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Stringent Analysis of Particle Swarm Optimization Algorithms

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Abstract: **Swarm algorithms, although initially introduced for simulating human social behaviors, have nowadays become very popular as efficient search and optimization techniques. This paper provides an overview on two distinctive approaches of particle swarm optimization algorithms, one of the swarm intelligent groups. It begins with a foundation of classical Particle Swarm Optimization algorithm, and gradually examines the various extensions of the classical algorithm. It starts with static parameter analysis and then followed by the methods of dynamic parameter analysis. It also presents the development of PSO dynamics and the different methods adopted for the analysis of algorithm in a nutshell.**

Keywords: Swarm intelligence, pso, algorithm, static, dynamic,

I. Introduction

PSO is a population based stochastic search algorithm, aimed at obtaining optimal solution to a complex non-linear optimization problem. PSO has many fundamental properties of classical swarm algorithm, such as proximity, diversity and adaptability. In PSO, particles are pseudo entities having two components, position and velocity. The position of a particle is a trial solution to a given optimization problem. Particle fly in the search space and adapt their velocity and positions based on its own and companions' historical behavior. The position of a particle in a simple PSO algorithm is determined by three factors, inertia, and the personal and global best position vectors. The inertia component is responsible for the motion of the particle in the direction of the velocity of previous iteration. The personal and global best position vectors act like two dynamic attractors. In fact, the velocity of a particle is an algebraic sum of three vectors: the inertial component and the positional difference of the particle with respect to its personal and global best positions. The personal best position ($pbest$) is the historical best position of a particle in its lifetime until the current iteration. The global best position is the best among the personal best positions of all the particles. Both the personal and the global best positions are updated over the iterations and the particle velocity and position are updated using their dynamic personal and global best positions until a criteria for convergence is satisfied. The particle with the best fitness measure of the objective function is declared as the solution to the optimization problem.

Researchers are keen to hybridize PSO with other evolutionary technique. For instance, selection, crossover and mutation operations in GA have been introduced into the PSO by some researchers. By the selection operation, the particles with the best performance are copied into the next generation to keep the best performing particles. Crossover operation is used in PSO to exchange information between a pair of individual particles to have the ability to jump to the new search areas like other evolutionary algorithms. The mutation operation is borrowed from evolutionary algorithm with the idea that PSO will increase its ability to escape from local optima.

PSO has attracted a good number of researchers from diverse domains of science, engineering and humanities, particularly for its following characteristics:

- ***Simplicity:*** PSO is simple, and can be easily implemented in any high level programming language. Main body of a PSO program comprises a few lines of code. This particular feature of PSO attracts researchers from different disciplines with minimum programming skill.
- ***Good Performance:*** PSO outperforms binary coded GA, and has comparable performance with real coded GA. It is found to give to good accuracy in determining optima for uni-modal, multi-modal, and functions with very rough surface. PSO is better than the differential evolutionary (DE) algorithms on occasions. DE employs a greedy search as the parameter vectors in current iteration is either better or of similar quality in DE. In PSO, particles move away from its historical best iteration, and thus it is not a greedy algorithm.
- ***Few Control Parameters:*** PSO has few control parameters; the inertial co-efficient, the local and the global acceleration constants. Extensive research has already been undertaken to study the performance of PSO on the selection of parameters. There is however scope of further research on this issue particularly, selection of control parameters to have better

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exploration in the first phase, and faster convergence in the exploitation phase.

- ***Low Space Complexity:*** Because of its low space complexity, PSO is preferred to its competitive counterparts, such as CMA-ES [1]. The low space complexity is a useful feature for PSO for complex optimization of high dimensional search problems.

The paper is organized to analize the static and dynamic parameter analysis of PSO algorithm.

