International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-02, May 2018

Circle related reverse-graphoidal Magic strength

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Abstract Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G .The magic labelling f of G, there is a constant $c(f)$ such that $f(x) + f(y) + f(xy)$, , for every edge $xy \in E(G)$. The magic strength of G is defined as $m\,(G) = min\,\{c\,(f)$: if f is a magic labeling of $G\}$.In this paper we determine reverse process of graphoidal of a magic strength is called reverse- graphoidal magic strength and proved reverse - graphoidal magic strength of C_n, Parachute $W_{n,2}$, Armed Crown $C_n \Theta P_n$, graph $K_{1,n} \times K_2$.

Keywords: Graphoidal Constant, Magic Graphoidal, Magic Srength, reverse magic graphoidal, reverse- grahoidal magic strength.

I. INTRODUCTION

B.D. Acharya and E. Sampath Kumar [1] defined graphoidal covering of graph. Selvam,Vasuki, Jeyanthi [9] introduced the concept of magic strength of a graph .

A graph G is said to be magic if there exist a bijection $f: V \cup E \rightarrow \{1,2,3 \ldots \ldots m+n\}$; where '$n$' is the number of vertices and 'm' is the number of edges of a graph. Such that for all edges xy, $f(x) + f(y) + f(xy)$ is a constant. Such a bijection is called a magic labeling of G. Let P be a path $\{v_1, v_2, \ldots \ldots, v_n\}$ in ψ with $f^*(P) = f(v_1) + f(v_n) + \sum_{i=1}^{n-1} f(v_i v_{i+1}) = k$ is a constant, where f^* is the induced labeling on ψ. . Then, we say that G admits ψ - magic graphoidal total labeling of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labelling of G.

Here combination of graphoidal and magic strength we introduced a new concept (ie. Reverse) process of graphoidal magic strength is called ***reverse- graphoidal magic strength***.

Definition 1.1

The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges

Definition1.2

A ***Cycle*** C_n is a closed path of length atleast 1 with n vertices

Definition 1.3

An ***Armed crown*** $C_n \theta P_n$ is a graph obtained from a cycle C_n by attaching a path P_n at each vertex of C_n.

Definition 1.4

The ***Direct product*** $K_{1,n} \times K_2$,whose vertex set is $V(K_{1,n}) \times V(K_2)$ and for which vertices (x, y) and (x', y') are adjacent precisely if $(xx') \in E(K_{1,n})$ and $(yy') \in E(K_2)$.

II.MAIN RESULTS

Definition 2. 1

A reverse magic graphoidal labeling of a graph G is one-to-one map f from $V(G) \cup E(G) \rightarrow \{1,2,3, \ldots \ldots \ldots, m+n\}$where '$n$' is the number of vertices of a graph and 'm' is the number of the edges of a graph, with the property that , there is an integer constant 'μ'_{rmgc} such that

$f^*(P) = \sum_{i=1}^{n-1} f(v_i\, v_{i+1}) - \{f(v_1) + f(v_n)\} = \mu_{rmgc}$, is a contant

Then the reverse methodology of magic graphoidal labeling is called reverse- magic graphoidal labeling (rmgl). Reverse process of magic graphoidal of a graph is called reverse-magic graphoidal graph.(rmgg).

Selvam and Vasuki [9] made a note, Let f be a magic labeling of G with constant $c\,(f)$.Then adding all the constant obtained at each edge. We have

$$\mathcal{E}\, c(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$$

From the above equation we introduce the concept of reverse process of graphoidal of a magic strength and it is called ***reverse- graphoidal magic strength*** and it is denoted as $rgms(G)$, is defined as the minimum of all μ_{rmgc} where the minimum is taken over all reverse magic graphoidal total labeling f of (G).

To proceed further, we make the following equation.

