Optimizing and Marketing Policy for Non-Instantaneous Deteriorating Items with Generalized Type Deterioration Rates Under Two-Warehouse Management and Inflation

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Abstract - Today's business environment has become highly competitive due to globalization of the market and therefore it has become necessary to consider the essential marketing parameters affecting the inventory cost of retailers. Selling Price and advertisement cost are one of the most important marketing parameters which must be taken into consideration while modelling for an inventory system. This paper considers an inventory system with deteriorating item under the two-warehouse storage concept. In this paper the demand is a function of advertisement of an item and selling price. This paper assist the retailer in maximizing the total profit by determining the optimal inventory policy and marketing parameters. In contrast to previous inventory models, an arbitrary deterioration rate has been incorporated to provide general framework of the model. First a mathematical model is developed and then numerical examples are included to illustrate and validate the model applicability and effects of key parameters are studied to analyse the behaviour of the model. Sensitivity analysis is performed to study the change in the value of profit function while changing in the value of a parameter at a time and keeping the values of other parameters unchanged. Three cases are discussed depending on the value of no-deterioration period.

Key words: Two-warehouse, inventory, two components dependent demand rate, generalised type deterioration rates.

I. INTRODUCTION

In the present scenario of business, advertisement of product is playing a crucial role in raising sales of product and is a major parameter affecting the marketing policy and it has become the common trend to use the advertising policy to promote a product through print media, electronic media or other means to attract the customers. Considering the effect of price and advertising on demand rate various inventory models under single ware house inventory system have been developed by the researchers like Subramanyam and Kumaraswamy [1] and so forth.

Storage is the major need of an inventory system. In the past era many models are developed using single warehouse concept for stocking goods with consideration that the capacity of warehouse is unlimited. In practical life, it is not true since warehouses are of limited capacity. In the single warehouse inventory system the limited storage is a major practical problem for real situation due to the lack of large storage space at an important market places, forcing retailers to own a small warehouse. However, to reduce the problem of storage retailers prefer to rent a house for a limited period. In case deteriorating items, specially equipped storage facility is required to reduce the amount of deterioration. To handle this situation the requirement of another storage space, providing the required facilities become necessity and therefore it become essential to use an additional warehouse on rental basis for a sort time period according to necessity of business. Additionally rented warehouse is abbreviated as RW and the owned warehouse as OW while studying the two-warehouse inventory system. Generally, it is assumed that the rented ware house provides better storage facilities as compared to own ware-house and due to this the rate of deterioration in RW is smaller than OW which results more holding cost at RW therefore it is necessary to consume the goods kept in RW first and thereafter the demand of customers is fulfilled from OW.

Deterioration in the product is a common phenomenon in an inventory system and products are deteriorated during its normal storage period. The rate of deterioration depends on the nature of the product and the facilities provided in the warehouse. The problem of deteriorating inventory has received considerable attention in recent years and to control and maintain the inventory of the deteriorating

 DOI : 10.18231/2454-9150.2018.0416

items to satisfy the demand of customers or retailers, it becomes an integral part of inventory system to study about deterioration. Ghare and Schrader [2] were first to propose the inventory model where the factor of deterioration was considered on a two-warehouse inventory model and they observed the significant effect of deterioration on the inventory system. Later, there are many authors who use the phenomenon of deterioration with different type of deterioration rates to develop the inventory model considering single warehouse system such as Roy [3], and Covert and Philip [4] and so forth. Some authors studied inventory system under two storage facilities for deteriorating items such as Wee et. al. [5], Yang [6], Sarma [7], Banarjee and Agrawal [8] and Yang [9] and so forth.

In recent years inventory problems for deteriorating items have been widely studied considering different type of deterioration rates under single ware house system and many researchers are using this phenomenon to develop an inventory system considering two-warehouse facilities. Most of the classical inventory models consider the demand of inventories is either constant or time dependent. Many researchers have developed the model considering this type of demand pattern with various combination of business environment but demands of certain type of products depend on the price fixed for selling product. In a competitive market it is general trend that whenever selling price of an item reduces the demand increases and hence sales revenue increases. In essence, the lower selling price raises the selling rates whereas the higher selling price has reverse effect. Several scholars have developed inventory models under consideration with price sensitive demand rates and single warehouse system such as Mukopadhyay et.al. [10], Begam et. al. [11] and Sana [12].Abad [13] has developed an inventory model considering price sensitive demand rates under single warehouse system.

In, traditional models, deterioration rate is considered to be constant and deterioration of product starts as soon as inventory enters into the system but in practical this is not true as some product starts deteriorating after a time interval known as no-deterioration period. The items deteriorated after a fixed time period are known as Non-instantaneous deteriorating items Dye [14] has proposed a joint pricing and ordering policy for deteriorating items and N.H. Shah et. al. [15] has proposed an inventory model for non-instantaneous deteriorating items with generalised type deterioration and holding cost rates to optimize the inventory and marketing policy. In practical, deterioration rate for some products, such as hardware, steel ,wooden made and plastic toys, is so low in the beginning and starts deteriorates after some time period i.e. deterioration rates may vary with time. Covert and Philip developed an inventory model for deteriorating products with variable rate of deterioration.