II. ANALYSIS OF PSO ALGORITHM

Since pioneered by Eberhart and Kennedy in 1995 [2], PSO is analyzed by so many researchers. The first PSO dynamics had only two parameters, the local and the global best positions $p_i^l(t)$ and $p^g(t)$ respectively. The then dynamics had no inertia factor ω as given in (1), where each particle is treated as a point in a 1-dimensional space.

$$\vec{v}_i(t+1)=\vec{v}_i(t)+\alpha^l(t)(\vec{p}_i^l(t)-\vec{x}_i(t))+\alpha^g(t)(\vec{p}^g(t)-\vec{x}_i(t)) \quad (1)$$

$$\vec{x}_i(t+1)=\vec{x}_i(t)+\vec{v}_i(t+1) \quad (2)$$

The inertia component in the velocity adaptation is used for local exploration, while the cognitive component and the social components together are used for global exploration in the search landscape.

In the year 1998, Shi and Eberhart [3] introduced a new parameter called inertia weight into the original PSO dynamics as in (3).

$$\vec{v}_i(t+1)=\omega\vec{v}_i(t)+\alpha^l(t)(\vec{p}_i^l(t)-\vec{x}_i(t))+\alpha^g(t)(\vec{p}^g(t)-\vec{x}_i(t))$$

$$\vec{x}_i(t+1)=\vec{x}_i(t)+\vec{v}_i(t+1) \quad (3)$$

The inertia factor is capable to control the local/global exploration. If ω is set to zero, all particles would virtually converge at the same point in the search space. The larger the ω in $[0,1]$, the higher is the spread of search from the initial search position on the search landscape.

Static parameter analysis: The first deterministic study of the particle trajectory was undertaken by Clerc and Kennedy in 2002 [4]. They considered the velocity update equation as in (4).

$$\vec{v}_i(t+1)=\vec{v}_i(t)+\alpha^l(t)(\vec{p}_i^l(t)-\vec{x}_i(t))+\alpha^g(t)(\vec{p}^g(t)-\vec{x}_i(t)) \quad (4)$$

$$\vec{x}_i(t+1)=\vec{x}_i(t)+\vec{v}_i(t+1) \quad (5)$$

where, the inertia factor ω is assumed to be 1, and $\alpha^l(t)$ and $\alpha^g(t)$ are random positive numbers. By considering

$$\alpha(t)=\alpha^l(t)+\alpha^g(t) \text{ and}$$

$$p=\frac{\alpha^l(t)p_i^l(t)+\alpha^g(t)p^g(t)}{\alpha^l(t)+\alpha^g(t)},$$

the dynamics for a population of 1-Dimensional deterministic particle, can be reduced to (6)-(7).

$$v_i(t+1)=v_i(t)+\alpha(t)(p-x_i(t)) \quad (6)$$

$$x_i(t+1)=x_i(t)+v_i(t+1) \quad (7)$$

where, p and $\alpha(t)$ are constants. The velocity and position being defined in one dimension needs are scalars, and so the vector symbols are dropped from (6)-(7).

The basic system dynamics can be expressed in a state equation as (8)

$$v_i(t+1)=v_i(t)+\alpha(t)y(t) \quad (8)$$

where, $y(t)=p-x_i(t)$. Then

$$\begin{aligned} y(t+1) &= p-x_i(t+1) \\ &= p-(x_i(t)+v_i(t+1)) \text{ by (6)} \\ &= y(t)-v_i(t)-\alpha(t)y(t) \text{ by (7)} \\ &= -v_i(t)+(1-\alpha(t))y(t) \end{aligned}$$

Let, $P_t=\begin{bmatrix} v_i(t) \\ y(t) \end{bmatrix}$, then the system matrix M is defined as in the form $P_{t+1}=MP_t$,

$$M=\begin{bmatrix} 1 & \alpha(t) \\ -1 & 1-\alpha(t) \end{bmatrix}.$$

The eigen-values of M are

$$e_1=1-\frac{\alpha(t)}{2}+\frac{\sqrt{\alpha^2(t)-4\alpha(t)}}{2},$$

$$e_2=1-\frac{\alpha(t)}{2}-\frac{\sqrt{\alpha^2(t)-4\alpha(t)}}{2}.$$

Now the behavior of the trajectory for different cases of $\alpha(t)$ is as follows:

Case 1: For $0<\alpha(t)<4$, i.e.; roots are complex conjugate, then the trajectory of the particles are cyclic.

