International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-03, June 2018

Operations on relations of a Tadpole Graph

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Abstract: In this paper, we apply the operations like converse, matrix, complement, composition, associated directed graph on relations of a Tadpole graph to its line graph to analyze how they are interlinked each other. In and out degrees of the associated digraph show the nature of the given graph.

Keywords: Graph, Tadpole graph, relations, complement, matrix, directed graph.

I. INTRODUCTION

The graph (V, E) is denoted by $G = G(V, E)$ where V is called the vertex set and E is called the edge set [1]. A Tadpole graph is denoted by $T_{m,n}$ we mean the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge [2].

Example. $T_{3,2}$ a typical Tadpole graph is in Figure 1.

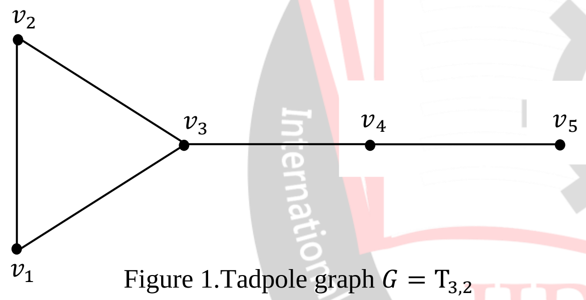


Figure 1.Tadpole graph $G = T_{3,2}$

A line graph of a simple graph $G = G(V, E)$ is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge if and only if the corresponding edges of G have a vertex in common [3]. Line graph for the above graph is in Figure 2.

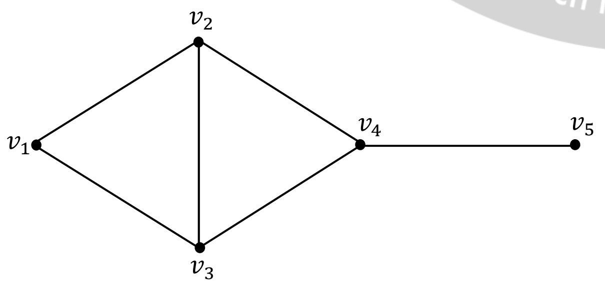


Figure 2.Line graph of a Tadpole graph $G = T_{3,2}$

Remark: To maintain the standardization of a graph (Figure 2.) we represent the e_i's (from Figure 1.) to v_i's (in Figure 2.) where $i = 1$ to 5.

Let A and B be two sets, then a subset of $A \times B$ is called a relation R from A to B where $A \times B = \{(a, b)/a \in A, b \in B\}$, Here $(a, b) \in R$ then $a \in A$ is related to $b \in B$ by R and can be written as aRb. Let R be a relation on a finite set A then the complement of a relation $\overline{R}$ is defined as $\overline{R} = (A \times A) - R$. Consider the sets $A = \{a_1, a_2 \dots \dots a_m\}$, $B = \{b_1, b_2 \dots \dots . . b_n\}$ of orders m and n respectively. Let R is a relation from A to B, then $m_{ij} = 1$, if $(a_i, b_j) \in R$ & $m_{ij} = 0$, if $(a_i, b_j) \notin R$ and matrix $m \times n$ formed by these m_{ij}'s is called matrix of the relation denoted by $M(R)$ where $1 \le i \le m, 1 \le j \le n$ [4].

Let R be a relation on a finite set A. Draw the circle for each element of A. These circles are called vertices. Draw an arrow called edges between the vertices. The resulting pictorial representation of R is called directed graph or digraph of R.

II. RESULTS AND DISCUSSION

In this section, we first find the two finite sets A and B from Figure 1. and Figure 2. to write the relations R & S. From Figure 1. we get the set of vertices say $A = \{v_1, v_2, v_3, v_4, v_5\}$ and from Figure 2. say $B = \{v_1, v_2, v_3, v_4, v_5\}$. Since a relation is a subset of the Cartesian product of two sets, we apply the set-theoretic operations to construct new relations from the arrived relations under the different cases of relations.

Case 1. R is a binary relation on A.

Let $A = \{v_1, v_2, v_3, v_4, v_5\}$, $A = \{v_1, v_2, v_3, v_4, v_5\}$ and

$$R = \left\{ \begin{matrix} (v_1, v_2), (v_1, v_3), (v_2, v_1), (v_2, v_3), (v_3, v_1), \\ (v_3, v_2), (v_3, v_4), (v_4, v_3), (v_4, v_5), (v_5, v_4) \end{matrix} \right\}$$

Then, the converse of R is

$$R^C = \left\{ \begin{matrix} (v_2, v_1), (v_3, v_1), (v_1, v_2), (v_3, v_2), (v_1, v_3), \\ (v_2, v_3), (v_4, v_3), (v_3, v_4), (v_5, v_4), (v_4, v_5) \end{matrix} \right\}$$

Here, we observe that $R = R^C$.