Inflation is another marketing parameter which directly reflects in the business. The total inventory cost is influenced by inflation. Inflation may occur in the market due to shortages of an item and high demand, low production due to natural calamities, shortages of raw materials, strikes etc.So it become necessary to consider the effect of inflation on the inventory system. Lo et. al. [16] proposed an integrated production-inventory model with imperfect production processes under inflation. Several authors have considered finite replenishment rate for inflationary inventory systems. Wee and Law [17] developed a deteriorating inventory model under inflationary conditions for determining economic production lot size when the demand rate is a linear decreasing function of selling price. Sarkar and Pan [18] studied the effect of inflation and the time value of money on the order quantity with finite replenishment rate. Some authors are developed the stock-dependent demand rate models. Vrat and Padmanabhan [19] determined optimal ordering quantity for stock-dependent consumption rate items and showed that as the inflation rate increases, ordering quantity and the total system cost increases. Hou and Lin [20] developed an inventory model under inflation and time discounting for deteriorating items with stock-dependent selling rate. The selling rate is assumed to be function of the current inventory level and the rate of deterioration is assumed to be constant. These models considers single warehouse. Maiti et. al. [21] developed an inventory model with stock-dependent rate and two storage facilities under inflation and time value of money.

Based on above discussion and motivated by above papers, the proposed inventory model for non-instantaneous deteriorating items under two-warehouse management considers the selling price and frequency of advertisement dependent demand rate and the general type of deterioration rate. The general type of deterioration rates assumes that up to a fix point time there is no deterioration and thereafter deterioration starts depending upon time. The model is developed with the objective of maximizing the profit of the retailers under above consideration and different cases depending on the time up to which an item does not deteriorate while stocked in the warehouse are discussed separately. Numerical examples are provided to demonstrate the developed model and effect of marketing parameters is studied.

II. ASSUMPTIONS AND NOTATIONS

The mathematical model of the two-storage inventory problem is based on the following assumptions and notations:

2.1 Assumptions

- Replenishment rate is infinite and instantaneous.
- Storage capacity of RW is considered to be unlimited.
- The lead time is negligible and initial inventory level is zero.

 DOI : 10.18231/2454-9150.2018.0416

- Shortages are not permitted.
- Demand rate $d(p,A)$ is a function of marketing parameters with the frequency of advertisement (A) and the selling price (p).In this paper, power form of selling price and the frequency of advertisement for demand function is considered; i.e. $d(p,A) = A^{\sigma} a\, p^{-b}$ where a (>0) is scaling factor, b (>1) is index of price elasticity, and σ is the shape parameter, where $0 \le \sigma < 1$.
- In both warehouses, general type of deterioration rate has been considered.
- Holding cost rates has taken as constant in both warehouses.
- The deteriorated units cannot be repaired or replaced during the storage period.
- Deterioration occurs after a fix time period when items are entered into inventory system.
- Inventory system considers a single item.

2.2 Notations

The following notations are used throughout the paper:

M	Inventory level in RW at $t = 0$;
W	Capacity of OW.
$\theta_1(t)$	Deterioration rate in RW.
$\theta_2(t)$	Deterioration rate in OW.
C_a	Cost for each advertisement.
c	Purchase cost per unit of item.
p	Selling price per unit.
h_1	Holding cost per unit per unit time in RW at time t.
h_2	Holding cost per unit per unit time in OW at time t.
Q_{max}	The maximum ordered quantity for a cycle length.
$I_r(t)$	Inventory level at any time t for r =1, 2, 3 etc. in RW and OW.
μ	Time at which inventory level in RW vanish.
T	Length of replenishment cycle.
r	Rate of inflation.
γ	Time up to which there is no deterioration.
P (p, A, μ, T)	Total profit per unit time of inventory system.

III. MATHEMATICAL MODEL

In the beginning of each cycle a M_{max} units of items arrive in the inventory system. W units of items are stored in OW and rest items (M) are stocked into RW. Depending on the values of γ three cases viz.$0 < \gamma \le \mu$, $\mu < \gamma \le T$ and $T < \gamma$ arise. These cases are discussed in details as follows:

Case 1: $0 < \gamma \le \mu$

The graphical representation for this case is shown in figure 1.In this case, the inventory level during the time interval $[0 \quad \mu]$ in RW decreasing to zero due to combined effect of demand and deterioration. Based on above information the status of inventory level at any time t is represented by differential equations

$$\frac{d\, I_1(t)}{dt} = -\, d(p,a); \qquad 0 \le t \le \gamma \qquad (1)$$

$$\frac{d\, I_2(t)}{dt} = -\, d(p,a) - \theta_1(t)\, I_1(t); \; \gamma \le t \le \mu \quad (2)$$

with boundary conditions $I_1(0) = M$ at $t = 0$ and $I_2(\mu) = 0$ at $t = \mu$.The solution of eqs. (1) and (2) is, respectively

$$I_1(t) = M - d(p,a)\, t; \quad 0 \le t \le \mu \qquad (3)$$

$$I_2(t) = d(p,A) f\,(t) \int_t^{\mu} 1/f(x)dx; \quad \gamma \le t \le \mu \qquad (4)$$

where $f(x) = e^{\int_x^{\mu} \theta_1(y)dy}$

From the continuity of $I_2(t)\, at\, t = \gamma$ we have $I_1(\gamma) = I_2(\gamma)$

It follows from eqs. 3 and 4 that order quantity for each cycle is

$$M = d(p,A)\,\{\gamma + f\,(\gamma) \int_{\gamma}^{\mu} 1/f(x)dx\} \qquad (5)$$

Substituting eq. 5 into eq. 3, we have

$$I_1(t) = d(p,A)\,\{\,(\gamma - t) + f\,(\gamma) \int_{\gamma}^{\mu} 1/f(x)dx \qquad (6)$$

In the beginning, inventory level in OW and in the interval $[0 \quad \gamma]$ remains constant as there is no deterioration and level of inventory in the interval $[\gamma \quad \mu]$ reduces due to deterioration only. In the interval $[\mu \quad T]$, the level of inventory reduces due to combined effect of deterioration and demand and reaches to zero at $t = T$. Based on above information the status of inventory level at any time t is represented by differential equations

$$\frac{d\, I_3(t)}{dt} = 0; \qquad 0 \le t \le \gamma \qquad (7)$$

$$\frac{d\, I_4(t)}{dt} = -\theta_2(t)\, I_4(t); \quad \gamma \le t \le \mu \qquad (8)$$

$$\frac{d\, I_5(t)}{dt} = -\, d(p,a) - \theta_2(t)\, I_5(t); \quad \mu \le t \le T \qquad (9)$$

with boundary conditions $I_3(0) = W$ at $t = 0$, $I_4(\gamma) = W$ at $t = \gamma$ and $I_5(T) = 0$ at $t = T$.The solution of eqs. (7), (8) and (9) is, respectively

$$I_3(t) = W; \quad 0 \le t \le \mu \qquad (10)$$

$$I_4(t) = W \frac{e^{\int_{\gamma}^{\mu} \theta_2(y)dy}}{e^{\int_t^{\mu} \theta_2(y)dy}}; \quad 0 \le t \le \mu \qquad (11)$$

$$I_5(t) = d(p,A)\; f\,(t) \int_t^{T} 1/f(x)dx; \quad \mu \le t \le T \qquad (12)$$

where $f(x) = e^{\int_x^{T} \theta_2(y)dy}$

The total order quantity in each cycle is given by

$$M_{max} = W + d(p,A)\,\{\gamma + f\,(\gamma) \int_{\gamma}^{\mu} 1/f(x)dx\}; \qquad (13)$$

 DOI : 10.18231/2454-9150.2018.0416

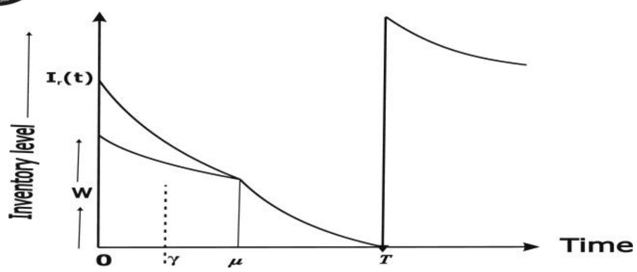

Figure 1: Graphical representation of inventory system for case 1

Thus the total present worth inventory cost during the cycle length, consist of the following costs elements

Ordering cost is K e^{-rT}

The advertisement cost is $C_a * A\, e^{-rT}$

Inventory holding cost in RW is

$$\int_0^\gamma e^{-rT} h_1 I_1(t)dt + \int_\gamma^\mu h_1 e^{-rT} I_1(t)dt$$
$$= e^{-rT}\int_0^\gamma h_1 d(p,A)\left\{(\gamma - t) + f(\gamma)\int_\gamma^\mu \frac{1}{f(x)}dx\right\}dt + \int_\gamma^\mu e^{-rT}\left\{h_1 d(p,A) f(t)\int_t^\mu \frac{1}{f(x)}dx\right\}dt$$

Inventory holding cost in OW is

$$\int_0^\gamma h_2 I_3(t)dt + \int_\gamma^\mu h_2(t) I_4(t)dt + \int_\mu^T h_2(t) I_5(t)dt$$
$$= \int_0^\gamma h_2 e^{-rT}\,\mathrm{W}\,dt + \int_\gamma^\mu h_2 e^{-rT} W\left\{\frac{e^{\int_\gamma^\mu \theta_2(y)dy}}{e^{\int_t^\mu \theta_2(y)dy}}\right\}dt + \int_\mu^T h_2 e^{-rT}\left\{d(c,a)\ f(t)\int_t^T \frac{1}{f(x)}dx\right\}dt$$