Case 2: For $\alpha(t)>4$ i.e., the roots are real numbers with no cyclic behavior.

Case 3: For $\alpha(t)=4$ i.e.; the roots are same value with -1. Then the system will oscillate infinitely.

In 2002,Trelea [5] analyzed the PSO algorithm on the basis of convergence, and parameter selection guidelines are derived from the dynamic system theory. He considered the one dimensional algorithm as in (9)

$$v_i(t+1)=\omega v_i(t)+\alpha^l(t)(p_i^l(t)-x_i(t))+\alpha^g(t)(p^g(t)-x_i(t)) \quad (9)$$

$$x_i(t+1)=x_i(t)+v_i(t+1). \quad (10)$$

Then (9) can be restructured like (6), as given in (11)

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$$v_i(t+1)=\omega v_i(t)+\alpha(t)(p-x_i(t)), \tag{11}$$

and (10) reduces to

$$x_i(t+1)=x_i(t)+v_i(t+1)$$
$$=\omega v_i(t)+(1-\alpha(t))x_i(t)+\alpha(t)p. \tag{12}$$

Now, by linear, discrete-time dynamic system theory, (11) and (12) can be represented by a state equation as,
$Y_{t+1}=AY_t+BP$

With, $Y_t=\begin{bmatrix}x_i(t)\\ v_i(t)\end{bmatrix}$, $A=\begin{bmatrix}1-\alpha(t) & \omega\\ -\alpha(t) & \omega\end{bmatrix}$ and

$$B=\begin{bmatrix}\alpha(t)\\ \alpha(t)\end{bmatrix}.$$

The eigen-values λ_1 and λ_2 of the system matrix A are the solutions of

$$\lambda^2-(\omega-\alpha(t)+1)\lambda+\omega=0.$$

Now, the necessary and sufficient condition for the system to be stable is that, both the eigen-values have magnitude less than 1. This gives,

$$\omega<0,\ \alpha(t)>0 \text{ and } 2\omega-\alpha(t)+2>0.$$

Case 1: For complex roots

$$\omega^2+\alpha^2(t)-2\omega\alpha(t)-2\omega-2\alpha(t)+1<0,$$

which, gives harmonic oscillation before the convergence.

Case 2: For roots with negative real parts

$$\omega<0,\ \omega-\alpha(t)+1<0,$$

which, gives zigzagging behavior before the convergence.

Dynamic parameter analysis: The first stability analysis of stochastic particle dynamics using Lyapunovs' analysis was undertaken by Kadirkamanathan in 2006 [7]. He considered the parameters are random in stead of non-random as presumed by the previous researchers. The values of this random numbers are increased when the inertia factor is reduced. The one dimensional particle dynamics is represented as (13) [7]:

$$v_i(t+1)=\omega v_i(t)+\alpha^l(t)(p_i^l(t)-x_i(t))+\alpha^g(t)(p^g(t)-x_i(t)) \tag{13}$$

$$x_i(t+1)=x_i(t)+v_i(t+1). \tag{14}$$

At equilibrium point $p^g(t)=p^l(t)=p$ is time invariant. Hence, at equilibrium, dropping the subscript,(13) reduces to

$$v(t+1)=\omega v(t)+\alpha(t)(p-x(t)) \tag{15}$$

$$x(t+1)=x(t)+v(t+1), \tag{16}$$

where, $\alpha(t)=\alpha^l(t)+\alpha^g(t)$, which satisfy $0<\alpha(t)<K$ and $K=c_1+c_2$, c_1,c_2 are constants called acceleration coefficients. Hence

$$x(t)=x(t)+\omega v(t)+\alpha(t)(p-x(t)). \tag{17}$$

Then the dynamics can be represented in state matrix form as (18)

$$\begin{pmatrix}x(t+1)\\ v(t+1)\end{pmatrix}=\begin{pmatrix}1 & \omega\\ 0 & \omega\end{pmatrix}\begin{pmatrix}x(t)\\ v(t)\end{pmatrix}+\begin{pmatrix}1\\ 1\end{pmatrix}u_t \tag{18}$$

$$y_t=\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}x(t)\\ v(t)\end{pmatrix}, \tag{19}$$

with $u_t=-\alpha(t)(y_t-p)$.

