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Markov Chain Analysis of Improved Round Robin CPU Scheduling Algorithm

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Abstract - **CPU scheduling is a fundamental operating system function that determines which of the process should be executed next when multiple run-able process is waiting in the ready queue. Round Robin scheduling algorithm is found efficient in case of time sharing system however an improved version of traditional RR algorithm had been given that provides priority to processes that are near to completion. This improved RR policy reduces the average waiting time and increases the throughput and maintains the same level of CPU utilization like traditional RR provides. In the proposed paper a Markov chain analysis is done in order to determine the performance of this suggested improved round robin algorithm. We have also proposed some other others ways to assign the scheduler to the next ready process. These efforts have found very efficient and useful. Further some numerical studies have been done to justify the proposed suggestions.**

Keywords* —*CPU Scheduling, Improved Round Robin Algorithm, Markov Chain Analysis, Round Robin Algorithm,

I. INTRODUCTION

Multiprogramming is one of the most important characteristics of operating systems. It requires several programs to be kept simultaneously in memory, the aim of which is maximum CPU utilization. The CPU scheduling decides which one among them to run first. Making this decision is CPU scheduling. CPU scheduling is the fundamental of multiprogramming systems. It mentions to a set of policies and mechanisms to control the order of work to be performed by an operating system, It is called the scheduler, using a CPU scheduling algorithm [4].

Scheduling algorithms are used for distributing resources among users which simultaneously and asynchronously request them. The main purposes of scheduling algorithms are to minimize resource starvation, to ensure fairness amongst the users utilizing the resources and to keep the CPU busy as much as possible by executing a (user) process and then switching to another process. Scheduling deals with the problem of deciding which of the outstanding requests is to be allocated resources.

The CPU scheduler executes the processes when they schedule on it. When there are number of processes in the ready queue, the algorithm which decides the order of execution of those processes is called *scheduling algorithm.* The various well-known CPU scheduling algorithms are First Come First Serve (FCFS), Shortest Job First (SJF), Shortest Remaining Time (SRT), Round-Robin (RR), Multi-Level Queue Scheduling (MLQ), Earliest Deadline First (EDF), and Priority scheduling algorithms. All the above algorithms are preemptive non-preemptive in nature. Shortest Remaining Time First (SRTF) and Round Robin (RR) are preemptive in nature. RR is most suitable for time sharing systems [15], [16], [17], [19].

The performance of all these scheduling algorithms are evaluated on the basis of criteria's that seems more important for the system. Some of the general criteria's are like CPU utilization, throughput, turnaround time, response time etc. It is recommended [4], [6], [9] that good scheduling algorithm must possess following characteristics:

- Minimum context switches.
- Maximum CPU utilization.
- Maximum throughput.
- Minimum turnaround time.
- Minimum waiting time.
- Minimum response time.

II. RELATED WORK

To carry out the proposed review work some of the studies are discussed, which had been previously undertaken in the field of Round Robin CPU Scheduling algorithm. Performed one scheduling scheme which is the mixture of FIFO and RR is found efficient in terms of model-based study using Markov chain model [8].

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Presented a general structure of transition scenario for the functioning of CPU scheduler in the presence of deadlock condition [10]. A new substitute of RR scheduling algorithm which is suitable for time shared systems, performed study to improve the RR algorithm using dynamic intelligent time slice and shortest remaining time next algorithm joint together to reduce the average waiting time, average turnaround time and the number of context switches [23], [1]. Researcher worked on existing round robin scheme to reduce the total waiting time of an any process which is spend in a ready queue and improve the performance of existing round robin algorithm to understand this waiting time difference using mathematical calculation [2]. Study about various RR algorithm and proposed a new improved RR algorithm; Shortest Remaining Burst Round Robin (SRBRR) by assigning the processor to processes with shortest remaining burst in round robin manner using the dynamic time quantum and also used same approaches to increase the performance of Shortest Remaining Burst Round Robin (SRBRR) scheduling algorithm and compare with different RR scheduling algorithms [24], [3]. described an improvement in RR; through preparing a simulator program and tested improved RR. After testing it has been found that the waiting time and turnaround time have been reduced drastically [25]. Proposed and enhanced a new round robin algorithm and also compare some other related algorithms and study priority based round robin CPU scheduling algorithm, it retains the advantage of round robin in reducing starvation and also integrates the advantage of priority scheduling [4], [6].

Presented a new priority driven scheduling algorithm based on burst time of processes which is reduces average waiting time, turnaround time, context switches and throughput of the simple round robin scheduling algorithm [7]. Developed a new RR algorithm which help to improve the CPU efficiency in real time and timesharing operating system. The proposed algorithm improves the drawback (context switch, average turnaround time, waiting time, etc.) of simple RR algorithm [9]. Compared an improved RR scheduling algorithm, with joining the two-scheduling algorithm (shortest job first and simple RR) [20]. Presented an improved RR CPU scheduling algorithm coined enhancing CPU performance using the features of SJF and RR scheduling with varying time quantum. The proposed algorithm is experimentally proven better than conventional RR [22].

The set of possible values of an individual random variable $X^{(n)}$ (or X(t)) of a stochastic process $\{X^{(n)}, n\geq1\}$, $\{X(t), t\in T\}$ is known as state space, The stochastic process $\{X^{(n)}, n=0,1,2\ldots\}$ is called Markov chain, if, for j, k, j1, … , j(n-1) € N (or any subset of I),

Medhi have given an elaborate study of a variety of stochastic processes and their applications in various fields and developed a Markov chain model for the study of uncertain rainfall phenomenon and also presented the use of stochastic process in the management of queues [26], [11], [26]. Naldi presented a Markov chain model for understanding the internet traffic sharing among various operators in a competitive market [5]. Researcher studied the use of Markov chain model for multilevel queue scheduler and also designed a scheduling scheme and compare through numerical based study [13], [14]. Proposed a linear data model-based study of improved RR CPU Scheduling algorithm with features of shortest job first scheduling with varying time quantum by using Markov chain model with different data set and performed some numerical based study [18], [21].

III. PROPOSED SYSTEM

In RR principle, processes are executed in the order of their arrival. However, unlike FCFS, the processes get only a fixed quantum of CPU time in each round. RR therefore avoids a long wait for first CPU response. A process may thus need several rounds for completion. A major drawback in RR policy is that even if a process is near completion, it is still placed at the rear end of Q1, which not only increases the total waiting time but also lowers the throughput.

In improved RR scheduling policy is combine the basic functions of RR with an improvement towards the priority assigned to the processes nearing completion. In view of (1), it is obvious that the time requirement for completion of a process P_i after $(r_i - 1)^{th}$ round will be at the most one-time quantum. We therefore, consider a priority queue (to be referred as Q2) in addition to the ready queue Q1. An additional queue has been used by Pandey et.al [4] for dispatching priority in context of FCFS scheduling. All processes, after being served by the CPU in penultimate round, are sent to the rear end of Q2 instead of Q1. Thus, the processes which need only one quantum or less will be terminated in the first round itself from Q1, while all others will be terminated on being dispatched from Q2. Therefore, processes going to CPU through Q1, if not terminated, may return back to the rear end of either Q1 or Q2. As shown in Fig. 3.1, this approach organizes the pending requests in two queues. The improved RR scheduling policy assume cycle of three queues (Q1, Q2, Q3) for the purpose of sequential allocation to CPU; it starts with two processes from Q1 one process from Q2 and one process from Q3 (waiting process).

