 0.4$ units and the average lead time, $\bar{L}_1, = 3.6749$ weeks, the mean number of outstanding orders at base 1 is

$$E(O_1) = 0.4 \times 3.6749 = 1.4700 \text{ units}$$

Similarly, the mean number of orders outstanding at base 2 and base 3 would be 2.1399 and 3.8099 units respectively. Next, we compute the variance if the number of backorders at the bases. The variance of the number of orders outstanding at the base i is given by

$$var(O_i) = \left(\frac{x_i}{x_c}\right)^2 var(B_c) + \left(\frac{x_i}{x_c}\right)\left(\frac{x_c - x_i}{x_c}\right)E(B_c) + x_iL_i\#(9)$$

For base 1, using Eq. 9 we get,

$$var(O_1) = \left(\frac{0.4}{2.4}\right)^2 \times 5.8162 + \left(\frac{0.4}{2.4}\right)\left(\frac{2.4 - 0.4}{2.4}\right) \times 4.0198 + 0.4 \times 2$$

or the variance of the number of backorders outstanding at base 1 is

$$var(O_1) = 1.5199 \text{ units}^2$$

Using Eq. 9, we can find the variance of the number of backorders outstanding at the other bases. Table 4 shows the cell references and formulae required for the computation. Fig. 6 shows the summary of the solution obtained – average lead time (in weeks), expected number of outstanding orders and variance of the number of outstanding orders - at each of the bases. Computation of remaining parameters – expected number of backorders and expected on-hand inventory - is explained later in this section.

	B	C	D	E	F
21	**Summary - Base**		Base		
22	Description	Variable	1	2	3
23	Average lead time (weeks)	L_i	3.6749	2.6749	3.1749
24	Expected No. of orders outstanding	$E(O_i)$	1.4700	2.1399	3.8099
25	Variance of no. of orders outstanding	$var(O_i)$	1.5199	2.3395	4.2590
26	Expected no. of backorders	$E(Bi)$	0.0675	0.1922	0.4139
27	Expected inventory on hand	$E(Ii)$	0.7929	1.1489	1.5243

Figure 6: Solution summary - base

STEP 6: Compute binomial distribution parameters, a and p

Graves approximates the expected number of outstanding orders at a base to a negative binomial distribution. The next step, therefore, is to determine the parameters of a negative binomial distribution a_i and p_i such that

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$$p_i = \frac{E(O_i)}{var(O_i)} \#(10)$$
#

and a_i is a positive integer which satisfies the following:

$$a_i = \frac{E(O_i)p_i}{(1-p_i)} \#(11)$$

We can compute the values for a_i and p_i for each of the bases using Eq. 10 and 11. Table 5 shows the cell references that may be used to perform these computations in a spreadsheet. Please note that the value of a_i is to be rounded-off up to the next highest integer. Fig 7 shows the values obtained for the binomial distribution parameters, a_i and p_i.

Table 4: Cell references to calculate binomial distribution parameters

Cell	Description	Base 1	Base 2	Base 3
D29…F29	Binomial parameter p	=D24/D25	=E24/E25	=F24/F25
D30…F30	Binomial parameter a	=ROUNDUP((D24*D31/(1-D31)),0)	=ROUNDUP((E24*E31/(1-E31)),0)	=ROUNDUP((F24*F31/(1-F31)),0)

Table 5: Cell references to compute lead-time, outstanding orders and its variance at each of the bases

Cell	Description	Base 1	Base 2	Base 3
D23…F23	Average lead-time (weeks)	=D$7+$D$18	=E$7+$D$18	=F$7+$D$18
D24…F24	Outstanding number of orders	=D6*D23	=E6*E23	=F6*F23
D25…F25	Variance of number of outstanding orders	=(((D$6/$G$6)^2)*$D$17)+((D$6/$G$6)*(($G$6-D$6)/$G$6)*$D$16)+(D$6*D$7)	=(((E$6/$G$6)^2)*$D$17)+((E$6/$G$6)*(($G$6-E$6)/$G$6)*$D$16)+(E$6*E$7)	=(((F$6/$G$6)^2)*$D$17)+((F$6/$G$6)*(($G$6-F$6)/$G$6)*$D$16)+(F$6*F$7)

Table 6: Cell references for computing expected backorders and on-hand inventory at base