Then by using Lyapunovs' stability analysis

1. For $|\omega|<1$ and $\omega\neq 0$,
2. $K<\left(\dfrac{2(1-2|\omega|+\omega^2)}{1+\omega}\right)$
3. For $\omega>0$, it reduces to $K<(2(1-\omega)^2)/(1+\omega)$
4. For $\omega<0$, it reduces to $K<2(1+\omega)$.

Violation of these conditions does not imply instability, but stability cannot be guaranteed.

In the year 2009, Poli [8], analyzes stochastically the PSO dynamics order-1 and order-2 stability by considering convergence analysis.

During stagnation each dimension of the PSO dynamics becomes independent and the dynamics in one dimension is given by (20) [8],

$$v_i(t+1)=\omega v_i(t)+\alpha^l(t)(p_i^l(t)-x_i(t))+\alpha^g(t)(p^g(t)-x_i(t)) \tag{20}$$

$$x_i(t+1)=x_i(t)+v_i(t+1) \tag{21}$$

where, $\alpha^l(t)$ and $\alpha^g(t)$ are constants whose elements are random numbers uniformly distributed in $[0,c_i]$. New random constants are drawn for each particle i and iteration t.During stagnation there is no fitness improvement, so the superscript i can be dropped.Now, from (21), $v(t)=x(t)-x(t-1)$ then we have

$$x(t+1)=x(t)(1+\omega)-x(t)(\alpha^l(t)+\alpha^g(t))-\omega x(t-1)$$
$$+\alpha^l(t)p^l(t)+\alpha^g(t)p^g(t).$$

By using the expectation operator E, the stability of the dynamic particle can be studies as follows:

$$E[x(t+1)]=(1+\omega)E[x(t)]-E[x(t)](E[\alpha^l(t)]+E[\alpha^g(t)])$$
$$-\omega E[x(t-1)]+E[\alpha^l(t)]p^l(t)+E[\alpha^g(t)]p^g(t).$$

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Since $\alpha_i(t)$ is uniformly distributed in $[0, c_i]$, we have $E[\alpha^l(t)] = \frac{c_1}{2}$, and $E[\alpha^g(t)] = \frac{c_2}{2}$ then,

$$E[x(t+1)] = E[x(t)]\left(1+\omega-\frac{c_1+c_2}{2}\right) - \omega E[x(t-1)]$$

$$+\frac{c_1}{2}p^l(t) + \frac{c_2}{2}p^g(t).$$

Considering p as a fixed point for this equation, we obtain

$$+\ p = \frac{c_1 p^l(t) + c_2 p^g(t)}{c_1 + c_2}.$$

For the sake of simplicity let us consider the case $c_1 = c_2 = c$, and $(1+\omega) = \omega'$. Then,

$$E[x(t+1)] = E[x(t)](\omega' - c) - \omega E[x(t-1)] + c\,\frac{p^l(t) + p^g(t)}{2}.$$

(22)

Then, the eigen-values of the above first order difference equation lie in the stable region. Similarly, for second order stable, the magnitude of the eigen-values of the system M must be less than 1. Where M is the system matrix of

$$z(t+1) = Mz(t) + b$$

where,

$$z(t) = \left(E[x(t)] \quad E[x(t-1)] \quad E[x^2(t)] \quad E[x(t)x(t-1)] \quad E[x^2(t-1)]\right)^T.$$

(23)

Poli analyzed the following lines for convergence [8]:

1. For a particle to converge, it requires

   $\lim_{t\to\infty} E[x(t)] = p$ and

   $\lim_{t\to\infty} StdDev[x(t)] = 0.$

2. The value of $\lim_{t\to\infty} StdDev[x(t)] = 0$ will be zero only if $p^l(t) = p^g(t).$

3. So, at $\lim_{t\to\infty} StdDev[x(t)] = 0.$, if the PSO is not an optimizer, still effectively a conjecture.

Recently, in 2010 Samal et al. [9], analyze the parameter selection by Jury's stability analysis and root locus technique.