The scheduling policy can further be improved by adopting some different cycle. Precise idea is to appropriately choose a pair of numbers p and q (p>q) that determine the number of processes from Q1 and Q2 for allocation to CPU in the cycle. An optimal choice may however, depend on the number of processes and the size of their CPU bursts. In the present work, we shall confine our discussion to p and q. This policy provides better estimates than the conventional RR policy in respect of all performance measures, including the throughput, without any significant increase in the overheads [2].

Generalized Markov chain models in CPU scheduling

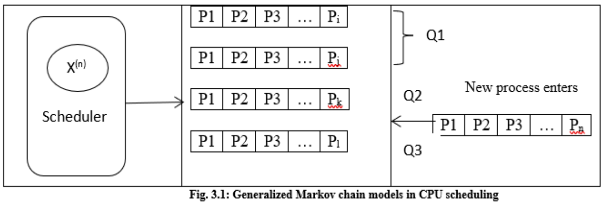

Fig. 3.1: Generalized Markov chain models in CPU Scheduling.

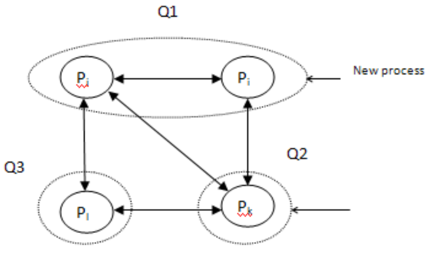

Fig. 3.2: Unrestricted transition diagram

Let $X^{(n)}$, n≥1, be a markov chain where $X^{(n)}$ denotes the state of the scheduling at the quantum of time. The state space for the random variable $X^{(n)}$ is {Q1, Q2, Q3} where Q1 = P_i, P_j are combine process in first queue, Q2 = P_k is second queue and Q3 = P_l is waiting state and scheduler X move stochastically over different processing states and waiting within different quantum of time. And fig. 3.2 shows the transition diagram performing transition from one state to another state according to CPU scheduling algorithm. Unit step transaction probability matrix for $X^{(n)}$ under general model is:

$$P = \begin{array}{c} \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} \\ P_j & S_{21} & S_{22} & S_{23} & S_{24} \\ P_k & S_{31} & S_{32} & S_{33} & S_{34} \\ P_l & S_{41} & S_{42} & S_{43} & S_{44} \end{array}}$$

Predefined selection for initial probabilities of states are:
$P [X^{(n)} = P_i] = P_{r1}$; $P [X^{(n)} = P_j] = P_{r2}$; $P [X^{(n)} = P_k] = P_{r3}$; $P [X^{(n)} = P_l] = 0$ ……… eq 1
Let S_{ij} (i, j = 1, 2, 3,…) be the unit step transition probabilities of scheduler over three states then transition probability depend on subject to condition:

$S_{14} = (1 - \sum_{i=1}^{3} S1i)$; $S_{24} = (1 - \sum_{i=1}^{3} S2i)$; $S_{34} = (1 - \sum_{i=1}^{3} S3i)$; $S_{44} = (1 - \sum_{i=1}^{3} S4i)$; & $0 \leq S_{ij} \leq 1$,

The state probabilities, after the first quantum can be obtained by a simple relationship:

$P [X^{(1)} = P_i] = P [X^{(0)} = P_i] P [X^{(1)} = P_i / X^{(0)} = P_i] + P [X^{(0)} = P_j] P [X^{(1)} = P_i / X^{(0)} = P_j] + P [X^{(0)} = P_k] P [X^{(1)} = P_i / X^{(0)} = P_k] +P [X^{(0)} = P_l] P [X^{(1)} = P_i / X^{(0)} = P_l]$

$P [X^{(1)} = P_i] = \sum_{i=1}^{3} Pri\ Si1$; $P [X^{(1)} = P_j] = \sum_{i=1}^{3} Pri\ Si2$; $P [X^{(1)} = P_k] = \sum_{i=1}^{3} Pri\ Si3$; $P [X^{(1)} = P_l] = \sum_{i=1}^{3} Pri\ Si4$ …………eq. 2

Similarly, state probabilities after second quantum can be obtained by simple relationship:

$P [X^{(2)} = P_i] = P [X^{(1)} = P_i] P [X^{(2)} = P_i / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_i / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_i / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_i / X^{(1)} = P_l]$

$P [X^{(2)} = P_i] = \sum_{i=1}^{4} (\sum_{j=1}^{3} Prj\ Sji) S_{i1}$; $P [X^{(2)} = P_j] = \sum_{i=1}^{4} (\sum_{j=1}^{3} Prj\ Sji) S_{i2}$; $P [X^{(2)} = P_k] = \sum_{i=1}^{4} (\sum_{j=1}^{3} Prj\ Sji) S_{i3}$; $P [X^{(2)} = P_l] = \sum_{i=1}^{4} (\sum_{j=1}^{3} Prj\ Sji) S_{i4}$ …………eq. 3

The generalized expressions for n quantum are:

$P [X^{(n)} = P_i] = \sum_{m=1}^{4} \cdots\cdots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} Prj\ Sji\ Sik\ Skl \ldots\ldots Sm1$;
$P [X^{(n)} = P_j] = \sum_{m=1}^{4} \cdots\cdots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} Prj\ Sji\ Sik\ Skl \ldots\ldots Sm2$;
$P [X^{(n)} = P_k] = \sum_{m=1}^{4} \cdots\cdots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} Prj\ Sji\ Sik\ Skl \ldots\ldots Sm3$;
$P [X^{(n)} = P_l] = \sum_{m=1}^{4} \cdots\cdots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} Prj\ Sji\ Sik\ Skl \ldots\ldots Sm4$ …………eq. 4

IV. SOME IMPROVED RR SCHEDULING SCHEMES

By imposing some restrictions and condition that can produce various scheduling schemes from above mentioned generalized IRR scheme. The three schemes are discussed as follows:

A. Scheme - I

At any stage, after dispatching two processes from Q1, if Q2 is found to be empty, another pair of processes will be dispatched from Q1. When process entry to first queue only – under process entry restriction, the scheme-1 is described in fig. 4.1.

- A new process can only enter to first queue Q1 and after executing the two processes Pi and Pj , if state Q2 (i.e. process Pk) is found to be empty, then another pair of processes (Pi and Pj) will be dispatched from state Q1. Scheduler comes to Q3 only if state Q1 and Q2 are empty.

- Define Q3 = P_l is a waiting state.