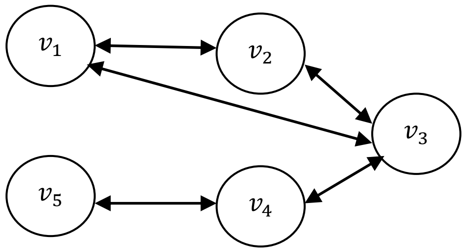
Matrix of R is

 DOI : 10.18231/2454-9150.2018.0379

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-03, June 2018

$$M(R) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} = M(R^C)$$

Directed graph of $R = R^C$:



From the directed graph of $R = R^C$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	2	2	3	2	1
Out degree	2	2	3	2	1

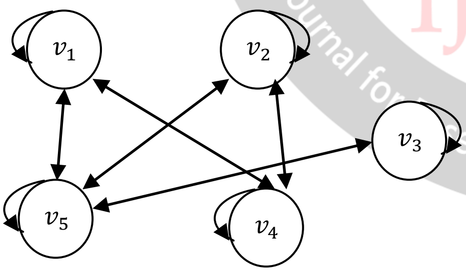
From the above table we observe that In and Out degrees of each vertex are same.

Complement of R is

$$\overline{R} = \begin{Bmatrix} (v_1, v_1), (v_1, v_4), (v_1, v_5), (v_2, v_2), (v_2, v_4), \\ (v_2, v_5), (v_3, v_3), (v_3, v_5), (v_4, v_1), (v_4, v_2), \\ (v_4, v_4), (v_5, v_1), (v_5, v_2), (v_5, v_3), (v_5, v_5) \end{Bmatrix}$$

$$M(\overline{R}) = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Directed graph of $\overline{R}$:



From the directed graph of $\overline{R}$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	3	3	2	3	4
Out degree	3	3	2	3	4

From the above table we observe that In and Out degrees of each vertex are same.

Case 2. S is a binary relation on B

Let $B = \{v_1, v_2, v_3, v_4, v_5\}$, $B = \{v_1, v_2, v_3, v_4, v_5\}$ and

$$S = \begin{Bmatrix} (v_1, v_2), (v_1, v_3), (v_2, v_1), (v_2, v_3), (v_2, v_4), (v_3, v_1), \\ (v_3, v_2), (v_3, v_4), (v_4, v_2), (v_4, v_3), (v_4, v_5), (v_5, v_4) \end{Bmatrix}$$

Then, the converse of S is

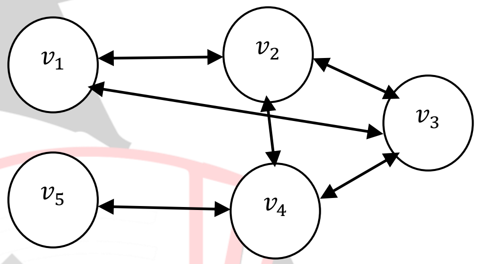
$$S^C = \begin{Bmatrix} (v_2, v_1), (v_3, v_1), (v_1, v_2), (v_3, v_2), (v_4, v_2), \\ (v_1, v_3), (v_2, v_3), (v_4, v_3), (v_2, v_4), (v_3, v_4), \\ (v_5, v_4), (v_4, v_5) \end{Bmatrix}$$

Here, we observe that $S = S^C$.

Matrix of S is

$$M(S) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} = M(S^C)$$

Directed graph of $S = S^C$:



From the directed graph of $S = S^C$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	2	3	3	3	1
Out degree	2	3	3	3	1

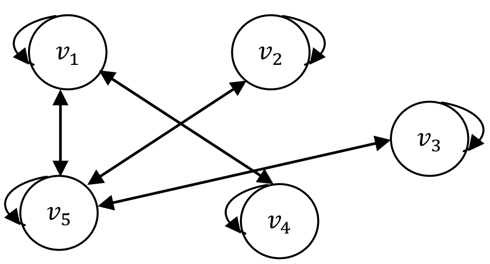
From the above table we observe that In and Out degrees of each vertex are same.

Complement of S is

$$\overline{S} = \begin{Bmatrix} (v_1, v_1), (v_1, v_4), (v_1, v_5), (v_2, v_2), \\ (v_2, v_5), (v_3, v_3), (v_3, v_5), (v_4, v_1), \\ (v_4, v_4), (v_5, v_1), (v_5, v_2), (v_5, v_3), (v_5, v_5) \end{Bmatrix}$$

$$M(\overline{S}) = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Directed graph of $\overline{S}$:



 DOI : 10.18231/2454-9150.2018.0379

From the directed graph of $\overline{S}$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	3	2	2	2	4
Out degree	3	2	2	2	4

From the above table we observe that In and Out degrees of each vertex are same.