Let f be a reverse magic graphoidal labeling of G with the constant μ_{rmgc}. Then ,adding all constant obtained at each edge, we get

$$rgms\,(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

 DOI : 10.18231/2454-9150.2018.0183

Theorem 2.1

$rgms\,(C_n) = \dfrac{n\,(n-7)}{2}$

Proof:

Let $V(G) = \{v_1, v_2, \ldots\ldots\ldots\ldots\ldots, v_n\}$

and $E\,(G) = \{(v_1v_2), (v_2v_3), (v_3v_4), \ldots\ldots, (v_{n-1}v_n), (v_nv_1)\}$

Here $m + n = 2n$

Define $f : V \cup E \rightarrow \{1,2\ldots\ldots m+n\}$ by

$f(v_1) = 2n$

$f(v_1v_2) = 1, f(v_2v_3) = 2, \ldots\ldots\ldots\ldots, f(v_{n-1}v_n) = n-1, f(v_nv_1) = n$

Let $\psi = \{P = (v_1v_2), (v_2v_3), (v_3v_4), \ldots\ldots\ldots, (v_{n-1}v_n), (v_nv_1)\}$

We have the equation,

$$\mu_{rmgc}(f) = \sum_e f(e) - \left[\sum_v d(v).f(v)\right]$$

Then the equation becomes,

$$= 1 + 2 + \ldots\ldots\ldots\ldots\ldots + n - \{2 \times 2n\}$$

$$= \sum_{i=1}^{n} i - \{2 \times 2n\}$$

$$= \frac{n\,(n+1)}{2} - \{2 \times 2n\} = \frac{n\,(n+1) - 8n}{2}$$

$$= \frac{n^2 + n - 8n}{2}$$

$$= \frac{n^2 - 7n}{2} = \frac{n\,(n-7)}{2} \qquad (1)$$

From(1),we conclude that

$$\mu_{rmgc}(f) = \frac{n(n-7)}{2}$$

$$\therefore\ rgms\,(f) = \frac{n(n-7)}{2}$$

Theorem 2.2

$rgms\,(W_{n,2}) = \left\{\dfrac{n^2 + 3n + 2}{2}\right\}$

Proof :

Let $V(G) = \{u_1, u_2, \ldots\ldots\ldots\ldots, u_{n+1}\}$

and $E(G) = \{(u_1u_2)(u_3u_4)\ldots\ldots\ldots\ldots, (u_n\,u_{n+1}), (u_{n+1}u_1), (u_1u_3)\}$

Here $m + n = 2n + 3$

Define $f : V \cup E \rightarrow \{1,2\ldots\ldots\ldots \mathrm{m+n}\}$ by

$f\,(u_1) = 1$

$f\,(u_3) = 2n + 3$

$f\,(u_1u_2) = 2,\quad f\,(u_2u_3) = 3, f\,(u_3u_4) = 4, \ldots\ldots\ldots\ldots, f\,(u_nu_{n+1}) = n + 1$

$f(u_{n+1}u_1) = n + 2$

$f(u_1u_3) = n + 3$

Let $\psi = \{P = (u_1u_2),\ (u_2u_3), \ldots\ldots\ldots\ldots, (u_{n+1}u_1), (u_1u_3)\}$

We have the equation

$$\mu_{rmgc}(f) = \sum_e f(e) - \left[\sum_v d(v).f(v)\right]$$

$$\mu_{rmgc}f(P) = f\,(u_1u_2) + f\,(u_2u_3) + f\,(u_3u_4) + \cdots\ldots\ldots\ldots + f\,(u_nu_{n+1}) + f\,(u_{n+1}u_1) + f(u_1u_3) - \{1 \times f\,(u_1) + 1 \times f(u_3)\}$$

$= 2 + 3 + 4 + \ldots\ldots n+1 + n+2 + n+3 - \{1 \times 1 + 1 \times (2n+3)\}$

$$= \frac{n^2 + 7n + 12 - 2}{2} - (2n+4)$$

$$= \frac{n^2 + 7n + 10 - 4n - 8}{2}$$

 DOI : 10.18231/2454-9150.2018.0183

$$= \frac{n^2 + 3n + 2}{2} \quad \underline{\hspace{6cm}}(1)$$

From equation (1) we conclude that

$$\mu_{rmgc}(W_{n,2}) = \frac{n^2 + 3n + 2}{2}$$
$$\therefore rgms\ (W_{n,2}) = \frac{n^2 + 3n + 2}{2}$$

