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Some Trigonometric Entropy for Fuzzy Rough Set and their Application in Medical Area

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Abstract: Entropy is an information theoretic measure which is applied to measure an uncertain degree of any data or information. There are two approaches used to determine entropy, the first one approach proposed by Shannon [1948] which is probabilistic in nature and second one proposed by Zadeh [1965] which is non-probabilistic in nature. In this paper, some non-probabilistic trigonometric entropy measures have been proposed for fuzzy rough sets. The validity of these proposed entropy measure has also been checked. At last, the proposed measures have been used in an application for medical diagnosis and data reduction problems by using a hypothetical example related to medical field.

Keywords: Fuzzy rough sets, similarity measures, entropy, fuzzy rough entropy

I. INTRODUCTION

In the literature related to the uncertainty problems, it is observed that most of the time the researcher's community has not embraced the concept of uncertainty. According to the traditional views of science, uncertainty represents an undesirable state which must be handled at all costs. To solve uncertainty related problem, it is significant to measure the degree of uncertainty. Shannon [1948] used "entropy" to measure an uncertain degree of the randomness in a probability distribution. The information contained in this experiment is given by $H(P)=-\sum_{i=1}^{n} p_i \log p_i$, which is known as Shannon Entropy. Later Zadeh [1965] proposed the concept of fuzzy set and defined the non-probabilistic entropy of a fuzzy set which was different from the classical Shannon's entropy as no probabilistic concept was needed in order to define it. Fuzzy entropy is a measure of fuzziness of a set which arises from the intrinsic ambiguity or vagueness carried by the fuzzy set.

Along with fuzzy set theory, there is another concept known as the rough set theory proposed by Pawalk [1982], it is used to handle the unclear and uncertain information but their foundations and highlights are distinct from fuzzy sets theory. As fuzzy sets deal with vague data, while rough sets handle the incomplete information thus both theories are the complement to each other. Since datasets obtained from real-world applications are inherently prone to contain both vague and incomplete information like patients having the same symptoms but suffering from the different diseases. This implies that no definite diagnosis can be applied based on the symptoms knowledge. Thus, to handle the problems which involve vagueness or incompleteness, Nakamura [1988] and Dubois and Prade [1990] gave the idea of hybridization of these two models into fuzzy rough sets and it has been widely used by Nanda and Majumdar [1992]. Fuzzy-rough sets encapsulate the distinct concept of the vagueness of fuzzy sets and indiscernibility relation of rough sets. Both are complementary and can be encountered in the real-life problems. To reduce the information loss caused by the discretization in rough sets in real-valued data sets we can be prefer fuzzy rough sets concept which is more advantageous over rough sets. A large number of machine learning problems are handled by the fuzzy rough approach. So far fuzzy rough sets approach was focused towards attribute selection algorithms to reduce the number of attributes describing the elements in a dataset to obtain a speed up and possible performance gain of posterior learning algorithms. To model the strength of individual attributes and to guide the search for an optimal attribute subset, fuzzy-rough sets are used. Thus hybridization of rough and fuzzy set theory allows performing data analysis on information systems with real-valued datasets directly. The entropy of probability distribution as defined by Shannon [1948] gives a measure of uncertainty about its actual structure. It has been a significant mechanism for characterizing the information content in different modes and applications in many various fields. Several authors like Kosko [1986], Kapur [1997], Pal and Pal [1989, 1999], Gupta et al. [2014] and Sharma et al. [2017] have used Shannon's concept and its variants to measure uncertainty in the rough set and fuzzy set theory. Also, Sharma et al. [22] proposed logarithmic entropy measure for fuzzy soft matrices and discussed their application in decision making and data reduction problems. Some authors like Gupta et al. [2016], Sharma et al. [2017]. Liu [1992] systematically gave the axiomatic

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definition of entropy, distance measure and similarity measure of fuzzy sets and discuss some basic relations between these measures. Also, Gupta et al. [2016] studied entropy, distance and similarity measures under interval-valued intuitionistic fuzzy environment.

In this paper, some new information entropy measures are proposed in fuzzy rough sets. Also, their axiomatic definition is given. The relationship between similarity and entropy measure has been used to derive the proposed entropy. By using existing entropy some trigonometric entropy are derived. The rest of paper is organized as:

In section 2 some theoretical concepts are discussed which have been used in this paper. Some trigonometric entropy has been proposed for fuzzy rough values by using the existing one entropy in section 3 and their validity also proved in the same section. In section 4 corresponding trigonometric entropy has been proposed for fuzzy rough sets. Section 5 deals with the weighted trigonometric entropy for fuzzy rough sets. In next section, an application of proposed measures is also discussed which is related to the medical diagnosis and data reduction in a fuzzy rough environment. At last, the conclusion of the paper has been drawn.

II. THEORETICAL GROUND

In this section we discuss some related concepts and terms of this paper.

Fuzzy Sets: A fuzzy set F on U is characterized by a membership function $\mu_F(x) : U \to [0,1]$, as $F = \{x, \mu_F(x) : x \in U\}$, where U is Universe of discourse. Then $F^C = \{x, 1-\mu_F(x) : x \in U\}$. where F^C is the complement of fuzzy set F.

