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Fibonacci Prime Labeling Of Cycle Related Graphs

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Abstract - A Fibonacci prime labeling of a graph $G = (V(G), E(G))$ with $|V(G)| = n$ is an injective function $g: V(G) \to \{f_2, f_3, \ldots . f_{n+1}\}$, where f_n is the n^{th} Fibonacci number, that induces a function $g^*: E(G) \to N$ defined by $g^*(uv) = gcd\{g(u), g(v)\} = 1 \; \forall \, uv \in E(G)$. The graph G admits a Fibonacci prime labeling is called a Fibonacci prime graph. In this paper we prove that some cycle related graphs are Fibonacci prime graphs.

Keywords: Fibonacci prime graph, barycentric subdivision, crown graph.

I. INTRODUCTION

In this paper, only finite simple undirected connected graphs are considered. The graph G has vertex set V =V (G) and edge set E = E (G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [6].Two integers a and b are said to be relatively prime if their greatest common divisor is 1.Many researchers have studied prime graph. Fu.H[3] has proved that the path P_n on n vertices is a prime graph.Deretsky et al [2] have proved that the cycle C_n on n vertices is a prime graph .Around 1980 Roger Entringer conjectured that all trees have prime labeling which has not been settled so far.

II. PRELIMINARY DEFINITIONS

Definition 2.1

The Fibonacci number f_n is defined recursively by the equations $f_1 = 1$; $f_2 = 1$; $f_{n+1} = f_n + f_{n-1}$ $(n \geq 2)$.

Note 2.2

It is observe that $g,.c.d(f_n, f_{n+1}) = 1 \;\; \forall \, n \geq 1$, $g.c.d(f_n, f_{n+2}) = 1 \; \forall \, n \geq 1$.

Definition 2.3

A prime labeling of a graph G is an injective function $f: V(G) \to \{1,2, \ldots . . |V(G)|\}$ such that for every pair of adjacent vertices $u \; and \; v$, $\gcd\{f(u), f(v)\} = 1$.A graph which admits a prime labeling is called a prime graph.

Definition 2.4

A Fibonacci prime labeling of a graph $G = (V, E)$ with $|V(G)| = n$ is an injective function $g: V(G) \to \{f_2, f_3, \ldots \ldots . . f_{n+1}\}$,where f_n is the n^{th} Fibonacci number that induces a function $g^*: E(G) \to N$ defined by $g^*(uv) = g.c.d\{g(u), g(v)\} = 1 \; \forall \, uv \in E(G)$.

The graph which admits a Fibonacci prime labeling is called Fibonacci prime graph.

Definition 2.5

$< G, K_{1,m} >, m \geq 1$ is the graph obtained by attaching $K_{1,m}$ to one vertex of the graph G.

Definition 2.6

Let $G = (V, E)$ be a graph .Let $e = (uv)$ be an edge of G and w is not a vertex of G.The edge e is subdivided when it is replaced by edge $e' = uw$ and $e'' = vw$. If every edge of a graph G is subdivided then the resulting graph is called barycentric subdivision of a graph G.

III. MAIN RESULTS

Theorem 3.1

Cycle C_n is a Fibonacci prime graph for $n \geq 3$.

Proof:

Let $v_1, v_2, \ldots . . v_n$ be the vertices of the cycle C_n .

The edge set of C_n is $E(C_n) = \{ v_i v_{i+1} \,|\, 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.

Define $g: V(C_n) \to \{f_2, f_3 \ldots \ldots f_{n+1}\} \; as$

$$g(v_i) = f_{i+1}, 1 \leq i \leq n \; .$$

Then the induced function $g^*: E(G) \to N \; is \; defined \; by$ $g^*(xy) = g.c.d\{g(x), g(y)\} \; \forall \, xy \in E(G)$.

Now,$\gcd\{g(v_i), g(v_{i+1})\} = \gcd\{f_{i+1}, f_{i+2}\}$ =1 ,$1 \leq i \leq n-1$.

and $\gcd\{g(v_n), g(v_1)\} = \gcd\{(f_{n+1}, f_2\}$ =$\gcd\{f_{n+1,} 1\} = 1$

Thus $f^*(xy) = g.c.d\{f(x), f(y)\} = 1, \forall \, xy \in E(G)$.

Hence C_n is a Fibonacci prime graph.