Theorem 2.3

$rgms(K_{1,n} \times K_2) = -(5n + 4)$

Proof:

Let $V(G) = \{a, b, a_1, a_2, \ldots\ldots, a_n, b_1, b_2, \ldots., b_n\}$

And $E(G) = \{(ab), (aa_1), (aa_2), \ldots., (aa_n), (bb_1), (bb_2) \ldots., (bb_n), (a_1b_1), (a_2b_2), \ldots., (a_nb_n)\}$

Here $m + n = 5n + 3$

Define $f: V \cup E \rightarrow \{1, 2 \ldots \ldots \ldots \ldots \ldots, m + n\}$ by

$f(a) = 5n + 2$

$f(b) = 5n + 3$

$f(ab) = 5n + 1$

$f(aa_1) = 1, f(aa_2) = 2, f(aa_3) = 3, \ldots \ldots \ldots, f(aa_n) = n$

$f(a_1b_1) = n + 1, f(a_2b_2) = n + 2, f(a_3b_3) = n + 3, \ldots\ldots\ldots, f(a_nb_n) = 2n$

$f(bb_1) = 4n - 1, f(bb_2) = 4n - 3, f(bb_3) = 4n - 5, \ldots\ldots\ldots, f(bb_n) = 2n + 1$

Let $\psi = \{ P_1 = [(aa_1b_1b), (aa_2b_2b), \ldots \ldots \ldots, (aa_nb_n b)]$

$P_2 = [ab]\}$

We have the equation

$$\mu_{rmgc}(f) = \sum_e f(e) - \left[\sum_v d(v).f(v)\right]$$

Then the equation becomes,

$\mu_{rmgc} f(P_{1(1)}) = f(aa_1) + f(a_1b_1) + f(b_1b) - \{1 \times f(a) + 1 \times f(b)\}$

$= 1 + n + 1 + 4n - 1 - \{1 \times (5n + 2) + 1 \times (5n + 3)\}$

$= 5n + 1 - \{5n + 2 + 5n + 3\}$

$= -(5n + 4)$ ________________________(1)

$\mu_{rmgc} f(P_{1(2)}) = f(aa_2) + f(a_2b_2) + f(b_2b) - \{1 \times f(a) + 1 \times f(b)\}$

$= 2 + n + 2 + 4n - 3 - (5n + 2) + (5n + 3)$

$= -(5n + 4)$ ________________________(2)

Continuing this process,

$\mu_{rmgc} f(P_{1(k)}) = f(aa_n) + f(a_nb_n) + f(b_nb) - \{1 \times f(a) + 1 \times f(b)\}$

$= n + 2n + 2n + 1 - \{1 \times (5n + 2) + 1 \times (5n + 3)\}$

$= 5n + 1 - (10n + 5)$

$= -(5n + 4)$ ________________________(3)

$\mu_{rmgc} f(P_2) = f(ab) - \{1 \times f(a) + 1 \times f(b)\}$

$= 5n + 1 - \{1 \times (5n + 2) + 1 \times (5n + 3)\}$

$= 5n + 1 - (10n + 5)$

$= -(5n + 4)$ ________________________(4)

From (1), (2), (3) and (4) we conclude that

$$\mu_{rmgc}(K_{1,n} \times K_2) = -(5n + 4)$$
$$\therefore rgms\ (K_{1,n} \times K_2) = -(5n + 4)$$

Theorem 2.4

$rgms\ (C_n\ \theta\ P_n) = \dfrac{(n^3 - n^2 - 4)}{2}$

Proof:

 DOI : 10.18231/2454-9150.2018.0183

ISSN : 2454-9150 Vol-04, Issue-02, May 2018

Let G be armed crown. The vertex sets are $\{u_1, u_2, u_3, \ldots \ldots . u_{n^2}\}$ and edge sets are

$\{u_1u_2), (u_2u_{n+1}), (u_{n+1}u_{2n+1}), \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (u_{n(n-2)+1}u_{n(n-1)+1})\}$