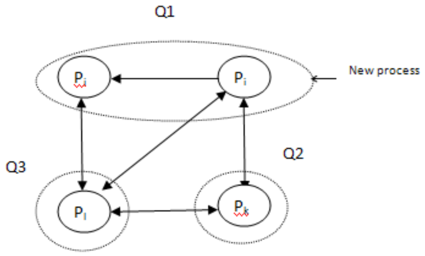

Fig.4.1: Restricted transition diagram

Thus, the initial probabilities under scheme-I are:

$P [X^{(0)} = P_i] = 1$; $P [X^{(0)} = P_j] = 0$; $P [X^{(0)} = P_k] = 0$; $P [X^{(0)} = P_l] = 0$

Unit step transaction probability matrix for $X^{(n)}$ under scheme-1 is:

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$$P = \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|c|c|c|c} & P_i & P_j & P_k & P_l \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} \\ \hline P_j & S_{21} & S_{22} & S_{23} & S_{24} \\ \hline P_k & S_{31} & S_{32} & S_{33} & S_{34} \\ \hline P_l & S_{41} & S_{42} & S_{43} & S_{44} \end{array}} = \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|c|c|c|c} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & S_{12} & S_{13} & S_{14} \\ \hline P_j & 0 & 0 & S_{23} & S_{24} \\ \hline P_k & S_{31} & 0 & 0 & S_{34} \\ \hline P_l & S_{41} & S_{42} & S_{43} & 0 \end{array}}$$

(rows: $\updownarrow X^{(n-1)}$)

By using eq. 2 the state probabilities after the first time quantum are:

$P [X^{(1)} = P_i] = 0$; $P [X^{(1)} = P_j] = S_{12}$; $P [X^{(1)} = P_k] = S_{13}$; $P [X^{(1)} = P_l] = S_{14}$

By using eq. 3 the state probabilities after the second time quantum are:

$P [X^{(2)} = P_i] = P [X^{(1)} = P_i] P [X^{(2)} = P_i / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_i / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_i / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_i / X^{(1)} = P_l]$

$P [X^{(2)} = P_i] = S_{13} S_{31} + S_{14} S_{41}$

$P [X^{(2)} = P_j] = P [X^{(1)} = P_i] P [X^{(2)} = P_j / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_j / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_j / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_j / X^{(1)} = P_l]$

$P [X^{(2)} = P_j] = S_{24} S_{42}$

$P [X^{(2)} = P_k] = P [X^{(1)} = P_i] P [X^{(2)} = P_k / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_k / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_k / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_k / X^{(1)} = P_l]$

$P [X^{(2)} = P_k] = S_{13} S_{31} + S_{34} S_{43}$

$P [X^{(2)} = P_l] = P [X^{(1)} = P_i] P [X^{(2)} = P_l / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_l / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_l / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_l / X^{(1)} = P_l]$

$P [X^{(2)} = P_l] = S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}$

Similarly, third time quantum are:

$P [X^{(3)} = P_i] = (S_{13} S_{31} + S_{34} S_{43}) S_{31} +(S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{41}$

$P [X^{(3)} = P_j] = (S_{13} S_{31} + S_{34} S_{43}) S_{12} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{42}$

$P [X^{(3)} = P_k] = (S_{13} S_{31} + S_{14} S_{41}) S_{13} +(S_{24} S_{42}) S_{23} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{43}$

$P [X^{(3)} = P_l] = (S_{13} S_{31} + S_{14} S_{41}) S_{14} +(S_{24} S_{42}) S_{24} + (S_{13} S_{31} + S_{34} S_{43}) S_{34}$

Similarly, fourth time quantum are:

$P [X^{(4)} = P_i] = \{ (S_{13} S_{31} + S_{14} S_{41}) S_{13} + (S_{24} S_{42}) S_{23} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{43}\} S_{31} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{14} + (S_{24} S_{42}) S_{24} + (S_{13} S_{31} + S_{34} S_{43}) S_{34}\} S_{41}$

$P [X^{(4)} = P_j] = \{ (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{41}\} S_{12} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{14} + (S_{24} S_{42}) S_{24} + (S_{13} S_{31} + S_{34} S_{43}) S_{34}\} S_{42}$

$P [X^{(4)} = P_k] = \{ (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{41}\} S_{13} + \{ (S_{13} S_{31} + S_{34} S_{43}) S_{12} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{42} \} S_{23} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{14} + (S_{24} S_{42}) S_{24} + (S_{13} S_{31} + S_{34} S_{43}) S_{34}\} S_{43}$

$P [X^{(4)} = P_l] = \{ (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{41}\} S_{14} + \{ (S_{13} S_{31} + S_{34} S_{43}) S_{12} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{42} \} S_{24} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{13} + (S_{24} S_{42}) S_{23} + (S_{14} S_{41} + S_{24} S_{42} + S_{34} S_{43}) S_{43}\} S_{34}$

Similarly, we can find fifth, sixth and so on time quantum.

B. Scheme - II

If Q1 is left with a single process, Q2 will have its turn immediately after the dispatch of the single process from Q1. When some transitions are restricted in the scheme-2 is described in fig. 4.2.

- A new process enters to Q1 only
- Scheduler can't jump to Q3 from Q1 without passing Q2
- If state Q1 is left with a single process, state Q2 will have its turn immediately after the dispatch of the single process from state Q1
- Resting of scheduler on state Q3 (process P_l) ends up only if a new process enters in Q1, otherwise resting continues.

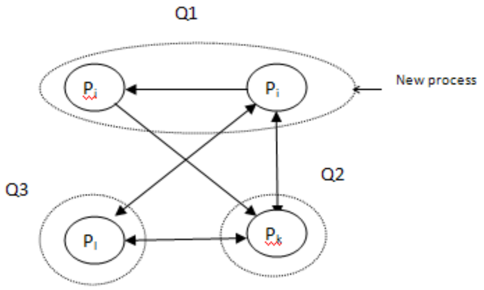

Fig.4.2: Restricted transition diagram

Thus, the initial probabilities under scheme-II are:

$P [X^{(0)} = P_i] = 1$; $P [X^{(0)} = P_j] = 0$; $P [X^{(0)} = P_k] = 0$; $P [X^{(0)} = P_l] = 0$

Unit step transaction probability matrix for $X^{(n)}$ under scheme-2 is:

$$P = \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|c|c|c|c} & P_i & P_j & P_k & P_l \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} \\ \hline P_j & S_{21} & S_{22} & S_{23} & S_{24} \\ \hline P_k & S_{31} & S_{32} & S_{33} & S_{34} \\ \hline P_l & S_{41} & S_{42} & S_{43} & S_{44} \end{array}} = \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|c|c|c|c} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & S_{12} & S_{13} & S_{14} \\ \hline P_j & 0 & 0 & S_{23} & 0 \\ \hline P_k & S_{31} & 0 & 0 & S_{34} \\ \hline P_l & S_{41} & 0 & S_{43} & 0 \end{array}}$$

(rows: $\updownarrow X^{(n-1)}$)

By using eq. 2 the state probabilities after the first time quantum are:

$P [X^{(1)} = P_i] = 0$; $P [X^{(1)} = P_j] = S_{12}$; $P [X^{(1)} = P_k] = S_{13}$; $P [X^{(1)} = P_l] = S_{14}$

By using eq. 3 the state probabilities after the second time quantum are:

$P [X^{(2)} = P_i] = P [X^{(1)} = P_i] P [X^{(2)} = P_i / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_i / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_i / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_i / X^{(1)} = P_l]$

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$P [X^{(2)} = P_i] = S_{13} S_{31} + S_{14} S_{41}$

$P [X^{(2)} = P_j] = P [X^{(1)} = P_i] P [X^{(2)} = P_j / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_j / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_j / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_j / X^{(1)} = P_l]$

$P [X^{(2)} = P_j] = 0$

$P [X^{(2)} = P_k] = P [X^{(1)} = P_i] P [X^{(2)} = P_k / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_k / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_k / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_k / X^{(1)} = P_l]$

$P [X^{(2)} = P_k] = S_{13} S_{31} + S_{34} S_{43}$

$P [X^{(2)} = P_l] = P [X^{(1)} = P_i] P [X^{(2)} = P_l / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_l / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_l / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_l / X^{(1)} = P_l]$

$P [X^{(2)} = P_l] = S_{14} S_{41} + S_{34} S_{43}$

Similarly, third time quantum are:

$P [X^{(3)} = P_i] = (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{34} S_{43}) S_{41}$
$P [X^{(3)} = P_j] = (S_{13} S_{31} + S_{14} S_{41}) S_{12}$
$P [X^{(3)} = P_k] = (S_{13} S_{31} + S_{14} S_{41}) S_{13} + (S_{14} S_{41} + S_{34} S_{43}) S_{43}$
$P [X^{(3)} = P_l] = (S_{13} S_{31} + S_{14} S_{41}) S_{14} + (S_{13} S_{31} + S_{34} S_{43}) S_{34}$

Similarly, fourth time quantum are:

$P [X^{(4)} = P_i] = \{ (S_{13} S_{31} + S_{14} S_{41}) S_{13} + (S_{14} S_{41} + S_{34} S_{43}) S_{43} \} S_{31} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{14} + (S_{13} S_{31} + S_{34} S_{43}) S_{34} \} S_{41}$

$P [X^{(4)} = P_j] = \{ (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{34} S_{43}) S_{41} \} S_{12}$

$P [X^{(4)} = P_k] = \{ (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{34} S_{43}) S_{41} \} S_{13} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{12} \} S_{23} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{14} + (S_{13} S_{31} + S_{34} S_{43}) S_{34} \} S_{43}$

$P [X^{(4)} = P_l] = \{ (S_{13} S_{31} + S_{34} S_{43}) S_{31} + (S_{14} S_{41} + S_{34} S_{43}) S_{41} \} S_{14} + \{ (S_{13} S_{31} + S_{14} S_{41}) S_{13} + (S_{14} S_{41} + S_{34} S_{43}) S_{43} \} S_{34}$

Similarly, we can find fifth, sixth and so on time quantum.

C. Scheme - III

If Q1 is left with no process, Q2 will function as a single ready queue. The following transition are restricted in this scheme-3 is described in fig. 4.3.

- A new process can only enter to Q2
- Transition from Q1 to Q3 is restricted
- Transition must occur in sequence from Q2 to Q1, Q1 to Q2, Q1 to Q3 and then Q2 to Q3.

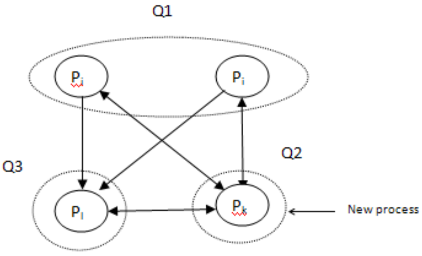

Fig.4.3: Restricted transition diagram

Thus, the initial probabilities under scheme-III are:

$P [X^{(0)} = P_i] = 0$; $P [X^{(0)} = P_j] = 0$; $P [X^{(0)} = P_k] = 1$; $P [X^{(0)} = P_l] = 0$

Unit step transaction probability matrix for $X^{(n)}$ under scheme-3 is:

$$P = X^{(n-1)} \updownarrow \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|c|c|c|c} & P_i & P_j & P_k & P_l \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} \\ \hline P_j & S_{21} & S_{22} & S_{23} & S_{24} \\ \hline P_k & S_{31} & S_{32} & S_{33} & S_{34} \\ \hline P_l & S_{41} & S_{42} & S_{43} & S_{44} \end{array}} = X^{(n-1)} \updownarrow \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|c|c|c|c} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & S_{13} & S_{14} \\ \hline P_j & 0 & 0 & S_{23} & S_{24} \\ \hline P_k & S_{31} & S_{32} & 0 & S_{34} \\ \hline P_l & 0 & 0 & S_{43} & 0 \end{array}}$$

By using eq. 2 the state probabilities after the first time quantum are:

$P [X^{(1)} = P_i] = 0$; $P [X^{(1)} = P_j] = 0$; $P [X^{(1)} = P_k] = S_{13}$; $P [X^{(1)} = P_l] = S_{14}$

By using eq. 3 the state probabilities after the second time quantum are:

$P [X^{(2)} = P_i] = P [X^{(1)} = P_i] P [X^{(2)} = P_i / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_i / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_i / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_i / X^{(1)} = P_l]$

$P [X^{(2)} = P_i] = S_{13} S_{31}$

$P [X^{(2)} = P_j] = P [X^{(1)} = P_i] P [X^{(2)} = P_j / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_j / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_j / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_j / X^{(1)} = P_l]$

$P [X^{(2)} = P_j] = S_{23} S_{32}$

$P [X^{(2)} = P_k] = P [X^{(1)} = P_i] P [X^{(2)} = P_k / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_k / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_k / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_k / X^{(1)} = P_l]$

$P [X^{(2)} = P_k] = S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}$

$P [X^{(2)} = P_l] = P [X^{(1)} = P_i] P [X^{(2)} = P_l / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_l / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_l / X^{(1)} = P_k] + P [X^{(1)} = P_l] P [X^{(2)} = P_l / X^{(1)} = P_l]$

$P [X^{(2)} = P_l] = S_{34} S_{43}$

Similarly, third time quantum are:

$P [X^{(3)} = P_i] = (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{31}$
$P [X^{(3)} = P_j] = (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{32}$
$P [X^{(3)} = P_k] = (S_{13} S_{31}) S_{13} + (S_{23} S_{32}) S_{23} + (S_{34} S_{43}) S_{43}$
$P [X^{(3)} = P_l] = (S_{13} S_{31}) S_{14} + (S_{23} S_{32}) S_{24} + (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{34}$

Similarly, fourth time quantum are:

$P [X^{(4)} = P_i] = \{ (S_{13} S_{31}) S_{13} + (S_{23} S_{32}) S_{23} + (S_{34} S_{43}) S_{43} \} S_{31}$

$P [X^{(4)} = P_j] = \{ (S_{13} S_{31}) S_{13} + (S_{23} S_{32}) S_{23} + (S_{34} S_{43}) S_{43} \} S_{32}$

$P [X^{(4)} = P_k] = \{ (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{31} \} S_{13} + \{ (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{32} \} S_{23} + \{ (S_{13} S_{31}) S_{14} + (S_{23} S_{32}) S_{24} + (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{34} \} S_{43}$

$P [X^{(4)} = P_l] = \{ (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{31} \} S_{14} + \{ (S_{13} S_{31} + S_{23} S_{32} + S_{34} S_{43}) S_{32} \} S_{24} + \{$

$(S_{13}\ S_{31})S_{13} + (S_{23}\ S_{32})\ S_{23} + (S_{34}S_{43})$ $S_{43}\}\ S_{34}$

Similarly, we can find fifth, sixth and so on time quantum.