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Example 3.2

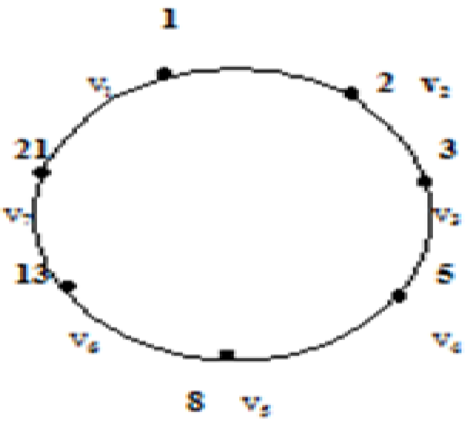

Figure: 1 C_6 **is a Fibonacci prime graph**

Theorem 3.3

The graph $< C_n, K_{1,m} >, m \geq 1$ is a Fibonacci prime graph.

Proof:

Let $G =< C_n, K_{1,m} >$.

The vertex set of the cycle C_n is $V(C_n) = \{u_1, u_2, \dots \dots u_n\}$.Let u_1 be the common vertex of C_n and $K_{1,m}$ .Let the remaining vertices of $K_{1,m}$ be $v_1, v_2, \dots \dots \dots \dots v_m$.

Hence the vertex set of G is $V(G) = \{u_1, u_2 \dots \dots \dots \dots u_n, v_1, v_2 \dots \dots \dots . v_m\}$.

Then$|V(G)| = n + m$ and $|E(G)| = n + m$.

The edge set of G is $E(G) = \{(u_i, u_{i+1}), 1 \leq i \leq n, u_{n+1} = u_1\} \cup \{(u_1, v_i), 1 \leq i \leq m\}$.

Define $g: V(G) \to \{f_2, f_3 \dots \dots \dots \dots f_{n+m+1}\}$ as follows

$$g(u_i) = f_{i+1}, 1 \leq i \leq n$$
$$g(v_i) = f_{n+i}, 1 \leq i \leq m + 1$$

Then the induced function $g^*: E(G) \to N$ is defined by $g^*(uv) = \gcd\{g(u), g(v)\} \forall uv \in E(G)$.

Clearly the vertex labels are distinct.

Now, $\gcd\{g(u_i), g(u_{i+1})\} = \gcd\{f_{i+1}, f_{i+2}\} = 1$ for $1 \leq i \leq n - 1$

$\gcd\{g(u_n), g(u_1)\} = \gcd\{f_{n+1}, f_2\} = g.c.d\{f_{n+1}, 1\} = 1$.

$\gcd\{g(u_1), g(v_i)\} = \gcd\{f_2, f_{n+i+1}\} = 1$ for $1 \leq i \leq m$.

Thus $g^*(uv) = \gcd\{g(u), g(v)\} = 1 \forall\, uv \in E(G)$.

Hence G admits a Fibonacci prime labeling. Hence G is a Fibonacci prime graph.

Example 3.4

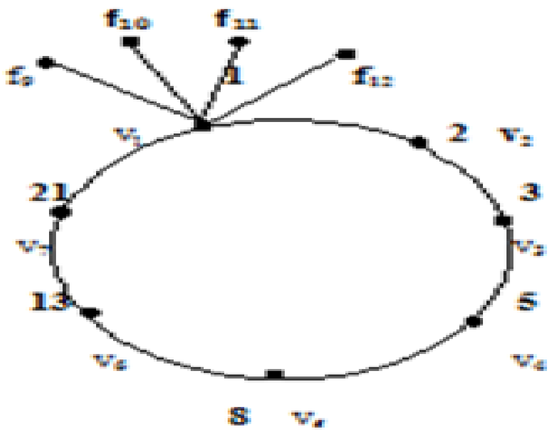

Figure: 2 Fibonacci prime labeling of $< C_7, K_{1,4} >$

Theorem 3.5

Barycentric subdivision of the Cycle $C_n[C_n]$ is a Fibonacci prime graph for all n.

Proof:

Let $\{u_1, u_2, \dots \dots \dots \dots . u_n\}$ be the vertices of cycle C_n and $\{u_1', u_2' \dots \dots \dots \dots u_n'\}$ be the newly inserted vertices to obtain barycentric subdivision of cycle C_n.Join each newly inserted vertices of incident edges by an edge we get new graph $C_n[c_n]$ .Let $G = C_n[c_n]$.G contains $2n$ vertices and $3n$ edges.

The vertices of G is $V(G) = \{u_i \;\;|\;\; 1 \leq i \leq n\} \cup \{u_i' \;\;|\;\; 1 \leq i \leq n\}$.

The edge set of G is $E(G) = \{u_i u_i' \;\;/\;\; 1 \leq i \leq n\} \cup \{u_{i+1}' u_i \;\;/\;\; 1 \leq i \leq n - 1\} \cup \{u_1' u_n\} \cup \{u_i' u_{i+1}' \;\;|\;\; 1 \leq i \leq n - 1\} \cup \{u_1' u_n'\}$.