The purchase cost is $c\, e^{-rT} * M_{max}$

The sale revenue for a cycle is $e^{-rT}\ p * d(p,A) * T$

Hence the average profit in the time interval $[0\ \ T]$ and for case-1, denoted by P_1 (p, A,μ, T), is given by

$$P_1(\mathrm{p},\mathrm{A},\mu,\mathrm{T}) = \frac{1}{T}\left[\begin{matrix}\text{Sales revenue} - \text{ordering cost} - \text{advertising cost} \\ -\text{holding cost} - \text{purchase cost}\end{matrix}\right] \qquad (14)$$

$$P_1(\mathrm{p},\mathrm{A},\mu,\mathrm{T}) = \frac{1}{T}\Bigg[p\,e^{-rT}d(p,A)T - (C_c\,A + K)\,e^{-rT} - e^{-rT}\int_0^\gamma h_1 d(p,A)\left\{(\gamma - t) + f(\gamma)\int_\gamma^\mu \frac{1}{f(x)}dx\right\}dt - \int_\gamma^\mu e^{-rT}\left\{h_1 d(p,A) f(t)\int_t^\mu \frac{1}{f(x)}dx\right\}dt - \int_0^\gamma h_2 e^{-rT}\,\mathrm{W}\,dt + \int_\gamma^\mu h_2 e^{-rT} W\left\{\frac{e^{\int_\gamma^\mu \theta_2(y)dy}}{e^{\int_t^\mu \theta_2(y)dy}}\right\}dt + \int_\mu^T h_2 e^{-rT}\left\{d(p,A)\ f(t)\int_t^T \frac{1}{f(x)}dx\right\}dt - c\,e^{-rT}M_{max}\Bigg] \qquad (15)$$

Case 2: $\mu < \gamma \leq T$

The graphical representation for this case is shown in figure 2.In this case, no deterioration period is greater than the point at which inventory vanishes in RW and less than the cycle length. Proceeding as case-1, the average profit in the time interval $[0\ \ T]$, denoted by $P_2(\mathrm{p},\mathrm{A},\mu,\mathrm{T})$ is given by

$$P_2(\mathrm{p},\mathrm{A},\mu,\mathrm{T}) = \frac{1}{T}\Bigg[p\,e^{-rT}d(p,A)T - (C_c\,A + K)\,e^{-rT} - \int_0^\mu h_1\,e^{-rT}d(p,A)\ (\mu - t)dt - \int_0^\mu h_2\,\mathrm{W}e^{-rT}dt - \int_\mu^\gamma h_2 e^{-rT}\{W + d(p,A)(\mu - t)\}dt - \int_\gamma^T h_2 e^{-rT}\left\{d(p,a)\ f(t)\int_t^T \frac{1}{f(x)}dx\right\}dt - c\,e^{-rT}M_{max}\Bigg]; \qquad (16)$$

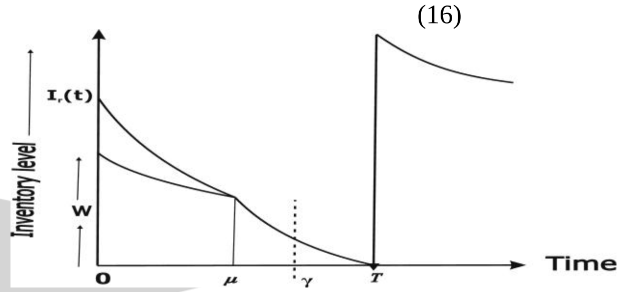

Figure 2: Graphical representation of inventory system for case 2

In this case, the total amount of inventory purchased is given by

$$M_{max} = W + d(p,A)\mu$$

Case 3: $T < \gamma$

The graphical representation of this case is shown in figure 3. In this case, no deterioration period is longer than cycle length. Thus the model become traditional inventory model for non –deteriorating items and the average profit in the time interval $[0\ \ T]$, denoted by $P_2(\mathrm{p},\mathrm{A},\mu,\mathrm{T})$ is given by

$$P_2(\mathrm{p},\mathrm{A},\mu,\mathrm{T}) = \frac{1}{T}\Bigg[p\,e^{-rT}\,d(p,A)\,T - (C_c\,A + K)\,e^{-rT} - \int_0^\mu h_1\,e^{-rT}d(p,A)\ (\mu - t)dt - \int_0^\mu h_2\,\mathrm{W}e^{-rT}dt - \int_\mu^T h_2 e^{-rT}\{d(p,A)\ (T - t)\}dt - c\,e^{-rT}M_{max}\Bigg]; \qquad (17)$$