Rough Sets: A brief recall of rough set is given in next definition:

Definition: Let U be a non-empty universe of discourse and R an equivalent relation on U, which is called an indistinguishable relation, $U/R = \{X_1, X_2, \ldots\ldots, X_n\}$ is all the equivalent class derived from R. $W = (U, R)$ are called an approximation space. $\forall X \subseteq U$, Suppose $\underline{X} = \{x \in U | [x] \subseteq X\}$ and $\overline{X} = \{x \in U | [x] \cap X \neq \emptyset\}$, a set pairs $(\underline{X}, \overline{X})$ are called a rough set in W, and symbolized as $X = (\underline{X}, \overline{X})$; $\underline{X}$ and $\overline{X}$ are the lower approximation and the upper approximation of X on W respecectively.

Fuzzy Rough Sets: The approximation of a crisp set in a fuzzy approximation space is called a fuzzy rough set. We exclaim the pair of fuzzy set $\left(\underline{R}(X), \overline{R}(X)\right)$ a fuzzy rough set with reference set $X \subseteq U$. A fuzzy rough set is characterized by a crisp set and two fuzzy sets:

$$\mu_{\underline{R}(X)}(x) = inf\{1 - \mu_R(x,y) \mid y \notin A\},$$

$$\mu_{\overline{R}(X)}(x) = sup\{\mu_R(x,y) \mid y \in A\},$$

Definition: Let S be the set of the whole rough sets, $A = (\underline{A}, \overline{A}) \in S$, then a fuzzy rough set $X = (\underline{X}, \overline{X})$ in A can be expressed by a pair mapping $\underline{x}, \overline{x}$

$$\underline{x} : \underline{X} \to [0,1], \quad \overline{x} : \overline{X} \to [0,1].$$

Also $\underline{x} \leq \overline{x}$, $\forall x \in \overline{X}$. And then, a fuzzy rough set X in A could be signified by

$$X = \{\langle x, (\underline{x}, \overline{x})\rangle \mid \forall x \in \overline{X}\}.$$

and $\{\langle \underline{x}, \overline{x}\rangle \mid \forall x \in X\}$ is called the value of fuzzy rough of x in A, still written as x.

Suppose X is a fuzzy rough set in A, when A is a finite set, then

$$X = \sum_{i=1}^{n} \langle x, \underline{x_i}, \overline{x_i}\rangle \mid x_i, x_i \in \overline{X}.$$

When A is continuous, then

$$X = \int \langle \underline{x}, \overline{x}\rangle \mid x, \forall x \in \overline{X}.\ over\ A.$$

The whole fuzzy rough set in A is renowned by $F^R(X)$.

Let $X = (\underline{X}, \overline{X})$ be a fuzzy rough set in A, $X^c = \left(\underline{X}^c, \overline{X}^c\right)$ is called the complementary set of $X = (\underline{X}, \overline{X})$, where $\underline{x}^c = 1 - \underline{x}$, $\forall x \in \underline{X}$; $\overline{x}^c = 1 - \overline{x}$, $\forall x \in \overline{X}$.

The order relation in X is defined by the following condition:

$$x \leq y \Leftrightarrow \underline{x} \leq \underline{y}\ and\ \overline{x} \leq \overline{y}.$$

Max-Min-Max Composition: Let $A, B\ and\ C$ be the universal sets and $R\ (A \to B)\ \&\ S\ (B \to C)$ be two fuzzy rough relations. Then the max-min-max composition $Q = SoR$ is defined as the fuzzy rough relation from $A\ to\ C$, where $'o'$ means the composition of membership degrees of S and R in the max-min-max sense. Now the membership function for the lower approximation space and upper approximation space is defined as:

$$\underline{\boldsymbol{\mu_Q(a,c)}} = \max_{a\in A, b\in B, c\in C}\left[\min_{a\in A, b\in B, c\in C}\left(\underline{\boldsymbol{\mu_R(a,b)}}, \underline{\boldsymbol{\mu_S(b,c)}}\right)\right] \quad \forall(\boldsymbol{a,c}) \in \boldsymbol{A \times C}$$

$$\overline{\boldsymbol{\mu_Q(a,c)}} = \min_{a\in A, b\in B, c\in C}\left[\max_{a\in A, b\in B, c\in C}\left(\overline{\boldsymbol{\mu_R(a,b)}}, \overline{\boldsymbol{\mu_S(b,c)}}\right)\right] \quad \forall(\boldsymbol{a,c}) \in \boldsymbol{A \times C}$$

Min-Max-Min Composition: Let $A, B\ and\ C$ be the universal sets and $R\ (A \to B)\ \&\ S\ (B \to C)$ be two fuzzy rough relations. Then the max-min-max composition $Q = SoR$ is defined as the fuzzy rough relation from

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$$\underline{\mu_Q(a,c)} = \min_{a\in A, b\in B, c\in C}\left[\max_{a\in A, b\in B, c\in C}\left(\underline{\mu_R(a,b)}, \underline{\mu_S(b,c)}\right)\right] \quad \forall (a,c) \in A\times C$$

$$\overline{\mu_Q(a,c)} = \max_{a\in A, b\in B, c\in C}\left[\min_{a\in A, b\in B, c\in C}\left(\overline{\mu_R(a,b)}, \overline{\mu_S(b,c)}\right)\right] \quad \forall (a,c) \in A\times C$$

Remark:

- In some situation the max-min-max or min- max-min composition of fuzzy rough relation do not satisfy the condition $\underline{x} \le \overline{x}$ i.e. in that case $\overline{x} < \underline{x}$. To handle the problem we consider fuzzy rough value as a crisp value [0,0]. On the other hand we can use $min - min$ composition for lower approximations and $max - max$ composition for upper approximation, instead of $max - min - max$ composition.
- We may use any one composition according to the circumstances of the problem.