Define $g: V(G) \to \{f_2, f_3 \dots \dots \dots . . f_{2n+1}\}$ as follows

$$g(u_i') = f_{2i} \;\;,\;\; 1 \leq i \leq n$$
$$g(u_i) = f_{2i+1} \;,\;\; 1 \leq i \leq n\,.$$

Then the induced function $g^*: E(G) \to N$ is defined by $g^*(uv) = \gcd\{g(u), g(v)\} \forall uv \in E(G)$.

Now,$\gcd\{g(u_i), g(u_i')\} = \gcd\{f_{2i+1}, f_{2i}\} = 1$

$\gcd\{g(u_i), g(u_{i+1}')\} = \gcd\{f_{2i+1}, f_{2i+2}\} = 1$

$\gcd\{g(u_1'), g(u_n)\} = \gcd\{f_2, f_{2n+1}\} = \gcd\{1, f_{2n+1}\} = 1$

$\gcd\{g(u_i'), g(u_{i+1}')\} = \gcd\{f_{2i}, f_{2i+1}\} = 1$

$\gcd\{g(u_1), g(u_n')\} = \gcd\{f_2, f_{2n}\} = 1$

Thus $g^*(uv) = \gcd\{f(u), f(v)\} = 1 \forall uv \in E(G)$.

Hence G admits Fibonacci prime labeling .Hence G is a Fibonacci prime graph.

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Example 3.6

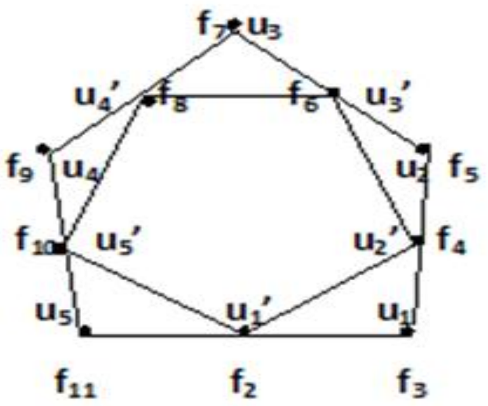

Figure: 3 Fibonacci prime labeling of $C_5[C_5]$

Theorem 3.7

The crown graph C_n^* is a Fibonacci prime graph.

Proof

Let G be a crown graph C_n^* .

Let $V(G) = \{u_1, u_2 \ldots \ldots . . u_n, v_1, v_2 \ldots \ldots \ldots \ldots v_n\}$.

The edge set

$E(G) = \{u_i v_i | 1 \leq i \leq n\} \cup \{u_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{u_n u_1\}$.

Then $|V(G)| = 2n$ *and* $|E(G)| = 2n$.

Define $g: V(G) \to \{f_2, \ldots \ldots \ldots \ldots . f_{2n+1}\}$

$$by\ g(u_i) = f_{2i+1}, 2 \leq i \leq n$$

$$g(v_i) = f_{2i+1}, 2 \leq i \leq n$$

.

The induced function $g^*: E(G) \to N$ *by*

$$g^*(uv) = \gcd\{g(u), g(v)\} \forall e = uv \in E(G)$$

$$Now,\ \gcd\{g(u_i), g(v_i)\} = \gcd\{f_{2i}, f_{2i+1}\}$$

$$= 1 \quad for\ 1 \leq i \leq n$$

$$g.c.d\{g(u_i), g(v_i)\} = g.c.d\{f_{2i}, f_{2i+1}\} = 1 \quad for\ 1 \leq i \leq n$$

$$g.c.d\{g(u_i), g(u_{i+1})\} = g.c.d\{f_{2i}, f_{2i+1}\} = 1 \quad for\ 1 \leq i \leq n.$$

Thus $g^*(uv) = \gcd\{f(u), f(v)\}$ $=1 \forall uv \in E(G)$.

Thus G admits a Fibonacci prime labeling.

Hence G is a Fibonacci prime graph.

Example 3.8

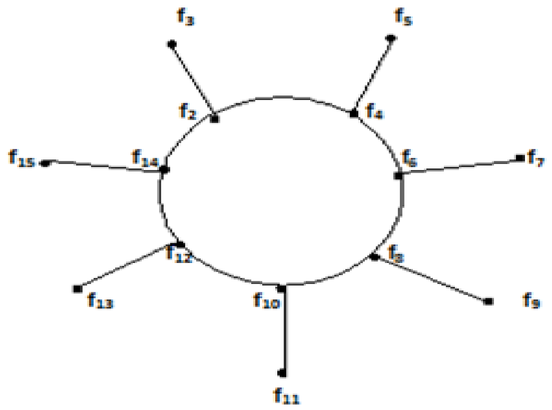

Figure: 4 Fibonacci prime labeling of C_7^*

IV. Conclusion

We proved that cycle related graphs, crown graph are all Fibonacci prime graphs .We extend the study to other families of graph.

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