Cell	Description	Formula
P6…P11	Index *j*. Number of rows to be created depends on the order-up-to inventory at each of the bases.	
Q6…S11	Expected on-hand inventory at base for a given value of *j*.	=(D$8-$P6)*(FACT(D$32+$P6-1)/(FACT($P6)*FACT(D$32-1)))*((D31^D32)*((1-D$31)^$P6)) Copy this formula to cell Q6 and drag the cell handle down to Q8. (See Eq. 13). While keeping the cells Q6…Q11 selected, also drag the handle from S11 to complete copying the formula to all the cells in the range Q6…S11. Cells returning a negative value must be deleted.
Q12…S23	Expected backorder at the base, for a given value of *j*	=($P15-D$8)*(FACT(D$32+$P15-1)/(FACT($P15)*FACT(D$32-1)))*((D31^D32)*((1-D$31)^$P15)) Copy this formula to cell Q15 and drag the cell handle down to Q27. (See Eq. 12). While keeping the cells Q12…Q27 selected, also drag the handle from S27 to complete copying the formula to all the cells in the range Q12…S27. Cells returning a negative value must be deleted.
Q13…S13	Expected on-hand inventory for each of the bases	=SUM(Q6:Q11). Copy this formula in columns R and S.
Q30…S30	Expected backorders at each of the bases.	=SUM(Q18:Q28). Copy this formula in columns R and S.

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	B	C	D	E	F
29			Base		
30	Description	Variable	1	2	3
31	Binomial parameter p	p_i	0.96717	0.91469	0.89456
32	Binomial parameter a	a_i	44	23	33

Figure 7: Calculation of binomial distribution parameters

STEP 7: Compute expected number of backorders and expected inventory on-hand at bases

Graves (1985) computes the expected number of backorders at base i using the following

$$E(B_i) = \sum_{j=S_i+1}^{\infty} (j - S_i)\, \frac{(a_i + j - 1)!}{j!\,(a_i - 1)!}\, p_i^{\,a_i}(1 - p_i)^j \#(12)$$

while the expected inventory on hand at base i is given by

$$E(I_i) = \sum_{j=0}^{S_i-1} (S_i - j)\, \frac{(a + j - 1)!}{j!\,(a - 1)!}\, p^a(1 - p)^j \#(13)$$

Table 6 shows the cell references and formulae to be used in the computation expected number of backorders and on-hand inventory at each of the bases.

As can be seen from Eq. 12 and 13, the expected number of backorders and the expected inventory at Base i is a function of the order-up-to inventory, S_i, maintained at that base. These values can be computed using the in-built functions in MS Excel (shown in Table 6). Sample computation for $E(B_i)$ and $E(I_i)$ are shown in Fig. 8. (For example, as can be seen from Fig. 8, $E(B_1) = 0.0675$ units and $E(I_1)$ is 0.7929 units).

STEP 8: Compute total inventory costs

The final step is to compute the total inventory costs for the multiechelon inventory system with given order-up-to inventory levels, i.e., $S_c = 2, S_1 = 2, S_2 = 3, \text{and } S_3 = 5$. Recollect that the total inventory cost includes the ordering cost, carrying cost and shortage costs. Ignoring the ordering cost, the total inventory cost (TIC) for this system is given by

$$TIC = \sum_{i=1}^{N} E(I_i)C_h + \sum_{i} \sum_{i=1}^{N} E(B_i)C_s + E(I_c)C_h + E(B_c)C_s \#(14)$$

where C_h is the carrying cost per unit per period and C_s is the shortage cost per unit short per period. For the given scenario, we can substitute the values for expected inventory and number of backorders, at the depot and each of the bases in Eq. 14 to compute the TIC. Fig. 9 shows sample TIC calculation for the scenario under consideration. As can be seen from the figure, the TIC is \$40.7504 per week.

IV. RESULTS & DISCUSSIONS

In the previous section, we reviewed the steps involved in computing the TIC for a multiechelon system for repairable items with one depot and three non-identical bases. We assumed the target inventory at the depot and the bases being maintained at different levels. Further we assumed a one-for-one replenishment policy.

In this section, we perform a *what-if analysis* and compute TIC for different values of target inventory at the depot keeping the target inventory at the base equal to the original plan. i.e., $S_1 = 2, S_2 = 3, \text{and } S_3 = 5$. All other variables (such as expected demand, lead times etc.) are retained at the same as in the previous section. The objective is to determine the level of target inventory at the depot that results in minimal TIC. This can be performed easily by simply changing the values in the input parameters section of the spreadsheet.