Similarity Measures: A similarity measure or similarity function is a real-valued function that enumerated the similarity between two objects. Although no specific definition of a similarity measures subsisted, usually such measures are some implication of the inverse of distance measures. Similarity measures are exploited in system configuration. Higher scores are given to more-similar quality, and lower or negative scores for dissimilar quality.

Proposition 2.1: Let S be a similarity measure on F, then entropy corresponding to S be as follows:

$$e(A) = S(A, A^c), \forall\, A \in F. \qquad (2.1)$$

where e is entropy on F and is called the entropy generated by similarity measure S. It is denoted by $e(S)$.

III. Entropy for Fuzzy Rough Values

The entropy quantifies the degree of uncertainty. Zadeh [1965] introduced the fuzzy entropy for the first time, de Luca and Termini [1972] introduced the axiom construction of entropy of fuzzy sets. There are several non-probabilistic entropies proposed in literature [6, 7, 8, 9, 10, 11, and 12]. Based on the axioms for the entropy of fuzzy sets [2], we give the definition of entropy for fuzzy rough sets.

For $A \in F^R(X)$, where $F^R(X)$ is the set of all fuzzy rough sets and A is the fuzzy rough set then entropy on A is defined in –(3.1) as follows

Definition 3.1: A real function $e: A \to [0, +\infty)$ is called entropy on A, if e has the following properties for all $x\ \&\ y \in A$ as:

1. $e(x) = 0, if\ x = [0,0] or\ x = [1,1]\ or\ x = [0,1]\ i.e., \underline{x} = 0\ or\ 1\ \&\ \overline{x} = 0\ or\ 1,$
2. $e(x) = e(x^c),$
3. $e(x) = 1$, assume a unique maximum if $x = [0.5, 0.5]\ i.e., \underline{x} = 0.5\ \&\ \overline{x} = 0.5,$
4. $e(x) \ge e(y), if\ y\ is\ crisper\ or\ sharper\ than\ x, i.e.\ y \le x\ for\ x \le 0.5\ (\overline{x} \le 0.5) and\ y \ge x, for\ x \ge 0.5\ (\ \underline{x} \ge 0.5).$

Remarks: In property (1) case [1, 0] is omitted because in fuzzy rough situation this condition does not exist.

In this section entropy is proposed by applying proposition (2.1) using existing similarity measure for fuzzy rough set. In literature various similarity measures for fuzzy rough set are proposed by different researchers. Chengyi et al. [2004] gave the similarity measure between two fuzzy rough sets and fuzzy rough values as follows.

Definition 3.2: Let $A \in F^R(X)$, $x = \langle \underline{x}, \overline{x}\rangle, y = \langle \underline{y}, \overline{y}\rangle$ the fuzzy rough values in A. The degree of similarity between the fuzzy rough values x and y can be evaluated by the function M_Z

$$M_Z(x,y) = 1 - \frac{1}{2}\left(\left|\underline{x} - \underline{y}\right| - \left|\overline{x} - \overline{y}\right|\right) \qquad (3.1)$$

Qi and Chengyi [2008] suggest some rules which are considered when they proposed a similarity measures between fuzzy rough sets and its elements. The similarity measures defined by them as:

Definition 3.3: Let $A \in F^R(X)$, $x = \langle \underline{x}, \overline{x}\rangle, y = \langle \underline{y}, \overline{y}\rangle$ the fuzzy rough values in A. The similarity degree between x and y can be evaluated by the function M,

$$M(x,y) = 1 - \frac{1}{2}(\rho_{xy} + \sigma_{xy}), \qquad (3.2)$$

where $\rho_{xy} = \left|\rho_x - \rho_y\right|$ and $\sigma_{xy} = \left|\sigma_x - \sigma_y\right|$ and $\tau_x = \overline{x} - \underline{x}$ is called the degree of indeterminacy of the element $x \in A$, $\rho_x = \underline{x} + \tau_x\underline{x} = (1 + \tau_x)\underline{x}$ is called the degree of

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favor $x \in A$ and $\sigma_x = 1 - \overline{x} + \tau_x(1 - \overline{x})$ is called the degree of favor $x \in A$.

Corresponding to similarity measure – (3.2) of the elements of Fuzzy rough set Sharma and Gupta [10] define trigonometric similarity measures of fuzzy rough sets elements as:

$$M_{sin}(x, y) = \sin\left[\frac{\pi}{2}\left(1 - \frac{1}{2}\left(\rho_{xy} + \sigma_{xy}\right)\right)\right], \qquad (3.3)$$

Some more similarity measures for fuzzy rough sets are also proposed by Sharma et al. [2017]

Now corresponding to (3.1) by using proposition (2.1) Sharma et al. [21] define entropy for a fuzzy rough set value as:

$$e(x) = 1 - \frac{1}{2}\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right), \forall x \in A, where\ A \in F^R(X)\ \&\ x = \langle \underline{x}, \overline{x} \rangle \quad (3.4)$$