V. FORMULATE AND CALCULATE THE EQUAL VALUE TRANSITION PROBABILITIES

Consider equal transition probability matrix for a constant number ‘d’, $0 \le d \le 1$.
Case of equal value transition probabilities:

Therefore, the nth quantum under scheme-I is determined as:

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad X^{(n-1)} \updownarrow \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & d & d & d & 1\text{-}3d \\ P_j & d & d & d & 1\text{-}3d \\ P_k & d & d & d & 1\text{-}3d \\ P_l & d & d & d & 1\text{-}3d \end{array}$$

$P[X^{(0)} = P_i] = d$; $P[X^{(0)} = P_j] = d$; $P[X^{(0)} = P_k] = d$; $P[X^{(0)} = P_l] = 1-3d$

The equal transition matrix for scheme-I is expressed as:

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad X^{(n-1)} \updownarrow \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & d & d & 1\text{-}2d \\ P_j & 0 & 0 & d & 1\text{-}d \\ P_k & d & 0 & 0 & 1\text{-}d \\ P_l & d & d & d & 0 \end{array}$$

Table 5.1: (Seven quantum transition probabilities under scheme-I)

States				
No. of Qntm	**P_i** $P[X^{(n)} = P_i]$	**P_j** $P[X^{(n)} = P_j]$	**P_k** $P[X^{(n)} = P_k]$	**P_l** $P[X^{(n)} = P_l]$
n =1	0	d	d	1-2d
n =2	$d-d^2$	$d-d^2$	d	$3d-4d^2$
n =3	$4d^2-4d^3$	$4d^2-5d^3$	$5d^2-6d^3$	$3d-6d^2+3d^3$
n =4	$3d^2-d^3-3d^4$	$3d^2-2d^3-d^4$	$3d^2+2d^3-6d^4$	$13d^2-32d^3+19d^4$
n =5	$16d^3-30d^4+13d^5$	$16d^3-33d^4+16d^5$	$19d^3-35d^4+15d^5$	$9d^2-13d^3-8d^4+13d^5$
n =6	$9d^3+6d^4-43d^5+28d^6$	$9d^3+3d^4-38d^5+26d^6$	$9d^3+19d^4-71d^5+42d^6$	$51d^3-165d^4+172d^5-57d^6$
n =7	$60d^4-146d^5+101d^6-15d^7$	$60d^4-159d^5+129d^6-29d^7$	$69d^4-156d^5+46d^6-3d^7$	$27d^3-20d^4-162d^5+291d^6-124d^7$

The equal transition matrix for scheme-II is expressed as:

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad X^{(n-1)} \updownarrow \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & d & d & 1\text{-}2d \\ P_j & 0 & 0 & 1 & 0 \\ P_k & d & 0 & 0 & 1\text{-}d \\ P_l & d & 0 & d & 0 \end{array}$$

Table 5.2: (Seven quantum transition probabilities under scheme-II)

States				
No. of Qntm	**P_i** $P[X^{(n)} = P_i]$	**P_j** $P[X^{(n)} = P_j]$	**P_k** $P[X^{(n)} = P_k]$	**P_l** $P[X^{(n)} = P_l]$
n =1	0	d	d	1-2d
n =2	$d-d^2$	0	d	$2d-3d^2$
n =3	$3d^2-3d^3$	d^2-d^3	$3d^2-4d^3$	$2d-4d^2+2d^3$
n =4	$2d^2-d^3-2d^4$	$3d^3-3d^4$	$3d^2-2d^3-d^4$	$6d^2-16d^3+10d^4$
n =5	$9d^3-18d^4+5d^5$	$2d^3-d^4-2d^5$	$11d^3-22d^4+10d^5$	$5d^2-10d^3+2d^4+4d^5$
n =6	$5d^3+d^4+24d^5+14d^6$	$9d^4-18d^5+5d^6$	$7d^3-2d^4-18d^5+9d^6$	$20d^3-69d^4+73d^5-20d^6$
n =7	$27d^4-71d^5+55d^6-11d^7$	$5d^4+d^5+24d^6+14d^7$	$34d^4-86d^5+102d^6-6d^7$	$12d^3-18d^4+6d^5-7d^6-37d^7$

The equal transition matrix for scheme-III is expressed as:

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad X^{(n-1)} \updownarrow \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & d & 1\text{-}d \\ P_j & 0 & 0 & d & 1\text{-}d \\ P_k & d & d & 0 & 1\text{-}2d \\ P_l & d & 0 & 1 & 0 \end{array}$$

Table 5.3: (Seven quantum transition probabilities under scheme-III)

States				
No. of Qntm	**P_i** $P[X^{(n)} = P_i]$	**P_j** $P[X^{(n)} = P_j]$	**P_k** $P[X^{(n)} = P_k]$	**P_l** $P[X^{(n)} = P_l]$
n =1	0	0	d	1-d
n =2	d^2	d^2	$1-2d+2d^2$	1-2d
n =3	$d-2d^2+2d^3$	$d-2d^2+2d^3$	$1-2d+2d^3$	$1-4d+8d^2-6d^3$
n =4	$d-2d^2+2d^4$	$d-2d^2+2d^4$	$1-4d+10d^2-10d^3+4d^4$	$1-2d-2d^2+10d^3-8d^4$
n =5	$d-4d^2+10d^3-10d^4+4d^5$	$d-4d^2+10d^3-10d^4+4d^5$	$1-2d+6d^3-8d^4+4d^5$	$1-4d+12d^2-26d^3+28d^4-12d^5$
n =6	$d-2d^2+6d^4-8d^5+4d^6$	$d-2d^2+6d^4-8d^5+4d^6$	$1-4d+14d^2-34d^3+48d^4-32d^5+8d^6$	$1-2d-6d^2+34d^3-60d^4+48d^5-16d^6$
n =7	$d-4d^2+14d^3-34d^4+48d^5-32d^6+8d^7$	$d-4d^2+14d^3-34d^4+48d^5-32d^6+8d^7$	$1-2d-4d^2+30d^3-60d^4+60d^5-32d^6+8d^7$	$1-4d+16d^2-58d^3+128d^4-156d^5+96d^6-24d^7$