By considering $\bar{x}(t) = p^g(t) - x_t$, the velocity update equation (1) reduces to

$$v(t+1) = \omega v(t) + \alpha(t)\bar{x}(t) + \alpha^l(t)(p^l(t) - p^g(t)).$$

Similarly, the (2) reduces to

$$\bar{x}(t+1) = (1-\alpha(t))\bar{x}(t) - \omega v(t) - \alpha^l(t)(p^l(t) - p^g(t)).$$

It can be represented in vector-matrix (state-space) form as in classical discrete control theory (Kuo, 1992) [10], given by

$X(t+1) = AX(t) + BU(t)$, is

$$\begin{pmatrix}\bar{x}(t+1)\\ v(t+1)\end{pmatrix} = \begin{pmatrix}1-\alpha(t) & -\omega\\ \alpha(t) & \omega\end{pmatrix}\begin{pmatrix}\bar{x}(t)\\ v(t)\end{pmatrix} + \begin{pmatrix}\alpha^l(t)\\ -\alpha^l(t)\end{pmatrix}\left(p^g(t) - p^l(t)\right)$$

(24)

Considering a closed loop system of the PSO dynamics, the transfer function of the system, using the theory of state equations (Shi and Eberhart, 1998) [11], in the form $G(z) = C(zI - A)^{-1}B$ which gives

$$G(z) = \frac{z\alpha^l(t)}{z^2 + z(\alpha_t - \omega - 1) + \omega}. \qquad (25)$$

Jury's test (Kuo, 1992) is a well-known method to test stability of a closed loop system from its characteristic equationGiven a characteristic equation of the form of a polynomial of z,

$$F(z) = a_n z^n + a_{n-1}z^{n-1} + \ldots\ldots\ldots\ldots + a_2 z^2 + a_1 z^1 + a_0 = 0,$$

where $a_0, a_1, \ldots\ldots\ldots\ldots\ldots, a_n$ are the real co-efficients, , the necessary and sufficient conditions for the polynomial $F(z)$ to have no roots on and outside the unit circle in the z-plane to ensure stability, as indicated below.

Necessary conditions:1. $F(1) > 0$, 2. $(-1)^n F(-1) > 0$,

Sufficient condition: $|a_0| < a_n$, for $n = 2$.

In root locus analysis, the trajectories of the roots of the *characteristic equation* are considered by varying the gain from zero to infinity. These trajectories, known as the locus of the roots (or root locus) can be used to determine the range of parameters of a closed loop system for guaranteed stability.

By using the above two stability analysis the selected range of parameters are given below:

(i) $\omega < 1$, (ii) $0 < \alpha^g(t) < 2(\omega+1)$,

(iii) $2\alpha^l(t) + \alpha^g(t) < 2(\omega+1)$,

(iv) $(1-\omega) < \alpha^l(t) < (1+\omega)$ and

(v) $(1+\omega) < (\alpha^l(t) + \alpha^g(t)) < (1+\sqrt{\omega})^2$ .Substitution of condition (iv) in (v) yields:(vi) $2\omega \le \alpha^g(t) \le 2\sqrt{\omega}$.

Since 1995, the PSO is analyzed by deterministically and stochastically by so many researchers for faster convergence at the global minima. The details of the evolution and analysis are summarized in Table 1 and Table 2. In Table 1, the developments of PSO are given in a chronological order,

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whereas in Table 2, different methods of analyses and selection of parameters are summarized.