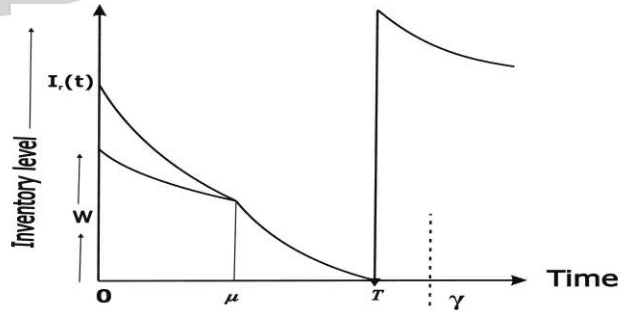

Figure 3: Graphical representation of inventory system for case 3

In this case, the total amount of inventory purchased is given by

$$M_{max} = W + d(p,A)\mu$$

Thus the total profit of the inventory system is given by

 DOI : 10.18231/2454-9150.2018.0416

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-03, June 2018

$$P(\mathrm{p,A},\mu,\mathrm{T}) = \begin{cases} P_1(\mathrm{p,A},\mu,\mathrm{T}) & if\ 0 < \gamma \le \mu \\ P_2(\mathrm{p,A},\mu,\mathrm{T}) & if\ \mu < \gamma \le T \\ P_3(\mathrm{p,A},\mu,\mathrm{T}) & if\ T < \gamma \end{cases} \quad (18)$$

IV. SOLUTION PROCEDURE

The necessary and sufficient condition to maximize the total average profit is

Maximize $P(\mathrm{p,A},\mu,\mathrm{T})$
Subject to: $(\mu > 0, T > 0)$

For known values of frequency of advertisement and selling price, the following condition must be satisfied to find the optimal solution

$$\frac{\partial P(\mathrm{p,A},\mu,\mathrm{T})}{\partial \mu}=0; \qquad \frac{\partial P(\mathrm{p,A},\mu,\mathrm{T})}{\partial \mathrm{T}} = 0; \qquad (19)$$

and

$$\left(\frac{\partial^2 P(\mathrm{p,A},\mu,\mathrm{T})}{\partial \mu^2}\right)\left(\frac{\partial^2 P(\mathrm{c,A},\mu,\mathrm{T})}{\partial T^2}\right) - \left(\frac{\partial P(\mathrm{p,A},\mu,\mathrm{T})}{\partial \mu\, \partial \mathrm{T}}\right)^2 < 0$$

Solving equation (19) for known values of A and p, the optimal value of decision variables can be obtained as μ^* and $\mathrm{T}^*$and with these values the total average profit can be calculated using eq. (18) depending on the value of parameter γ.

V. NUMERICAL EXAMPLES

To illustrate the developed model, deterioration rate has taken as the linear function of time in RW and OW are considered respectively as

$\theta_1(t) = \alpha t\ \ where\ \ \alpha > 0;$

and

$\theta_2(t) = \beta t\ where\ \beta > 0;$

Example-1: In this example the following set of values of parameters are considered as $\mathrm{K} = \$200$ per order, $\mathrm{b} = 2$, $\mathrm{a} = 1000$, $\mathrm{c} = \$5$ per unit, $h_1 = \$\,0.6$ per unit per year, $h_2 = \$0.4$ per unit per year, $\mathrm{A_c} = \$70$ per advertiesment, $\alpha = 0.04$, $\beta = 0.06$, $\sigma = 0.02$, $\mathrm{W} = 100$, $r = 0.03$ .The results obtained from using the model for two cases 1 and 2 are given in Table-1.

Table-1:Average profit

Case	p	A	μ^*(Week)	T*(Week)	Q_{max}	Average profit ($)
1 ($\gamma = 0.2$)	2	1	1.5434	5.5076	494.6	144.74
		2	1.5020	5.4259	488.9	147.00
		3	0.1179	2.7741	130.1	154.43
	3	1	0.3195	3.5379	135.2	123.81
		2	0.7548	4.3316	185.4	110.76
		3	Infeasible			
	4	1	Infeasible			
		2	Infeasible			
2 ($\gamma = 2$)	2	1	1.6605	3.5732	415.1	254.57
		2	1.6570	3.6905	420.0	242.52
		3	1.6536	3.8049	422.5	228.93
	3	1	1.6398	4.2703	182.2	154.12
		2	1.6344	4.4500	184.1	143.60
		3	1.6393	4.6221	185.0	132.26