Fig. 10 summarizes the TIC for different values of target inventory at the depot. As see from the figure, the TIC is minimum (\$66.2385) for $S_c = 2$ units.

Formula for cell Q6 is set to
=(D$8-$P6)*(FACT(D$30+$P6-1)/(FACT($P6)*FACT(D$30-1)))
((D29^D30)((1-D$29)^$P6))
Drag cell handle down first through to Q8 (if the order-up-to inventory = 2) and then to S11.

	P	Q	R	S
5	j	1	2	3
6	0	0.4604	0.3858	0.1265
7	1	0.3325	0.5047	0.3521
8	2	0.0000	0.2583	0.4733
9	3		0.0000	0.3882
10	4			0.1842
11	5			0.0000
12				
13	Total	0.7929	1.1489	1.5243
14				
15	2	0.0000		
16	3	0.1237	0.0000	
17	4	0.0954	0.1019	
18	5	0.0451	0.0938	0.0000
19	6	0.0161	0.0560	0.0960
20	7	0.0047	0.0264	0.1128
21	8	0.0012	0.0106	0.0892
22	9	0.0003	0.0037	0.0571
23	10	0.0001	0.0012	0.0316
24	11	0.0000	0.0003	0.0156
25	12	0.0000	0.0001	0.0071
26	13	0.0000	0.0000	0.0029
27	14	0.0000	0.0000	0.0011
28	15	0.0000	0.0000	0.0004
29				
30	Total	0.0675	0.1922	0.4139

Calculation of Expected Inventory Onhand. This is a function of order-up-to inventory maintained at the base.

Calculation of Expected Backorder at Base *i*. This is a function of order-up-to inventory maintained at the base.

Figure 8: Sample computations for E(Bi) and E(Ii)

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	B	C	D	E	F	G	H
35	Summary - Costs		Base				
36	Cost Head		1	2	3	Depot	TIC
37	Carrying cost		7.9294	14.3609	18.2916	0.16856	
38	Shortage cost		0.5397	1.5380	3.3112	20.0991	
39	Total cost (at a location)		8.4692	15.8989	21.6028	20.2677	40.7504

Figure 9: TIC - Sample calculation of inventory costs

A similar procedure can be carried out to determine the TIC, and hence the optimal target inventory at the depot, for different set of target inventory at the bases for a known set of input parameters (costs, lead times etc.). This way practitioners would be able to find the global minima for their multiechelon inventory problem.

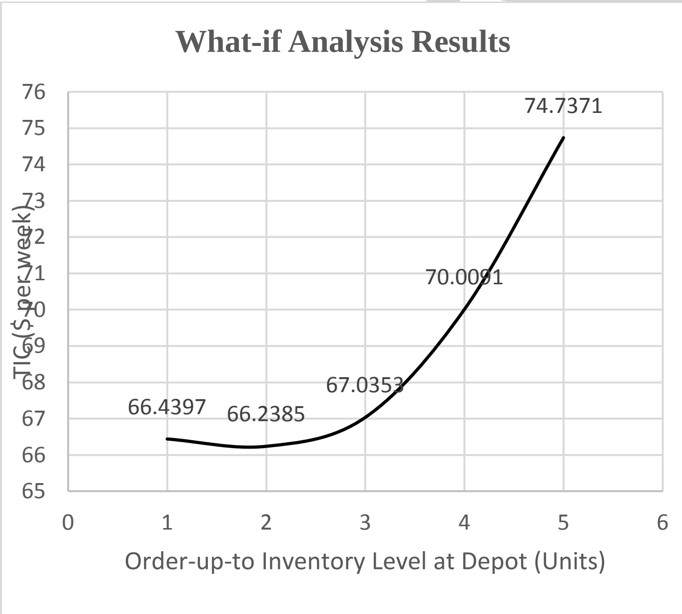

Figure 10: TIC for a given set of values of target inventory

As part of our initiative to determine the effectiveness of teaching this method (of using spreadsheets) to students, we conducted a survey amongst 35 students of an undergraduate class in which they were asked the following questions:

- Did you learn the theory of multiechelon better using this method, or using the traditional classroom method?
- Were you able to use spreadsheets to compute the inventory performance (i.e., compute the backorders at a stage) more efficiently?