Case 3. The relations R and S from A to B.
Let $A = \{v_1, v_2, v_3, v_4, v_5\}$, $B = \{v_1, v_2, v_3, v_4, v_5\}$

$$R = \begin{Bmatrix} (v_1,v_2),(v_1,v_3),(v_2,v_1),(v_2,v_3),(v_3,v_1), \\ (v_3,v_2),(v_3,v_4),(v_4,v_3),(v_4,v_5),(v_5,v_4) \end{Bmatrix} \text{ and}$$

$$S = \begin{Bmatrix} (v_1,v_2),(v_1,v_3),(v_2,v_1),(v_2,v_3),(v_2,v_4),(v_3,v_1), \\ (v_3,v_2),(v_3,v_4),(v_4,v_2),(v_4,v_3),(v_4,v_5),(v_5,v_4) \end{Bmatrix}$$

Then, the union of R and S is

$$R \cup S = \begin{Bmatrix} (v_1,v_2),(v_1,v_3),(v_2,v_1),(v_2,v_3),(v_2,v_4), \\ (v_3,v_1),(v_3,v_2),(v_3,v_4),(v_4,v_2),(v_4,v_3), \\ (v_4,v_5),(v_5,v_4) \end{Bmatrix}$$

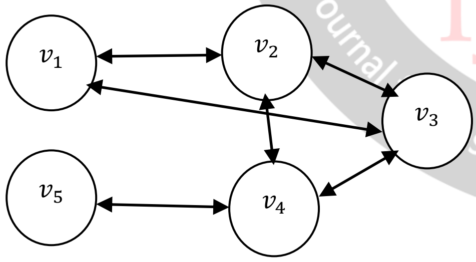
Note: $R \cup S = S$

Matrix of the union of R and S is

$$M(R \cup S) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Note: $M(R \cup S) = M(S)$

Directed graph of $(R \cup S)$:



Note: Directed graph of $(R \cup S)$ = Directed graph of S

From the directed graph of $(R \cup S)$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	2	3	3	3	1
Out degree	2	3	3	3	1

Note: In and Out degrees of the directed graph of $(R \cup S)$ is same as In and Out degrees of the directed graph of S .

The intersection of R and S is

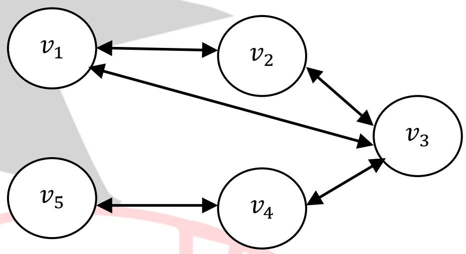
$$R \cap S = \begin{Bmatrix} (v_1,v_2),(v_1,v_3),(v_2,v_1),(v_2,v_3),(v_3,v_1), \\ (v_3,v_2),(v_3,v_4),(v_4,v_3),(v_4,v_5),(v_5,v_4) \end{Bmatrix}$$

Note: $R \cap S = R$
Matrix of the intersection of R and S is

$$M(R \cap S) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Note: $M(R \cap S) = M(R)$

Directed graph of $(R \cap S)$:



Note: Directed graph of $(R \cap S)$ = Directed graph of R

From the directed graph of $(R \cap S)$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	2	2	3	2	1
Out degree	2	2	3	2	1

Note: In and Out degrees of the directed graph of $(R \cap S)$ is same as In and Out degrees of the directed graph of R .

The composition of R and S is

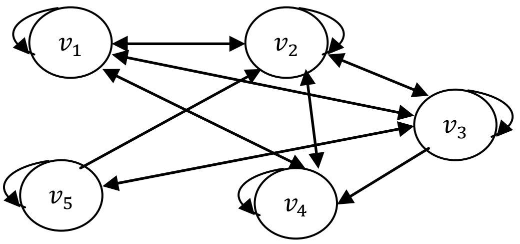
$$RoS = \begin{Bmatrix} (v_1,v_1),(v_1,v_2),(v_1,v_3),(v_1,v_4),(v_2,v_1), \\ (v_2,v_2),(v_2,v_3),(v_2,v_4),(v_3,v_1),(v_3,v_2), \\ (v_3,v_3),(v_3,v_4),(v_3,v_5),(v_4,v_1),(v_4,v_2), \\ (v_4,v_4),(v_5,v_2),(v_5,v_3),(v_5,v_5) \end{Bmatrix}$$

Matrix of the composition of R and S is

$$M(RoS) = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Directed graph of (RoS):

 DOI : 10.18231/2454-9150.2018.0379



From the directed graph of (RoS), we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	4	5	4	4	2
Out degree	4	4	5	3	3