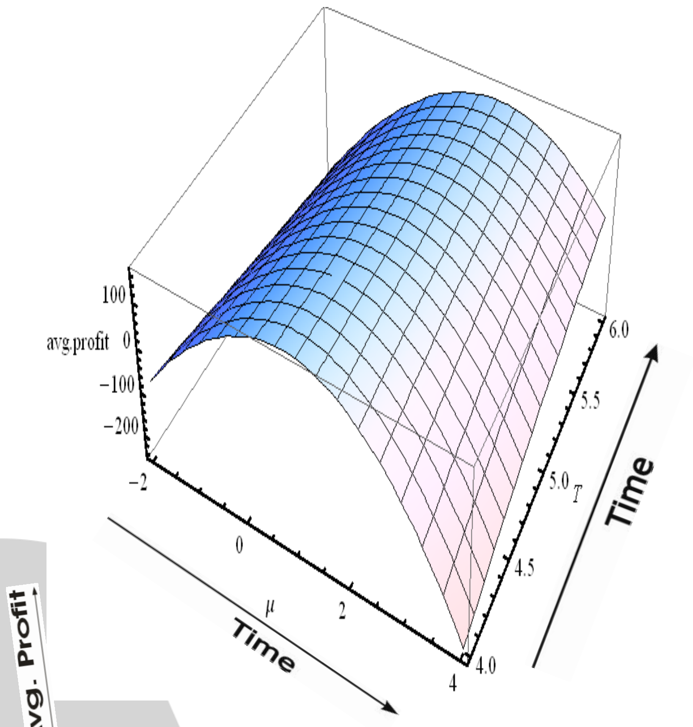

Figure 4: Graphical representation of inventory system case 1

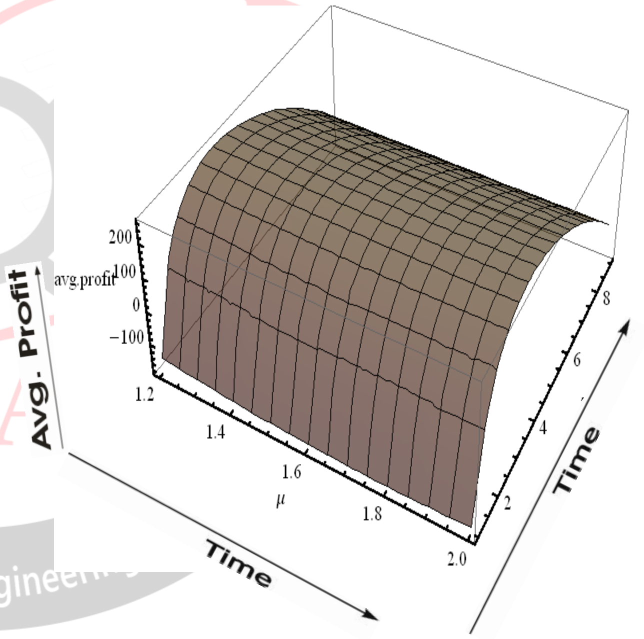

Figure 5: Graphical representation of inventory system case 2

Example 2: In this example, same set of values of parameters are considered as given in example 1.Since in this case the model becomes traditional model as no deterioration period is larger than the cycle length. The result obtained in this case is given in Table-2.

Table-2:Average profit

Case	A	p	μ^*(week)	T*(Week)	Q_{max}	Average profit ($)
3	1	2	5.8366	18.6043	1559.1	560.91
		3	5.5460	15.7448	716.2	291.0
		4	5.4218	13.8528	438.8	186.40

 DOI : 10.18231/2454-9150.2018.0416

	2	2	5.8507	18.7170	1583.9	567.37
	3	2	5.8654	18.8302	1598.9	570.53
	4	3	5.6351	16.315	743.7	293.09

Example-3: In this example the values of parameters are considered same as in case 1 of example 1 except the values of c= $21 and $A_c = \$60$ and rate of deterioration is considered to be constant. The results obtained using the model for two cases 1 and 2 are given in Table-3.

Table-3: Average profit

Case	p	A	μ^*(week)	T^*(Week)	Q_{max}	Average profit ($)
1 ($\gamma = 0.2$)	21	1	35.1426	139.76	101.0	2.73
2 ($\gamma = 100$)	21	1	35.3639	190.59	100.83	20.1484
3	21	Infeasible				

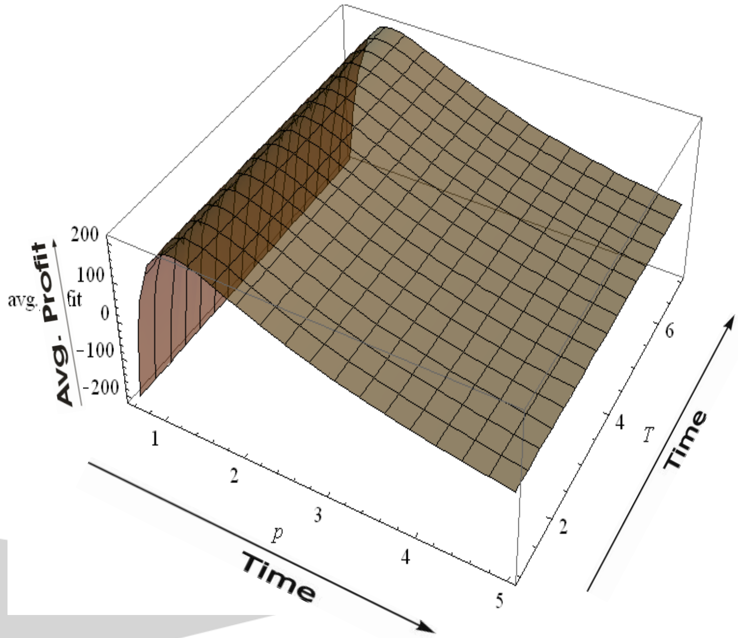

Figure 7: Graphical representation of inventory system for case 1 (p, T verses average profit)