Now corresponding to entropy measure (3.4) we proposed some trigonometric entropy measures for fuzzy rough sets as:

$$e_{sin}(x) = \sin\left[\frac{\pi}{2}\left(1 - \frac{1}{2}\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right)\right)\right], \forall x \in A, where\ A \in F^R(X)\ \&\ x = \langle \underline{x}, \overline{x} \rangle (3.5)$$

$$e_{cos}(x) = \cos\left[\frac{\pi}{4}\left(\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right)\right)\right], \forall x \in A, where\ A \in F^R(X)\ \&\ x = \langle \underline{x}, \overline{x} \rangle \qquad (3.6)$$

$$e_{tan}(x) = \tan\left[\frac{\pi}{4}\left(1 - \frac{1}{2}\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right)\right)\right], \forall x \in A, where\ A \in F^R(X)\ \&\ x = \langle \underline{x}, \overline{x} \rangle (3.7)$$

To prove the validity of proposed entropy measures, the four properties defined in definition (3.1) must satisfy. The validity of proposed entropy is checked by theorem (3.1) as:

Theorem 3.1: The measure $e_{sin}(x), e_{cos}(x)\ \&\ e_{tan}(x)$ defined in (3.5), (3.6) and (3.7) is entropy for fuzzy rough values of set A.

Proof: To prove the theorem, first prove the following lemmas:

Lemma 1: If $A \in F^R(X)$, then $\forall x \in A, where\ x = \langle \underline{x}, \overline{x} \rangle$, the measures defined in (3.5), (3.6) and (3.7) is equal to zero for crisp values that is $if\ x = [0,0] or\ x = [1,1]\ or\ \underline{x} = 0\ or\ 1\ \&\ \overline{x} = 0\ or\ 1$.

Proof: The proposed measures are zero when $\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right) = 2$, it is possible only when $\left|2\underline{x} - 1\right| = 1\ \&\ |2\overline{x} - 1| = 1$, it is possible if, $2\underline{x} - 1 = -1\ or\ 2\underline{x} - 1 = 1\ \&\ 2\overline{x} - 1 = -1\ or\ 2\overline{x} - 1 = 1$ it is possible when $\underline{x} = 0\ or\ 1\ \&\ \overline{x} = 0\ or\ 1$ or for crisp values.

Lemma 2: If $A \in F^R(X)$, then $\forall x \in A, where\ x = \langle \underline{x}, \overline{x} \rangle$, the measure defined in (3.5), (3.6) and (3.7) is equal to their complement.

Proof: The proposed measures is equal to its complement when $\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right)$ is equal to $\left(\left|2(1 - \underline{x}) - 1\right| + |2(1 - \overline{x}) - 1|\right)$. For this evaluate $\left(\left|2(1 - \underline{x}) - 1\right| + |2(1 - \overline{x}) - 1|\right) = \left(\left|1 - 2\underline{x}\right| + |1 - 2\overline{x}|\right) = \left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right)$. Thus the proposed measures is equal to their complement.

Lemma 3: If $A \in F^R(X)$, then $\forall x \in A, where\ x = \langle \underline{x}, \overline{x} \rangle$, the measure defined in (3.5), (3.6) and (3.7) is equal to one i.e., assume a unique maximum if $x = [0.5, 0.5]\ i.e., \underline{x} = 0.5\ \&\ \overline{x} = 0.5$.

Proof: The proposed measures is equal to one when $\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right) = 0$. Thus for $x = [0.5, 0.5]\ i.e., \underline{x} = 0.5\ \&\ \overline{x} = 0.5$, we have $\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right) = (|2(0.5) - 1| + |2(0.5) - 1|) = 0$. Hence proposed measures attains maximum value one.

Lemma 4: If $A \in F^R(X)$, then $\forall x\ \&\ y \in A, where\ x = \langle \underline{x}, \overline{x} \rangle\ \&\ y = \langle \underline{y}, \overline{y} \rangle$, we have $e_{sin}(x) \geq e_{sin}(y)\ \&\ e_{cos}(x) \geq e_{cos}(y)\ \&\ e_{tan}(x) \geq e_{tan}(y), if\ y\ is\ crisper\ or\ sharper\ than\ x, i.e.\ y \leq x\ for\ x \leq 0.5\ (\overline{x} \leq 0.5) and\ y \geq x, for\ x \geq 0.5\ (\underline{x} \geq 0.5)$. where $e(x)$ defined as (3.4).

Proof: In first case let $y \leq x\ for\ x \leq 0.5\ (\overline{x} \leq 0.5)$, then $\underline{y} \leq \overline{y} \leq \overline{x} \leq 0.5\ \&\ \underline{y} \leq \underline{x} \leq \overline{x} \leq 0.5$, then we have $|2\overline{x} - 1| \leq |2\overline{y} - 1|\ \&\ \left|2\underline{x} - 1\right| \leq \left|2\underline{y} - 1\right|$, so we have $\frac{1}{2}\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right) \leq \frac{1}{2}\left(\left|2\underline{y} - 1\right| + |2\overline{y} - 1|\right)$, then $-\frac{1}{2}\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right) \geq -\frac{1}{2}\left(\left|2\underline{y} - 1\right| + |2\overline{y} - 1|\right)$, thus we have $1 - \frac{1}{2}\left(\left|2\underline{x} - 1\right| + |2\overline{x} - 1|\right) \geq 1 - \frac{1}{2}\left(\left|2\underline{y} - 1\right| + |2\overline{y} - 1|\right)$, hence $e_{sin}(x) \geq e_{sin}(y)\ \&\ e_{cos}(x) \geq e_{cos}(y)\ \&\ e_{tan}(x) \geq e_{tan}(y)$, because sine and tangent is increasing function and cosine is decreasing function.