VI. SIMULATION STUDY WITH NUMERICAL ANALYSIS

In order to analyze three schemes mentioned in section 4 (A, B & C) under markov chain model with equal and unequal transition elements in section 5 using different data sets:

A. *Data Set – I*

Scheme I: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad X^{(n-1)} \updownarrow \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.2 & 0.3 & 0.5 \\ P_j & 0 & 0 & 0.4 & 0.6 \\ P_k & 0.35 & 0 & 0 & 0.65 \\ P_l & 0.45 & 0.3 & 0.25 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad X^{(n-1)} \updownarrow \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.2 & 0.2 & 0.6 \\ P_j & 0 & 0 & 0.2 & 0.8 \\ P_k & 0.2 & 0 & 0 & 0.8 \\ P_l & 0.2 & 0.2 & 0.6 & 0 \end{array}$$

Table 6.1.1: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal					Equal			
	P_i	P_j	P_k	P_l		P_i	P_j	P_k	P_l
n = 1	0	0.2	0.3	0.5		0	0.2	0.2	0.6
n = 2	0.33	0.18	0.268	0.568		0.16	0.18	0.52	0.76
n = 3	0.3494	0.2364	0.313	0.4472		0.256	0.184	0.524	0.656
n = 4	0.311	0.204	0.311	0.52		0.236	0.182	0.482	0.72
n = 5	0.343	0.218	0.305	0.48		0.24	0.191	0.516	0.673
n = 6	0.323	0.213	0.31	0.501		0.238	0.183	0.49	0.71
n = 7	0.334	0.215	0.307	0.491		0.24	0.19	0.51	0.68

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Scheme II: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.2 & 0.3 & 0.5 \\ P_j & 0 & 0 & 1.0 & 0 \\ P_k & 0.35 & 0 & 0 & 0.65 \\ P_l & 0.45 & 0 & 0.55 & 0 \end{array}$$

Equal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.2 & 0.2 & 0.6 \\ P_j & 0 & 0 & 1.0 & 0 \\ P_k & 0.2 & 0 & 0 & 0.8 \\ P_l & 0.2 & 0 & 0.8 & 0 \end{array}$$

Table 6.1.2: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal					Equal			
	P_i	P_j	P_k	P_l		P_i	P_j	P_k	P_l
n = 1	0	0.2	0.3	0.5		0	0.2	0.2	0.6
n = 2	0.33	0	0.463	0.583		0.16	0	0.68	0.76
n = 3	0.424	0.066	0.42	0.466		0.288	0.032	0.64	0.64
n = 4	0.357	0.085	0.45	0.485		0.256	0.058	0.602	0.685
n = 5	0.376	0.071	0.459	0.471		0.257	0.051	0.657	0.635
n = 6	0.373	0.075	0.443	0.486		0.258	0.051	0.61	0.68
n = 7	0.374	0.075	0.454	0.474		0.258	0.052	0.647	0.643

Scheme III: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & 0.3 & 0.7 \\ P_j & 0 & 0 & 0.4 & 0.6 \\ P_k & 0.35 & 0.25 & 0 & 0.4 \\ P_l & 0 & 0 & 1.0 & 0 \end{array}$$

Equal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & 0.2 & 0.8 \\ P_j & 0 & 0 & 0.2 & 0.8 \\ P_k & 0.2 & 0.2 & 0 & 0.6 \\ P_l & 0 & 0 & 1.0 & 0 \end{array}$$

Table 6.1.3: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal					Equal			
	P_i	P_j	P_k	P_l		P_i	P_j	P_k	P_l
n = 1	0	0	0.3	0.7		0	0	0.2	0.8
n = 2	0.105	0.1	0.605	0.4		0.04	0.04	0.68	0.6
n = 3	0.212	0.151	0.472	0.376		0.136	0.136	0.616	0.472
n = 4	0.165	0.118	0.5	0.428		0.123	0.123	0.526	0.587
n = 5	0.175	0.125	0.525	0.386		0.1052	0.1052	0.636	0.512
n = 6	0.184	0.131	0.489	0.408		0.127	0.127	0.554	0.55
n = 7	0.171	0.122	0.516	0.403		0.111	0.111	0.601	0.536

B. Data Set – II

Scheme I: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.35 & 0.25 & 0.4 \\ P_j & 0 & 0 & 0.4 & 0.6 \\ P_k & 0.45 & 0 & 0 & 0.55 \\ P_l & 0.4 & 0.3 & 0.25 & 0 \end{array}$$

Equal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.3 & 0.3 & 0.4 \\ P_j & 0 & 0 & 0.3 & 0.7 \\ P_k & 0.3 & 0 & 0 & 0.7 \\ P_l & 0.3 & 0.3 & 0.4 & 0 \end{array}$$

Table 6.2.1: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal					Equal			
	P_i	P_j	P_k	P_l		P_i	P_j	P_k	P_l
n = 1	0	0.35	0.25	0.4		0	0.3	0.3	0.4
n = 2	0.273	0.21	0.25	0.508		0.21	0.21	0.37	0.61
n = 3	0.316	0.273	0.279	0.373		0.294	0.246	0.37	0.49
n = 4	0.275	0.241	0.281	0.444		0.258	0.235	0.358	0.549
n = 5	0.304	0.252	0.276	0.409		0.272	0.242	0.368	0.518
n = 6	0.288	0.25	0.279	0.425		0.266	0.237	0.361	0.536
n = 7	0.295	0.25	0.278	0.419		0.269	0.241	0.365	0.525

Scheme II: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.35 & 0.25 & 0.4 \\ P_j & 0 & 0 & 1.0 & 0 \\ P_k & 0.45 & 0 & 0 & 0.55 \\ P_l & 0.4 & 0 & 0.6 & 0 \end{array}$$

Equal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.3 & 0.3 & 0.4 \\ P_j & 0 & 0 & 1.0 & 0 \\ P_k & 0.3 & 0 & 0 & 0.7 \\ P_l & 0.3 & 0 & 0.7 & 0 \end{array}$$

Table 6.2.2: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal					Equal			
	P_i	P_j	P_k	P_l		P_i	P_j	P_k	P_l
n = 1	0	0.35	0.25	0.4		0	0.3	0.3	0.4
n = 2	0.273	0	0.443	0.49		0.21	0	0.58	0.61
n = 3	0.395	0.096	0.362	0.353		0.357	0.063	0.49	0.49
n = 4	0.304	0.138	0.407	0.357		0.294	0.107	0.513	0.486
n = 5	0.326	0.106	0.428	0.345		0.3	0.088	0.535	0.477
n = 6	0.331	0.114	0.395	0.366		0.304	0.09	0.512	0.495
n = 7	0.324	0.116	0.416	0.35		0.302	0.091	0.528	0.48

Scheme III: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & 0.45 & 0.55 \\ P_j & 0 & 0 & 0.35 & 0.65 \\ P_k & 0.4 & 0.35 & 0 & 0.25 \\ P_l & 0 & 0 & 1.0 & 0 \end{array}$$