Of the 35 students surveyed, more than 82% of the respondents favored the spreadsheet modelling method of solving multiechelon problems. Please note that the students surveyed had very minimal background in mathematics, including probability and statistics.

V. CONCLUSION

In this paper, we have successfully demonstrated a method to model a complex yet practical multiechelon inventory system for repairable items using spreadsheets that are available in almost every organization. We have comprehensively described all the steps required to create a low-cost, re-usable tool. Inventory managers and practitioners with minimal knowledge of calculus and probability theory can use this to implement an inventory management system within their organization. They can also perform what-if analysis to study the cost impact of making changes to their system. This tool would help eliminate the need for organizations to invest in high cost, off-the-shelf software.

Also, operations management students can use this to learn multiechelon theory faster. We studied the effectiveness of using this method of learning amongst a group of students using a simple survey. More than 82% of the students suggested that the spreadsheet modelling helped them learn the complex theory better than just the classroom lecture. Results obtained are consistent with those presented by past researchers in operations management.

A possible extension of this paper would be to apply the learnings to managing multiple items in a multiechelon system. Another improvement could be to introduce capacity constraints including limited repair facility constraint. Potential also exists to model variable transportation times between bases, including the possibility of lateral supplies between bases.

REFERENCES

[1] Öztürk, H., Eroglu, A., and Lee, G.M. (2015) An economic order quantity model for lots containing defective items with rework option, International Journal of Industrial Engineering, 22 (6), 683-704.

[2] López, J. A, Mendoza. A, and Masini, J. (2013). A classical and effective approach to inventory management, International Journal of Industrial Engineering, 20 (5-6), 372-386.

[3] Silver, E. A., Pyke, D. F. and Petersen, R. (1998). Inventory management and production planning and scheduling, Wiley, NY, USA.

[4] Ji, P., Chen, K.J., and Yan, Q. P. (2008). A Mathematical Model for a Multi-Commodity, Two-Stage Transportation and Inventory Problem, International Journal of Industrial Engineering, 15(3), 278-285.

[5] Diaz, A. and Fu, C.M. (1995). Multiechelon models for repairable items: a review, Working paper, University of Mayland at College Park.\, Maryland.

[6] Sherbrooke, C.C. (1968). Metric: A multiechelon technique for recoverable item control, Operations Research, 16(1), 122-141.

[7] Nahmias, S (1981). Managing repairable item inventory

 DOI : 10.18231/2454-9150.2018.0064

systems: A Review, In Multilevel Production / Inventory Control Systems – Theory and Practice, Studies in Management Sciences, 16, L B Shwarz (Ed.), North Holland, Amsterdam, 253-277.

[8] Vrat, P. (2014). Materials management – an integrated systems approach, Springer, New Delhi, India.

[9] Graves, S.C. (1985). A multiechelon inventory model for a repairable item with one-for-one replenishment, Management Science, 31(10), 1247-1256.

[10] Nahmias, S. and Olsen, T.L. (2015), Production and operations analysis, Waveland Press, 7th Ed., IL, USA.

[11] Chopra, S., and Meindl, P. (2012). Supply chain management: strategy, planning and options, Prentice Hall, USA.

[12] Axsater, S. (1990). Evaluation of (R, Q)-Policies for Two-Level Inventory Systems with Poisson Demand, Luleå University of Technology. Sweden.

[13] Barlow, J. F. (2005). Excel models for business and operations management, John Wiley & Sons, 2nd Ed., England, UK.

[14] Ragsdale, C. (2016), Spreadsheet Modelling and Decision Analysis – A Practical Introduction to Business Analytics, Cengage Learning, 8th Ed., CT, USA.

[15] Liu, Q., Zhang, X, Liu, Y., and Lin, L (2013). Spreadsheet inventory simulation and optimization models and their application in a national pharmacy chain. INFORMS Transactions on Education, 14(1),13-25.

[16] Cobb, B. R. (2013). Spreadsheet modelling of (Q,R) inventory policies, Decision Sciences Journal of Innovative Education, 11 (2): 175-184.

[17] Strakos, J. K. (2016). Using spreadsheet modeling to teach exchange curves (optimal policy curves) in inventory management, Decision Sciences Journal of Innovative Education, 14 (1): 51-66.

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