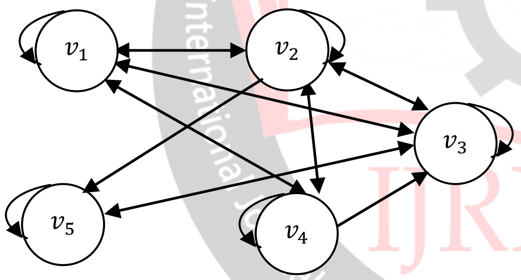
The composition of S and R is

$$SoR = \left\{\begin{matrix}(v_1,v_1),(v_1,v_2),(v_1,v_3),(v_1,v_4),(v_2,v_1),\\(v_2,v_2),(v_2,v_3),(v_2,v_4),(v_2,v_5),(v_3,v_1),\\(v_3,v_2),(v_3,v_3),(v_3,v_5),(v_4,v_1),(v_4,v_2),\\(v_4,v_3)(v_4,v_4),(v_5,v_3),(v_5,v_5)\end{matrix}\right\}$$

Matrix of the composition of S and R is

$$M(SoR) = \begin{pmatrix}1&1&1&1&0\\1&1&1&1&1\\1&1&1&0&1\\1&1&1&1&0\\0&0&1&0&1\end{pmatrix}$$

Directed graph of (SoR):



From the directed graph of (SoR), we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	4	4	4	4	3
Out degree	4	4	5	3	2

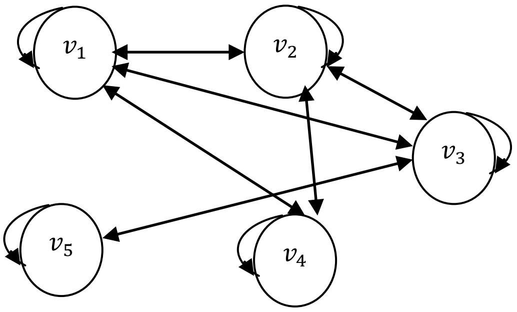
The composition of R and R is

$$RoR = \left\{\begin{matrix}(v_1,v_1),(v_1,v_2),(v_1,v_3),(v_1,v_4),(v_2,v_1),\\(v_2,v_2),(v_2,v_3),(v_2,v_4),(v_3,v_1),(v_3,v_2),\\(v_3,v_3),(v_3,v_5),(v_4,v_1),(v_4,v_2),(v_4,v_4),\\(v_5,v_3),(v_5,v_5)\end{matrix}\right\}$$

Matrix of the composition of R and R is

$$M(RoR) = M(R^2) = \begin{pmatrix}1&1&1&1&0\\1&1&1&1&0\\1&1&1&0&1\\1&1&0&1&0\\0&0&1&0&1\end{pmatrix}$$

Directed graph of $(RoR = R^2)$:



From the directed graph of $(RoR = R^2)$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	4	4	4	3	2
Out degree	4	4	4	3	2

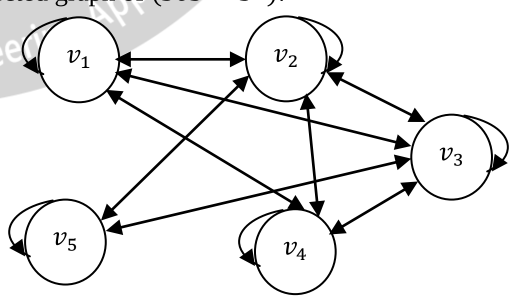
The composition of S and S is

$$SoS = \left\{\begin{matrix}(v_1,v_1),(v_1,v_2),(v_1,v_3),(v_1,v_4),(v_2,v_1),\\(v_2,v_2),(v_2,v_3),(v_2,v_4),(v_2,v_5),(v_3,v_1),\\(v_3,v_2),(v_3,v_3),(v_3,v_4),(v_3,v_5),(v_4,v_1),\\(v_4,v_2),(v_4,v_3),(v_4,v_4),(v_5,v_2),(v_5,v_3),\\(v_5,v_5)\end{matrix}\right\}$$

Matrix of the composition of S and S is

$$M(SoS) = M(S^2) = \begin{pmatrix}1&1&1&1&0\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&0\\0&1&1&0&1\end{pmatrix}$$

Directed graph of $(SoS = S^2)$:



From the directed graph of $(SoS = S^2)$, we have

Vertex	v_1	v_2	v_3	v_4	v_5
In degree	4	5	5	4	3
Out degree	4	5	5	4	3

From the above table we observe that In and Out degrees of each vertex are same.

DOI : 10.18231/2454-9150.2018.0379

III. CONCLUSION

The above results show the application of set-theoretic operations and its analysis on a special graph called Tadpole graph with its line graph and nature of the given graph.

ACKNOWLEDGMENT

The authors are grateful to the referee for their valuable comments and suggestions for improving the paper.

The authors are also grateful to the managements of both the institutions for their support and guidance.

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 DOI : 10.18231/2454-9150.2018.0379