$(u_2, u_3), (u_3, u_{n+2}), (u_{n+2}, u_{2n+1}), \ldots \ldots \ldots \ldots . \ldots . . (u_{n(n-2)+2}, u_{n(n-1)+2})\}$

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Here, $m + n = 2n^2$

Define $f: V \cup E \rightarrow \{1,2, \ldots \ldots , 2n^2\}$ by

$f(u_1) = 1 ; \ f(u_2) = 2, f(u_3) = 3, \ldots \ldots \ldots \ldots , f(u_{n-1}) = n - 1, f(u_n) = n$

$f(u_{n(n-1)+1}) = 2n^2, f(u_{n(n-1)+2}) = 2n^2 - 1, f(u_{n(n-1)+3}) = 2n^2 - 2, \ldots \ldots \ldots ,$

$f(u_{n^2-1}) = 2n^2 - n + 2$

$f(u_{n^2}) = 2n^2 - n + 1$

$f(u_1u_2) = n + 1, \quad f(u_2u_3) = n + 2, \ldots \ldots . . \ldots \ldots , f(u_{n-1}u_n) = 2n - 1$

$f(u_2u_{n+1} = 2n + 1, \quad f(u_3u_{n+2}) = 2n + 2, \ldots \ldots . . \ldots \ldots , f(u_nu_{2n-1}) = 3n - 1$

$f(u_{n+1}u_{2n+1}) = 3n + 1, . . f(u_{n+2}u_{2n+2}) = 3n + 2, \ldots \ldots . . \ldots \ldots , f(u_{2n}u_{3n-1}) = 4n - 1$

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$f(u_{n(n-2)+1}u_{n(n-1)+1}) = n(2n - 1), f(u_{n(n-2)+2}u_{n(n-1)+2}) = 2n(n - 1), \ldots \ldots \ldots .,$

$f(u_{n(n-1)-1}u_{n^2-1}) = n^2 + 2n - 2$

$f(u_nu_1) = 2n , f(u_1u_{2n}) = 3n, f(u_{2n}u_{3n}) = 4n, \ldots \ldots . . \ldots \ldots , f(u_{n(n-1)})u_{n^2}) = n^2 + n - 1$

Let $\psi = P_I = (u_1u_2) \cup (u_2u_{n+1}) \cup (u_{n+1}u_{2n+1}) \cup, \ldots \ldots \ldots . . , \cup (u_{n(n-2)+1} \ u_{n(n-1)+1})$

$(u_2u_3) \cup (u_3u_{n+2}) \cup (u_{n+2}u_{2n+2}) \cup, \ldots \ldots \ldots . . , \cup (u_{\,n(n-2)+2}u_{n(n-1)+2})$

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$(u_{n-1}u_n) \cup (u_nu_{2n-1}) \cup (u_{2n}u_{3n-1}) \cup, \ldots \ldots \ldots . . , \cup (u_{n^2-n-1}u_{n^2-1})\}$

$P_2 = (u_nu_1) \cup (u_1u_{2n}) \cup (u_{2n}u_{3n}) \cup, \ldots \ldots \ldots . . , \cup (u_{n(n-1)})u_{n^2})\}$

And we have the equation

$$\mu_{rmgc}(f) = \sum_e f(e) - \left[\sum_v d(v).f(v)\right]$$

Then the equation becomes,

$$\mu_{rmgc} f\left(P_{1(1)}\right) = f(u_1u_2) + f(u_2u_{n+1}) + f(u_{n+1}u_{2n+1}) + \cdots \ldots \ldots \ldots \ldots . . + f(u_{n(n-2)+1})f(u_{n(n-1)+1}) - \{1 \times f(u_1) + 1 \times f(u_{n(n-1)+1})\}$$

$$= n + 1 + 2n + 1 + 3n + 1 + (n - 1)n + \cdots \ldots . . + n(2n - 1) - \{1 \times 1 + 2n^2 \times 1\}$$