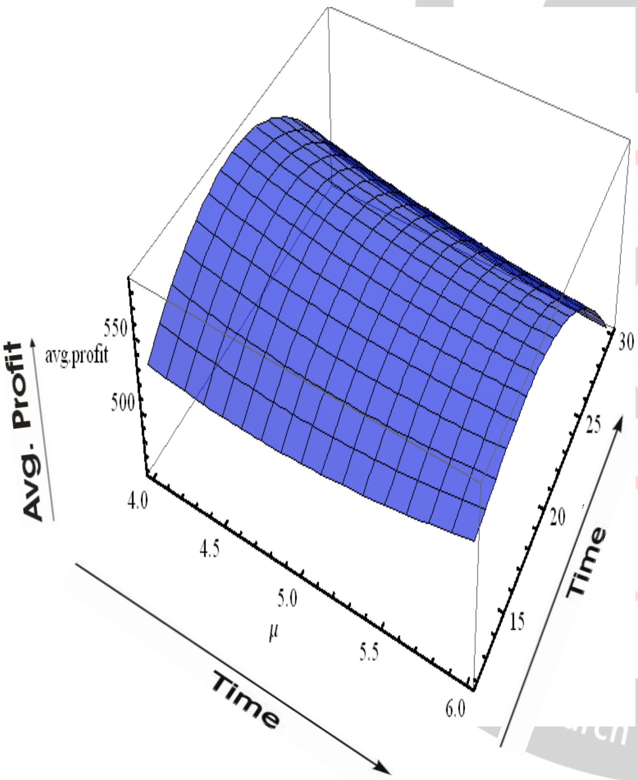

Figure 6: Graphical representation of inventory system of example-2 (case 3)

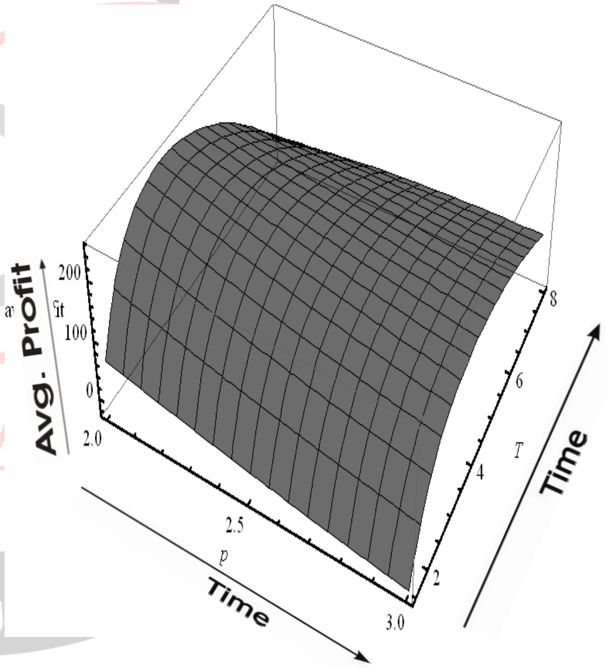

Figure 8: Graphical representation of inventory system for case 2 (p, T verses average profit)

 DOI : 10.18231/2454-9150.2018.0416

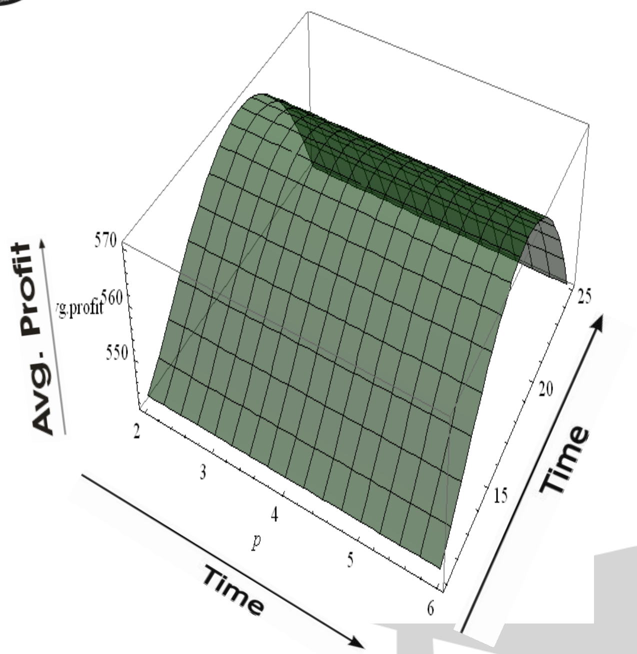

Figure 9: Graphical representation of inventory system for case 3
(p, T verses average profit)

VI. Sensitivity Analysis

To study the effect of the developed model, sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values.