Equal

$$P = \begin{array}{c} \\ X^{(n-1)} \updownarrow \end{array} \begin{array}{c|cccc} & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & 0.3 & 0.7 \\ P_j & 0 & 0 & 0.3 & 0.7 \\ P_k & 0.3 & 0.3 & 0 & 0.4 \\ P_l & 0 & 0 & 1.0 & 0 \end{array}$$

Table 6.2.3: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal					Equal			
	P_i	P_j	P_k	P_l		P_i	P_j	P_k	P_l
n = 1	0	0	0.45	0.55		0	0	0.3	0.7
n = 2	0.18	0.123	0.553	0.25		0.09	0.09	0.58	0.4
n = 3	0.221	0.194	0.374	0.317		0.174	0.174	0.454	0.358
n = 4	0.15	0.131	0.484	0.341		0.136	0.136	0.462	0.425
n = 5	0.194	0.169	0.454	0.289		0.139	0.139	0.507	0.375
n = 6	0.181	0.159	0.435	0.33		0.152	0.152	0.458	0.397
n = 7	0.174	0.152	0.467	0.312		0.137	0.137	0.488	0.396

C. Data Set – III

Scheme I: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

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Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad \updownarrow X^{(n-1)} \quad \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.38 & 0.42 & 0.2 \\ P_j & 0 & 0 & 0.46 & 0.54 \\ P_k & 0.48 & 0 & 0 & 0.52 \\ P_l & 0.42 & 0.4 & 0.18 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad \updownarrow X^{(n-1)} \quad \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.4 & 0.4 & 0.2 \\ P_j & 0 & 0 & 0.4 & 0.6 \\ P_k & 0.4 & 0 & 0 & 0.6 \\ P_l & 0.4 & 0.4 & 0.2 & 0 \end{array}$$

Table 6.3.1: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal				Equal			
	P_i	P_j	P_k	P_l	P_i	P_j	P_k	P_l
n = 1	0	0.38	0.42	0.2	0	0.4	0.4	0.2
n = 2	0.286	0.216	0.295	0.386	0.24	0.24	0.28	0.44
n = 3	0.304	0.263	0.289	0.327	0.288	0.272	0.28	0.36
n = 4	0.276	0.246	0.308	0.353	0.256	0.259	0.296	0.389
n = 5	0.296	0.246	0.293	0.348	0.274	0.258	0.284	0.384
n = 6	0.287	0.252	0.300	0.344	0.267	0.263	0.29	0.38
n = 7	0.289	0.247	0.298	0.344	0.268	0.259	0.288	0.385

Scheme II: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad \updownarrow X^{(n-1)} \quad \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.42 & 0.4 & 0.18 \\ P_j & 0 & 0 & 1.0 & 0 \\ P_k & 0.48 & 0 & 0 & 0.52 \\ P_l & 0.46 & 0 & 0.54 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad \updownarrow X^{(n-1)} \quad \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0.4 & 0.4 & 0.2 \\ P_j & 0 & 0 & 1.0 & 0 \\ P_k & 0.4 & 0 & 0 & 0.6 \\ P_l & 0.4 & 0 & 0.6 & 0 \end{array}$$

Table 6.3.2: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal				Equal			
	P_i	P_j	P_k	P_l	P_i	P_j	P_k	P_l
n = 1	0	0.42	0.4	0.18	0	0.4	0.4	0.2
n = 2	0.275	0	0.473	0.364	0.24	0	0.52	0.44
n = 3	0.394	0.116	0.307	0.295	0.384	0.096	0.36	0.36
n = 4	0.283	0.165	0.433	0.231	0.288	0.154	0.466	0.293
n = 5	0.314	0.119	0.403	0.276	0.304	0.115	0.445	0.337
n = 6	0.320	0.132	0.394	0.266	0.313	0.122	0.439	0.328
n = 7	0.312	0.134	0.404	0.262	0.307	0.125	0.444	0.326

Scheme III: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$ and $Pr_4 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad \updownarrow X^{(n-1)} \quad \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & 0.48 & 0.52 \\ P_j & 0 & 0 & 0.46 & 0.54 \\ P_k & 0.42 & 0.4 & 0 & 0.18 \\ P_l & 0 & 0 & 1.0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \quad \updownarrow X^{(n-1)} \quad \begin{array}{c|cccc} & P_i & P_j & P_k & P_l \\ \hline P_i & 0 & 0 & 0.4 & 0.6 \\ P_j & 0 & 0 & 0.4 & 0.6 \\ P_k & 0.4 & 0.4 & 0 & 0.2 \\ P_l & 0 & 0 & 1.0 & 0 \end{array}$$

Table 6.2.3: The transition probabilities P [$X^{(n)}$ = Qi] for unequal and equal cases:

Quantum No.	Unequal				Equal			
	P_i	P_j	P_k	P_l	P_i	P_j	P_k	P_l
n = 1	0	0	0.48	0.52	0	0	0.4	0.6
n = 2	0.202	0.184	0.566	0.18	0.16	0.16	0.52	0.2
n = 3	0.238	0.226	0.362	0.306	0.208	0.208	0.328	0.296
n = 4	0.152	0.145	0.524	0.311	0.131	0.131	0.462	0.315
n = 5	0.22	0.21	0.451	0.252	0.185	0.185	0.42	0.25
n = 6	0.189	0.180	0.454	0.309	0.168	0.168	0.398	0.306
n = 7	0.191	0.182	0.483	0.277	0.159	0.159	0.44	0.281

VII. GRAPHICAL ANALYSIS

Graphical analysis is performed under above mentioned three schemes in section 6 (A, B & C) with different data sets considering unequal and equal probability matrix to put various quantum values. Thus the analytical discussion on graphs about the variation P [$X^{(n)}$ = Qi] over three data sets are as follows:

A. *Data Set – I*

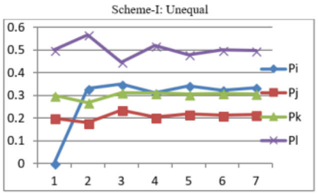

Fig.7.1

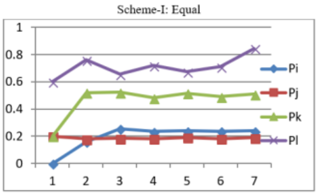

Fig.7.4

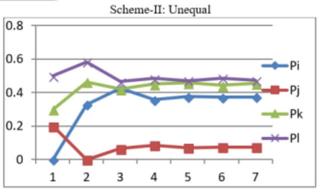

Fig.7.2

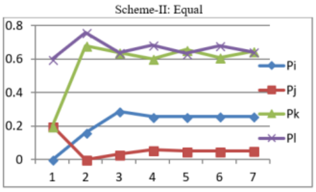

Fig.7.5

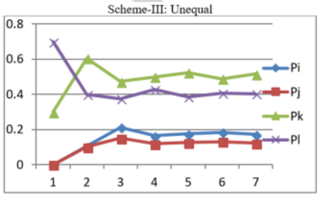

Fig.7.3

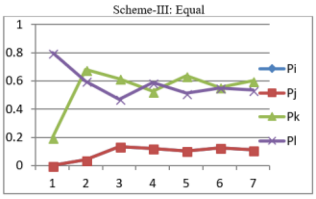

Fig.7.6

B. *Data Set – II*

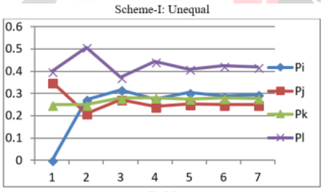

Fig.7.7

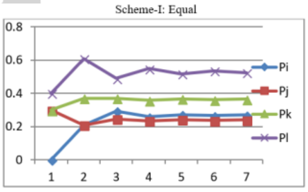

Fig.7.10

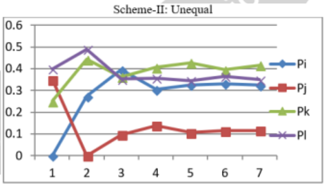

Fig.7.8

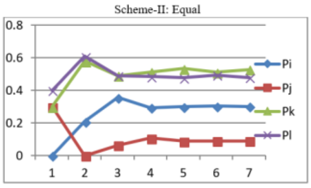

Fig.7.11

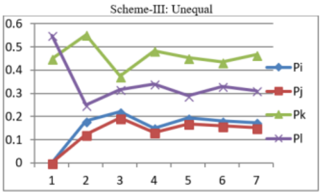

Fig.7.9

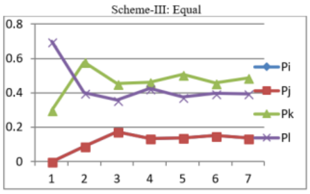

Fig.7.12

C. *Data Set – III*

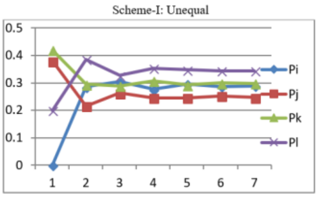
Scheme-I: Unequal

Fig.7.13

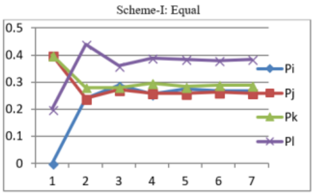
Scheme-I: Equal

Fig.7.16

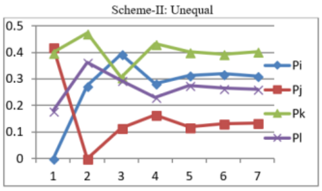
Scheme-II: Unequal

Fig.7.14

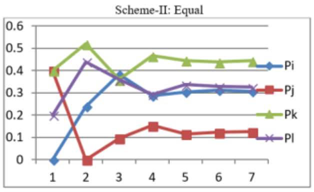
Scheme-II: Equal

Fig.7.17

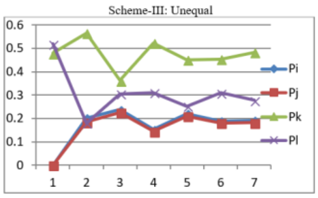
Scheme-III: Unequal

Fig.7.15

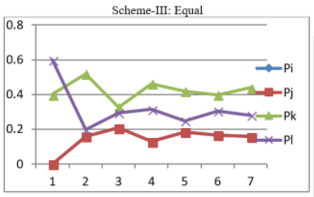
Scheme-III: Equal

Fig.7.18

Scheme - I

i. Unequal: Although the transition in the states P_i , P_j , P_k and P_l of the scheduler makes stable pattern when number of quantum n >= 3 but up to n = 3 it reflects changing in patterns. The remarkable point is that the probability of wait state P_l is higher in all data sets than other states especially in fig. 7.1 and fig. 7.2 but state P_i and P_k is flying equally high in fig. 7.3. This shows a loss of efficiency so that scheduling spends more time on the wait state than on working states. Therefore, less restricted scheduling scheme lead to a loss of CPU time.

ii. Equal: The graphical patterns (fig. 7.4, fig. 7.5 and fig. 7.6) state probabilities are moved independent of the quantum variation because the pattern of distribution of state probabilities is almost similar in these data sets.

Scheme - II

i. Unequal: The graphical pattern (fig. 7.7) reveals higher probabilities at the wait state than the other states (fig. 7.8 and fig. 7.9). This again leads to a lack of performance efficiency under these data sets due to more on waiting of the scheduler.

ii. Equal: The state probabilities are moved independent of the quantum variation because the pattern of distribution of state probabilities is almost similar in these fig. 7.10, fig.7.11 and fig. 7.12. So, the probabilities of wait state P_l in (fig. 7.10 and fig. 7.11) are flying comparatively much high. Therefore, it gives degrading in performance and CPU time in scheduling the processes. The special remark is that there are more chance for process contained in P_i and P_k to be processed than in P_j .

Scheme - III

i. Unequal: The probability of scheduler in the wait state P_l is lower than state P_k (it is slightly high value) over different quantum which is a sign of increase performance efficiency of the IRR scheduling in the data sets. The probability of state P_k is higher than the previous schemes. Most of the transition probabilities are almost equal in fig. 7.14 and fig. 7.15 and observed minor variation in fig. 7.13 in graphical pattern. The scheme-III provides more chance to job processing than waiting which gives good throughput comparatively to previous schemes.

ii. Equal: The transition states pattern in these graphs are identical in fig. 7.16, fig. 7.17 and fig. 7.18. But, the probability of scheduler in wait state is low, which results of good performance of the IRR scheduling in these data sets than scheme-I and scheme-II. Other state probabilities according to quantum variation… The special remark for this process-scheduling scheme-I, scheme-II and scheme-III is that probability for the state P_k is very high. Therefore, there are more chance for jobs contained in P_k to be processed than P_i and P_j.

VIII. CONCLUSION

In this paper we have done performance analysis and comparison between three schemes of the improved round robin scheduling using Markov chain model and by incorporating equal and unequal probability matrix with number of data sets which have functions of restriction in terms of some state transition. The equal transition probabilities precedence to quantum independency and the information overlapping in scheme-I and scheme-II which are less restricted scheduling. In the unequal probability matrix, elements make a better picture of transition within states. In these earlier scheduling schemes, the probability precedence the waiting state is high which show that a loss of system efficiency and serious downfall in performance of IRR. The graphical pattern does not depend much on quantum variation that is high effect of equal and unequal probability elements which gives very lesser chance for processing. Moreover, in these schemes, the different state has less probability which a good indication for poor scheduling algorithms. Therefore, both schemes are not recommended for further utilization. But in the scheme-III provide a stable pattern of probability variation over quantum almost in all the three data sets. For the variation becomes independent of changes in terms of quantum and wait state probabilities are decreased than other states in both equal an unequal transition matrix. Further, the pattern in having not much variation over changing data. This is an interesting function which leads to the stability of the whole system that is useful over the earlier two schemes. Therefore, efficiency of this highly imposing restricted scheduling scheme-III in terms of security measure is highly efficient, useful, and recommendable to improve the performance of study.

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