Table 1: Development of PSO

Researchers name and year	Developments of PSO	Remarks
Eberhart and Kennedy,1995 [2]	PSO with no inertia factor	Optimization of non-linear functions using PSO is introduced.
Shi and Eberhart, 1998 [3]	Introduction of inertia factor	Reaches the global minima with minimum iteration of time
Angeline, 1998 [12]	Introduction of hybrid swarm.	Optimize quickly but with a limiting search space.
Clerc, 1999 [13]	Introduction of Constriction factor	He defined a Swarm and Queen method with “no-hope” and “re-hope” convergence criterion. So that, the swarm re-initializes its position time to time.
Suganthan, 1999 [14]	Dynamic neighborhood	Dynamically increasing local neighborhood operator and *LBEST* were discussed.
Clerc, and van Den Bergh, 2002 [15]	Parameter selection by stability analysis	By the stable analysis, the inertia factor should be $\omega < 1$.
Kennedy and Mendes, 2002 [16]	Topological structuring in the neighborhood	A good solution is obtained but the process is slow.
CoelloCoello, Lechuga, 2002 [17]	Extend the heuristic called, Multi-objective PSO	The generation of non-dominated vectors and the mechanism to maintain diversity.

Table 2: Methods of analysis and parameter selections on PSO

Researchers name	Methods of analysis	Remarks on parameters
Shi and Eberhart (1999) [18]	Simulations of global and local exploration depends upon ω	Analyze the convergence as if $v_i^{max} \le 2,\ \omega = 1$ $v_i^{max} \ge 3,\ \omega = 0.8$, $\omega = [0.9, 1.2]$
Clerc and Kennedy (2002) [4]	The particles’ trajectory in discrete time and then progress it in continuous time.	Given the suitable range of parameter for stability, $\alpha^l(t) + \alpha^g(t) = \alpha(t) > 4$, where, with $\omega = 1$ (fixed).
Trela (2002) [5]	Eigen value analysis of PSO system state equation	The parametric condition for stability $\omega < 1$, $\alpha^l(t) + \alpha^g(t) > 0$, $2\omega - (\alpha^l(t) + \alpha^g(t)) + 2 > 0$. Designs strategy for balancing exploration and exploitation.
Kadirkam anathan (2006) [7]	Stability analysis of Stochastic particle dynamics using Lyapunov analysis.	Sufficient condition for asymptotic stability. $K < \left(\frac{2(1 - 2\lvert\omega\rvert + \omega^2)}{1 + \omega} \right)$ Violation of this condition, however, does not lead to instability.
Poli (2009) [8]	Stochastic order 1 and order 2 stability analysis of PSO dynamics by considering convergence. $\lim_{t\to\infty} x(t = p^g(t)$ and $\lim_{t\to\infty} StdDev[x(t)] \to 0$	Analysis confirms that eigen-values of the PSO dynamics do not depend on the location of $p^l(t)$ and $p^g(t)$ in the search space. So, convergence of PSO at $p^g(t)$ with optimizing $\lim_{t\to\infty} StdDev[x(t)] \to 0$ remains a conjecture.
Samal et al. (2010) [9]	Stability and Convergence analysis of PSO dynamics by control theoretic approach.	The recommended guidelines in parameter selection for a *gbest* PSO program is $\alpha^l(t)$ in $[(1-\omega), 1]$ $\alpha^g(t)$ in $[2\omega, (2\omega + \theta)]$, where $\theta < 2(1 - \alpha^l(t))$, ω randomly in [0.17, 0.25].

III. Conclusions

This paper surveys the research and development of the PSO algorithm with different environments. During the last dacade, it found considerable interest from the natural computing research community and has been seen to offer rapid and effective optimization of complex multidimensional search spaces, with adaptations to multi-objectives and constrained optimization.

IV. Future Directions

On the other hand, the search process of a PSO algorithm should be a process consisted of both contraction and expansion, so that, it could have the ability to escape from local minima, and eventually find good enough solutions. A mathematical foundation of PSO is in need to have a deep understanding of the dynamic process of PSO. There is also a need of a unique representation of the PSO topology, so that researchers can duplicate each other's work and compare their work with the others. Challenges

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remain, in areas such as dynamic environments, avoiding stagnation, handling constraints and multiple objectives. Like evolutionary algorithms, PSO has become an important tool for optimization and other complex problem solving. The next decade will no doubt see further refinement of the approach and integration with other techniques, as well as large applications moving out of the research laboratory and into industry and commerce. Further understanding of the relative strengths of PSO and other techniques, and the challenges in deploying a PSO based system are required. However, to the optimization toolbox PSO is certainly welcome as a better addition.

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