Now in second case let $y \geq x, for\ x \geq 0.5\ (\underline{x} \geq 0.5)$, then $0.5 \leq \underline{x} \leq \overline{x} \leq \overline{y}\ \&\ 0.5 \leq \underline{x} \leq \underline{y} \leq \overline{y}$, then we have $|2\overline{x} - 1| \leq |2\overline{y} - 1|\ \&\ \left|2\underline{x} - 1\right| \leq \left|2\underline{y} - 1\right|$, so we have

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$\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|) \leq \frac{1}{2}(|2\underline{y}-1|+|2\overline{y}-1|)$, then $-\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|) \geq -\frac{1}{2}(|2\underline{y}-1|+|2\overline{y}-1|)$, thus we have $1-\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|) \geq 1-\frac{1}{2}(|2\underline{y}-1|+|2\overline{y}-1|)$, hence $e_{sin}(x) \geq e_{sin}(y)\ \&\ e_{cos}(x) \geq e_{cos}(y)\ \&\ e_{tan}(x) \geq e_{tan}(y)$, because sine and tangent is increasing function and cosine is decreasing function.

Since above four lemmas are proved hence the proposed measure are entropies for fuzzy rough sets A.

IV. Entropy for Fuzzy Rough Sets

Let A be fuzzy rough set in $X = \{x_1, x_2, \dots\dots, x_n\}$, where $A = \sum_{i=1}^{n}[\langle \underline{x_i}, \overline{x_i}\rangle]/x_i \quad x_i \in X$, then corresponding to equation (3.5), (3.6) and (3.7) we proposed following entropies for fuzzy rough set as follows:

$$E_{sin}(A) = \frac{1}{n}\sum_{i=1}^{n} e_{A;\,sin}(x_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \sin\left[\frac{\pi}{2}\left(1-\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|)\right)\right], \forall x_i \in X, where\ A \in F^R(X); x_i = \langle \underline{x_i}, \overline{x_i}\rangle \quad (4.1)$$

$$E_{cos}(A) = \frac{1}{n}\sum_{i=1}^{n} e_{A;\,cos}(x_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \cos\left[\frac{\pi}{4}\left((|2\underline{x}-1|+|2\overline{x}-1|)\right)\right], \forall x_i \in X, where\ A \in F^R(X); x_i = \langle \underline{x_i}, \overline{x_i}\rangle \quad (4.2)$$

$$E_{tan}(A) = \frac{1}{n}\sum_{i=1}^{n} e_{A;\,tan}(x_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|)\right)\right], \forall x_i \in X, where\ A \in F^R(X); x_i = \langle \underline{x_i}, \overline{x_i}\rangle \quad (4.4)$$

It is obvious that proposed entropies lies in the interval [0,1]. Larger the value of entropies the more is uncertainty in A.

The following conclusions are obvious.

Proposition 1: If $A \in F^R(X)$, then $\forall x_i \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle$, is fuzzy rough value of A, then the proposed entropies equal to $zero\ if\ A\ is\ a\ crisp\ set\ i.e., \underline{x_i} = 0\ or\ 1\ \&\ \overline{x_i} = 0\ or\ 1$.

Proposition 2: If $A \in F^R(X)$, then $\forall x_i \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle$, is fuzzy rough value of A, then proposed entropies for A is equal to the entropies for A^c where $A^c = \langle 1-\underline{x_i}, 1-\overline{x_i}\rangle$.

Proposition 3: If $A \in F^R(X)$, then $\forall x_i \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle$, is fuzzy rough value of A, then proposed trigonometric entropies for A is equal to one that is assume a unique maximum if $x_i = [0.5, 0.5]\ i.e., \underline{x_i} = 0.5\ \&\ \overline{x_i} = 0.5$.

Proposition 4: If $A\ \&\ B \in F^R(X)$, then $\forall x_i\ \&\ x_j \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle\ \&\ x_j = \langle \underline{x_j}, \overline{x_j}\rangle$, are fuzzy rough values of $A\ \&\ B$ respectively then the proposed trigonometric entropies for A is greater than or equal to the entropies for B where B is crisper or sharper than$A, i.e. B \leq A\ for\ x_i \leq 0.5\ (\overline{x_i} \leq 0.5) and\ B \geq A, for\ x_i \geq 0.5\ \left(\underline{x_i} \geq 0.5\right)$.