$$= n + 2n + 3n + . \ldots \ldots . . + n(n - 1) + n(2n - 1) + (n - 1) - \{1 + 2n^2\}$$

$$= n(1 + 2 + 3 + \cdots \ldots . . + n - 1) + 2n^2 - n + (n - 1) - \{1 + 2n^2\}$$

$$= \frac{n(n(n-1))}{2} + 2n^2 - n + n - 1 - 1 - 2n^2$$

$$= \frac{n^2(n-1)}{2} - 2$$

$$= \frac{n^3 - n^2 - 4}{2} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)$$

$$\mu_{rmgc} f\left(P_{1(2)}\right) = f(u_2u_3) + f(u_3u_{n+2}) + \{f(u_{n+2}u_{2n+2}) + \ldots . . + f(u_{n(n-2)+2}u_{n(n-1)+2}) - \{1 \times f(u_2) + 1 \times f(u_{n(n-1)+2}\}$$

$$= n + 2 + 2n + 2 + 3n + 2 + \cdots \ldots . . + 2n(n - 1) + 1 - \{1 \times 2 + 1 \times (2n^2 - 1)\}$$

$$= [n + 2n + 3n + . \ldots \ldots . . + n(n - 1)] + 2n^2 - 2n + 1) + 2(n - 1) - \{2 + 2n^2 - 1\}$$

$$= \frac{n(n(n-1))}{2} + 2n^2 - 2n + 1 + 2n - 2 - 2n^2 - 1$$

$$= \frac{n^2(n-1)}{2} - 2$$

DOI : 10.18231/2454-9150.2018.0183

$$= \frac{n^3 - n^2 - 4}{2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

Continuing this process,

$$\mu_{rmgc} f\,(P_{1(k)}) = f(u_{n-1}u_n) + f(u_n u_{2n-1}) + \{ f(u_{2n}u_{3n-1}) + \ldots.. +$$

$$f(u_{n(n-1)-1})\, f(u_{n^2-1}) - \{1 \times f(u_{n-1}) + 1 \times f(u_{n^2-1})\}$$

$$= [2n - 1 + 3n - 1 + 4n - 1 + \cdots \ldots.. + n^2 + 2n - 2] - \{1 \times (n-1) + 1 \times 1(2n^2 - n + 2\}$$

$$= n[1 + 2 + \cdots \ldots.. + (n-1)] + n^2 - 2n + 1 + n^2 + 2n - 2 - 2n^2 - 1$$

$$= \frac{n(n(n-1))}{2} - 2$$

$$= \frac{n^3 - n^2 - 4}{4} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(3)$$

Again,

$$\mu_{rmgc} f\,(P_2) = f(u_n u_1) + f(u_1 u_{2n}) + f(u_{2n}u_{3n}) + \ldots.. + f\big(u_{n(n-1)}u_{n^2}\big) -$$

$$\{1 \times f(u_n) + 1 \times f(u_{n^2}\}$$

$$= 2n + 3n + 4n + \cdots \ldots.. + n^2 + n - 1 - \{1 \times n + 1 \times (2n^2 - n + 1)\}$$

$$= n + n + n + 2n + n + 3n + \ldots \ldots.. + n(n-1)n + \cdots \ldots.. + n^2 + n - 1 - \{n + 2n^2 - n + 1\}$$

$$= \frac{n(n(n-1))}{2} + n^2 - n + n^2 + n - 1 - (2n^2 + 1)$$

$$= \frac{n^3 - n^2 - 4}{2} = \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(4)$$

from the above equation (1), (2), (3), and (4) we conclude that

$$\mu_{rmgc}\,(C_n\,\theta\,P_n) = \frac{n^3 - n^2 - 4}{2}$$

$$\therefore rgms\,(C_n\,\theta\,P_n) = \frac{n^3 - n^2 - 4}{2}$$

III. CONCLUSION

The magic strength of a graph is one the most interesting area in graph theory. As all the graphs reverse techniques of magic strength is very interesting to investigate graphs or graph families which admit reverse- graphoidal magic strength. Here we reporting reverse- graphoidal magic strength of various graphs

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