Table-4: Change in average profit

Parameter	μ^*	T*	Q_{max}	Average profit ($)
p (1)	1.6345	5.5255	1776.9	-54.30
(3)	1.6355	5.5528	286.1	119.56
K (100)	1.5251	5.4681	498.3	143.36
(200)	1.5251	5.4681	498.3	112.78
h_1 (0.3)	0.2416	2.8844	161.8	154.81
(0.9)	1.3053	5.6083	438.8	114.55
h_2 (0.2)	0.8462	6.0612	317.8	208.85
(0.6)	0.4321	2.6517	210.5	95.82
A (2)	1.5020	5.4259	488.9	137.00
(4)	0.2666	3.0685	168.5	134.37
α (0.02)	1.4563	5.3209	479.6	128.05
(0.06)	1.4503	5.3423	478.0	127.65
β (0.03)	2.4242	7.6568	737.3	76.75
(0.09)	0.9399	4.1933	343.0	144.96
σ (0.01)	1.4480	5.3241	473.2	125.46
(0.03)	1.4585	5.3394	484.4	130.26
A (500)	Infeasible			
(800)	0.3360	3.2779	168.7	97.27
(1500)	1.5947	5.5326	726.17	236.45
b (1)	1.6552	5.6158	968.2	344.40
(3)	Infeasible			
(2.4)	0.4212	3.4235	177.7	79.98
W (50)	1.5289	5.3709	499.7	141.07
(150)	1.3768	5.2945	508.1	114.79
r (0.015)	1.3865	5.2756	460.0	145.98
(0.045)	1.5150	5.3802	495.62	109.7
A_c (35)	1.5283	5.4743	494.2	144.12
(105)	0.3515	3.2371	189.87	122.57

VII. Result Analysis

On the basis of the results of tables, the following observations can be made:-

1) The length of cycle and order quantity decreases and profit is maximum for A=, fixed value of p=2 and increasing number of advertisement in case-1.The profit is decreasing with increase in the value of selling price.
2) Length of cycle is increasing and profit is maximum for A=1,fixed value of p=2 and increasing number of advertisement in case-2.The profit is decreasing with increase in the value of selling price with increasing number of advertisement.
3) The length of cycle increases and profit is maximum for A=3,fixed value of p=3 and increasing number of advertisement in case-3 when the length of cycle is highest. The profit is decreasing with increase in the value of selling price and number of advertisement.
4) The concavity of developed inventory model for case-1(fixed value of $A = 1$ and $p = 2$)is depicted in Figure-4.
5) The concavity of developed inventory model for case-2 (fixed value of $A = 1$ and $p = 2$) depicted in Figure-5.
6) The concavity of developed inventory model for case-3(fixed value of $A = 3$ and $p = 2$) depicted in Figure-6.
7) From the Table-1 and case-2, it is observed that in case-3, the model is more profitable as compared to case-1 and case-2 even though the time period for rented warehouse is much higher than the case-1 and case-2.This shows if deterioration does not occur during storage period then the profit is maximum.
8) It is also observed in case-1, case-2 and case-3 that, as the frequency of advertisement and selling price increases profit decreases.
9) Concavity of profit function for case-1, case-2 and case-3 (for fixed value of μ^*) are shown in Figure-7, 8 and 9 respectively.
10) When deterioration rates are considered as constant parameter, the value of profit is very low and cycle length is longer even though the selling price is much higher as compared to time varying deterioration rate, it is observed fromTable-3.
11) From Table-4, it is observed that the profit function indirectly proportional to the p, K, h_1, h_2,b, W r, α , A_c and is directly proportional to the σ , β.

VIII. Conclusion

In this paper, a two-warehouse deterministic inventory model with generalized type deterioration rate and constant holding cost rate is developed to optimize the total profit of retailer. Price and advertisement dependent demand rate is evolved to decide the marketing policy for optimal solution. The optimization technique is used to derive the optimum replenishment policy and results are obtained using software Mathematica 9.0. This paper is focused on development of profit function for retailers under the change in the value of selling price and demand. From result analysis it is concluded that profit is found to be maximum, in case variable deterioration rate as compared

 DOI : 10.18231/2454-9150.2018.0416

to traditional model described in example-3 for each case .Thus the model is most useful when selling price is lower and inventory is purchased in bulk quantity and a warehouse is rented to stock goods. Since the profit function is directly proportional to the deterioration rate therefore increasing rate of deterioration maximizes the average profit. Finally, after study it is concluded that model is very useful when selling price of products to be lowered and deterioration period decreases. Further this paper can be enriched by incorporating other types of demand such as probabilistic, stochastic and generalized type holding cost rates in combination with time discounting and time value of money.

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 DOI : 10.18231/2454-9150.2018.0416