V. Weighted Entropy for Fuzzy Rough Sets

Let A be fuzzy rough set in $X = \{x_1, x_2, \dots\dots, x_n\}$, where $A = \sum_{i=1}^{n}[\langle \underline{x_i}, \overline{x_i}\rangle]/x_i\ x_i \in X$ and w_i is the weight of the element x_i, where $w_i \in [0,1]$, then the weighted trigonometric entropies for fuzzy rough set A can be evaluated by the following:

$$H_{sin}(A) = \frac{\sum_{i=1}^{n} w_i e_{A;\,sin}(x_i)}{\sum_{i=1}^{n} w_i}$$

$$= \frac{\sum_{i=1}^{n} w_i \sin\left[\frac{\pi}{2}\left(1-\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|)\right)\right]}{\sum_{i=1}^{n} w_i}, \forall x_i \in X;\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle \quad (5.1)$$

$$H_{cos}(A) = \frac{\sum_{i=1}^{n} w_i e_{A;\,cos}(x_i)}{\sum_{i=1}^{n} w_i}$$

$$= \frac{\sum_{i=1}^{n} w_i \cos\left[\frac{\pi}{4}(|2\underline{x}-1|+|2\overline{x}-1|)\right]}{\sum_{i=1}^{n} w_i}, \forall x_i \in X;\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle \quad (5.2)$$

$$H_{tan}(A) = \frac{\sum_{i=1}^{n} w_i e_{A;\,tan}(x_i)}{\sum_{i=1}^{n} w_i}$$

$$= \frac{\sum_{i=1}^{n} w_i \tan\left[\frac{\pi}{4}\left(1-\frac{1}{2}(|2\underline{x}-1|+|2\overline{x}-1|)\right)\right]}{\sum_{i=1}^{n} w_i}, \forall x_i \in X;\ x_i = \langle \underline{x_i}, \overline{x_i}\rangle \quad (5.3)$$

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It is obvious that proposed entropies lays in the interval [0,1]. Larger the value of entropies the more is uncertainty in A.

The following conclusions are obvious.

Proposition 5: If $A \in F^R(X)$, then $\forall x_i \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i} \rangle$, is fuzzy rough value of A, then the proposed weighted entropies equal to zero $if\ A\ is\ a\ crisp\ set\ i.e., \underline{x_i} = 0\ or\ 1\ \&\ \overline{x_i} = 0\ or\ 1$.

Proposition 6: If $A \in F^R(X)$, then $\forall x_i \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i} \rangle$, is fuzzy rough value of A, then proposed weighted entropies for A is equal to the entropies for A^c where $A^c = \langle 1 - \underline{x_i}, 1 - \overline{x_i} \rangle$.

Proposition 7: If $A \in F^R(X)$, then $\forall x_i \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i} \rangle$, is fuzzy rough value of A, then proposed weighted trigonometric entropies for A is equal to one that is assume a unique maximum if $x_i = [0.5, 0.5]\ i.e., \underline{x_i} = 0.5\ \&\ \overline{x_i} = 0.5$.

Proposition 8: If $A\ \&\ B \in F^R(X)$, then $\forall x_i\ \&\ x_j \in X, where\ x_i = \langle \underline{x_i}, \overline{x_i} \rangle\ \&\ x_j = \langle \underline{x_j}, \overline{x_j} \rangle$, are fuzzy rough values of $A\ \&\ B$ respectively then the proposed weighted trigonometric entropies for A is greater than or equal to the entropies for B where B is crisper or sharper than$A, i.e.\ B \le A\ for\ x_i \le 0.5\ (\overline{x_i} \le 0.5) and\ B \ge A, for\ x_i \ge 0.5\ \left(\underline{x_i} \ge 0.5\right)$.

VI. APPLICATION OF PROPOSED MEASURES

The theory of FRSs has been utilized to perform medical diagnosis in [1, 17, 19, 20]. Here, an example is discussed how to solve medical diagnosis problem with the fuzzy rough entropies by proposed entropies measures for FRSs.

For the practical point of view, we consider a medical diagnosis problem for illustration of the proposed measures. In the new medical technologies for medical diagnoses problem the information which is offered to the physicians consists of uncertainties and increased volume of information. The method of characterizing the distinctive set of symptoms under a single name of the disease is the very typical job. In some real life problems, there exists the possibility of each element within a lower and an upper approximation of fuzzy rough sets. It can deal with the medical diagnosis to gripping the more indeterminacy. In fact, this method is more flexible and easy to use. The proposed information measures among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis. The main feature of these proposed measures is that they consider lower approximation and upper approximation of each element between two approximations of fuzzy-rough sets by taking one-time inspection for diagnosis. Now, an example of a medical diagnosis is presented.

Let P be the set of the given patients represented as $P = \{P_1, P_2, P_3, P_4\}$ and $D = \{Viral\ Fever, Malaria, Stomach\ problem, Chest\ problem, Typhoi$ be a set of diseases and $S = \{S_1, S_2, S_3, S_4, S_5\}$ be a set of symptoms. Here, our aim is to examine the patient and to determine the disease of the patient in the fuzzy rough environment and remove the symptoms which is unnecessary for diagnosis the diseases.

Table 1 and table 2 shown below represent the relation $R\ and\ S$ between patients-symptoms and symptoms-diseases in fuzzy rough environment.

Table 1. (Fuzzy rough approximation space of patients for the considered symptoms)

$Relation\ R$	S_1	S_2	S_3	S_4	S_5
P_1	[0.4, 0.6]	[0.3, 0.7]	[0.5, 0.9]	[0.5, 0.8]	[0.6, 0.8]
P_2	[0.2, 0.4]	[0.3, 0.5]	[0.2, 0.3]	[0.7, 0.9]	[0.8, 1]
P_3	[0.1, 0.5]	[0, 0.4]	[0.4, 0.8]	[0.4, 0.9]	[0.3, 0.6]
P_4	[0.8, 0.8]	[0.9, 1]	[1, 1]	[0.7, 0.8]	[0.6, 0.6]

Table 2. (Fuzzy rough approximation space of symptoms for the considered diseases)

$Relation\ S$	$Viral\ fever$	$Malaria$	$Stomach\ problem$	$Chest\ Problem$	$Typhoid$
S_1	[0.4, 0.8]	[0.1, 0.5]	[0.3, 0.5]	[0.2, 0.4]	[0.7, 0.9]
S_2	[0.5, 0.7]	[0.2, 0.6]	[0.2, 0.4]	[0.1, 0.5]	[0.6, 0.8]
S_3	[0.4, 0.9]	[0.1, 0.3]	[0.4, 0.6]	[0.1, 0.3]	[0.2, 0.7]

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S_4	[0.5, 0.7]	[0.3, 0.5]	[0.1, 0.3]	[0.5, 0.7]	[0.3, 0.8]
S_5	[0.1, 0.8]	[0.1, 0.3]	[0.1, 0.3]	[0.4, 0.6]	[0.3, 0.7]

Table 3 shows the relation Q between patients-diseases by using max-min-max composition on relation R and S, $Q = SoR$ is following as:

Table 3. (Fuzzy rough approximation space of patients for the considered diseases)

Relation Q	*Viral fever*	*Malaria*	*Stomach problem*	*Chest Problem*	*Typhoid*
P_1	[0.5, 0.7]	[0.3, 0.6]	[0.4, 0.6]	[0.5, 0.6]	[0.4, 0.8]
P_2	[0.5, 0.7]	[0.3, 0.3]	[0.2, 0.5]	**[0,0]**	[0.3, 0.7]
P_3	[0.4, 0.7]	[0.3, 0.5]	[0.4, 0.4]	[0.4, 0.5]	[0.3, 0.7]
P_4	[0.5, 0.8]	[0.3, 0.8]	[0.4, 0.6]	[0.5, 0.6]	[0.7, 0.7]

Now, from table 3 calculate the uncertainties between patients and diseases by using proposed entropy measures which is shown in below as:

$$E_{sin}(P:D) = 0.849281; \quad E_{cos}(P:D) = 0.849281 \quad E_{tan}(P:D) = 0.616727$$

Now, find the uncertainties among patients and Diseases by using proposed measures which are shown in table 4 and graphically in figure 1.

Table 4. (Uncertainties among patients and diseases)

Relation Q	*Viral fever*	*Malaria*	*Stomach problem*	*Chest Problem*	*Typhoid*
$E_{sin}(P_1)$	0.951057	0.891007	0.951057	0.987688	**0.809017**
$E_{cos}(P_1)$	0.951057	0.891007	0.951057	0.987688	**0.809017**
$E_{tan}(P_1)$	0.726543	0.612801	0.726543	0.854081	**0.509525**
$E_{sin}(P_2)$	0.951057	0.809017	0.891007	**0**	0.809017
$E_{cos}(P_2)$	0.951057	0.809017	0.891007	**0**	0.809017
$E_{tan}(P_2)$	0.726543	0.509525	0.612801	**0**	0.509525
$E_{sin}(P_3)$	0.891007	0.951057	0.951057	0.987688	**0.809017**
$E_{cos}(P_3)$	0.891007	0.951057	0.951057	0.987688	**0.809017**
$E_{tan}(P_3)$	0.612801	0.726543	0.726543	0.854081	**0.509525**
$E_{sin}(P_4)$	0.891007	**0.707107**	0.951057	0.987688	0.809017
$E_{cos}(P_4)$	0.891007	**0.707107**	0.951057	0.987688	0.809017
$E_{tan}(P_4)$	0.612801	**0.414214**	0.726543	0.854081	0.509525

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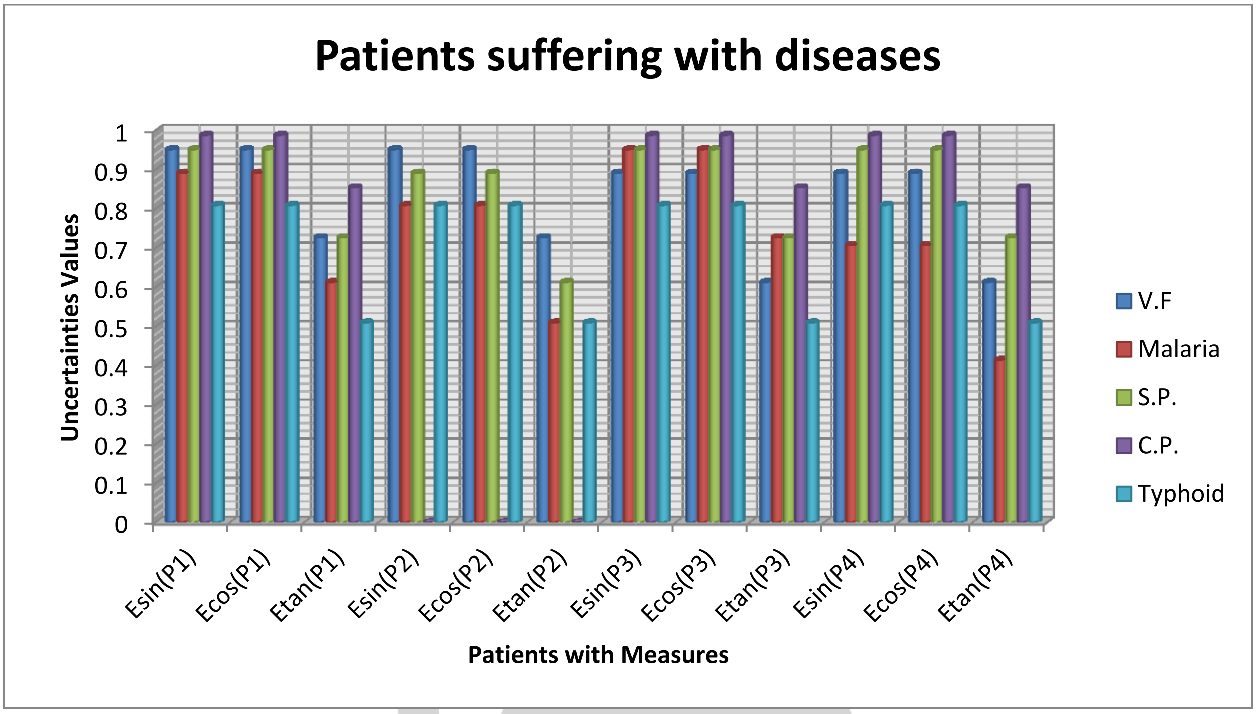

Figure 1: Patients suffering with diseases

From table 4 and figure 1 it is concluded that patient p_1& p_3 suffering from typhoid and p_2 suffering from chest problem whereas p_4 suffering from malaria and also sine and cosine measures are equivalent measure.

Now our goal is to reduce the symptoms which are unnecessary for diagnosis of the diseases of patients. After removal of each symptom individually and pair-wise it is found that although the uncertainties between patients and diseases is reduced but the diagnosis result of patients is alter in all cases except when $S_1$is reduced individually from the set of symptoms, also in that case the uncertainties in between patients and diseases is also minimized which is shown below:

$$E_{sin}(P:D) = 0.836349; \quad E_{cos}(P:D) = 0.836349; \quad E_{tan}(P:D) = 0.594210$$

Thus from figure 2 it is conclude that symptom S_1 does not alter the diagnosis results so it is irrelevant for diagnosis of patients thus it can be reduce from the set of symptoms.

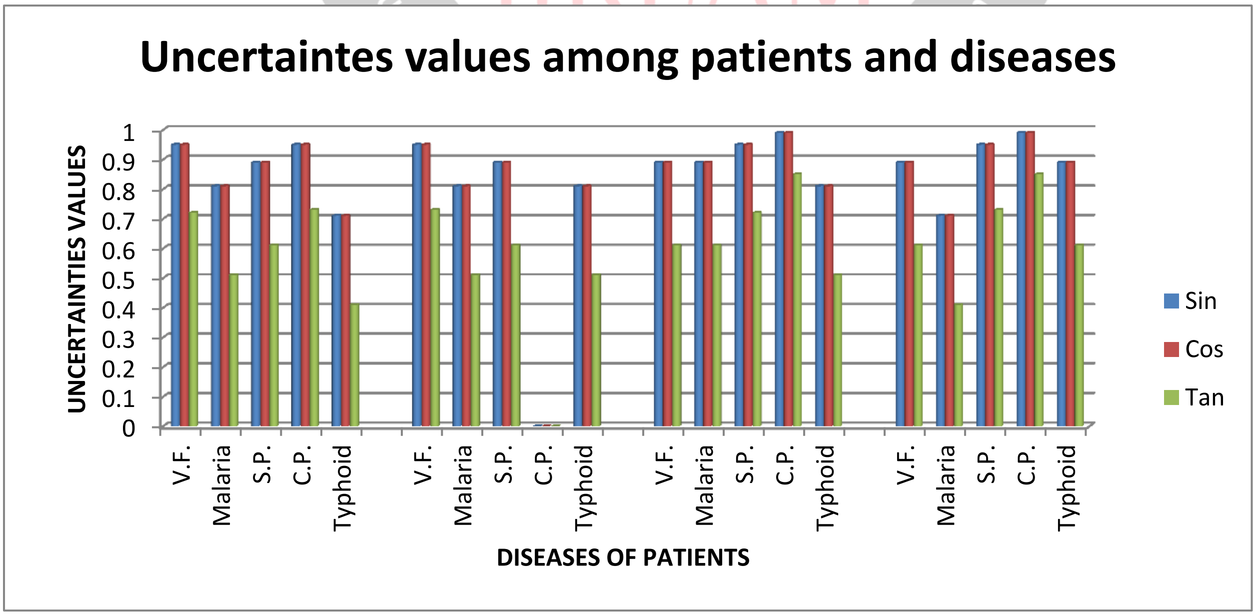

Figure 2: The uncertainties values between patients and diseases when S_1symptom is removed.

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VII. Conclusion

Fuzzy rough set theory offers the flexibility to deal with two types of uncertainty present in information related to decision making or data related problems in daily life. It incorporates fuzzy set theory which considers vagueness with in the rough set frame work handling uncertain information. A variety of measures and method for this integration have been proposed in the literature. In this paper some trigonometric entropy are proposed for fuzzy rough environment and their validity is also proved. At last, an application of these proposed measures have been used in medical diagnosis and data reduction problems. This shows the significance of proposed measures. These proposed measures can be used in other real life problem for fuzzy rough environment.

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