Comparison of Numerical Techniques for The Solution of Boundary Value Problem

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Abstract: A comprehensive study of solution of boundary value problem is attempted in this study. The technique of orthogonal collocation method (OCM) and orthogonal collocation on finite elements (OCFE) are used to solve boundary value problem numerically and analytically using Laplace transformation. It has been concluded that to solve stiff boundary value problems analytically is very time consuming and difficult to solve. The results are obtained by MATLAB ODE 15s system solver software. Comparison is shown both in tabulated and graphical form for different values of parameter .Relative error is used to check the efficiency of the technique.3-D graphs are used to specify the behaviour of solution for different values of parameter.

Keywords —Boundary value problems, collocation, Laplace, MATLAB ODE 15s, OCM, OCFE

I. INTRODUCTION

In the study of mathematical and scientific fields such as Geology (Modeling of Weather, Seismic wave detection), Astronomy (Celestial Mechanics), Physics (Fluid flow), Chemistry (Reaction rates), Biology (infectious diseases, genetic variation), ecology (population modeling, increase / decrease of population), economics (stock trends, interest rates, market equilibrium price changes) and various engineering applications (diffusion-dispersion, adsorption-desorption, Current Flow in Electric Circuits, Heat Dissipation in Solid Objects), the differential equations play a major role.

There are many types of differential equations such as diffusion equation, advection-diffusion equation, Burgers-Huxley equation, Burger-Fisher equation, Fisher-Kolmegnov equation, Fitzhogh-Nagumo equation, Kurmato-Sivashinsky equation, Kawahara equation etc., which describes the various physical problems.

Therefore differential equations play a vital role in the field of Mathematics, especially in modeling and simulation. The differential equations with an additional set of constraints are called boundary value problems.

Boundary value problems have been developed and used in science and engineering for more than 200 years, yet they remain very active and interesting research area because of their application in science and engineering. Computer solutions of boundary value problems spurred their study.

Increasing applications of mathematics in solving real life problems by using mathematical equations has given rise to the study of a new subject of mathematical modeling. It has also increased the applications of mathematics in industry and developed the concept of industrial mathematics. Numerous practical problems are solved by using mathematical equations and mathematical techniques. A variety of mathematical models has been used for improvement of production, increase of profits and understanding of complicated processes by many industries.

Study of boundary value problems involves another important step that is the solution of the boundary value problem. Number of numerical techniques has been developed time to time to solve the different type of boundary value problems. Laplace transform [Cuomo et al. 2007; Davis 1985; Liao & Shiau 2000 and Zheng & gu 1996], Fourier transform [Jaiswal et al. 2011; Kumar et al. 2012 and Nur Muhammad et al. 2005], Homotopy perturbation method [He 2003 and Zheng 1996], Least square method [Jia et al. 2013;Rasmuson & Nerettnieks 1980 and Solsvik & Jacobsen 2012], finite difference method [Caglar et al. 2006; Fletcher 1983;Gurarslan et al. 2013 and Zhao 2013],finite element method[Caldwell et al. 1981; Chen and Jiang 2004; Luo et al. 2013; Onah 2002; Robalo et al. 2013 and Sharma et al. 2012], spline collocation [Ali et al. 2009;Bialecki 2003; Bialecki & Faiweather 2001; Botella 2002; Danumjaya & Nandakumarab et al. 2006; Goh et al. 2012; Houstis et al. 1986; Johnson 2005;Mittal 2016; Mittal & Arora 2010; Morinishi et al. 2003; Pedas & Tamme 2006; Rashidinia et al. 2008; Sun et al. 2007 and Woo & Kim 2001], orthogonal collocation [Arden et al. 2007; Feiz 1997; Soliman 1998; Solsvik and Jacobsen 2012; Villadsen & Stewart 1967 and WU et al. 2011], orthogonal collocation on finite elements [Arora & Kaur 2015; Arora & Kaur 2016; Arora & Potucek 2012; Arora et al. 2005; Arora et al. 2006; Arora et al. 2006a; Arora et al. 2015; Arora et al. 2015a; Barrozo et al. 2006; Chang & Finlayson 1977; Ganaie et al. 2013; Ganaie et al. 2013a; Ganaie et al.

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2013b; Ganaie et al. 2014; Lang & Sloan 2002; Mittal et al. 2013 and Mittal et al. 2013a] etc.

Application of the numerical technique depends upon its rate of convergence. Higher the rate of convergence implies better will be the numerical technique. Laplace transform, Fourier transform are from the family of analytic techniques which are followed to solve the linear boundary value problems. However, finite difference method and collocation techniques are from the family of numerical techniques. The main difference between analytic and numerical techniques is that former solves the equation on an abstract set whereas later discretize the problem on a given set of points.

Collocation techniques are one of the weighted residual methods. In such type of techniques, the solution of the given problem is replaced by an approximating function. This approximating function is adjusted to the differential equation and the boundary conditions as well. The residual is set equal to zero at the collocation points. As the collocation techniques have easy adaptability to the computer codes and are simple than Galerkin method and finite difference method, these techniques have gained momentum in the solution of boundary value problems.

II. COLLOCATION POINTS

Choice of the collocation points is an important and sensitive part of orthogonal collocation method. In this method, collocation points are taken to be the zeros of orthogonal polynomials. Usually, the zeros of Jacobi polynomial are taken as collocation points. Legendre and Chebyshev polynomials are particular cases of Jacobi polynomial.

The collocation points are obtained by mapping the computational domain of the interval [-1, 1] to [0, 1] with the help of following relationship:

$$\xi_{m+3-j} = \frac{x_j}{2} + \frac{1}{2} \qquad (1)$$

where x_j is the j^{th} collocation point in the interval [-1, 1]. The m+2 interpolation points are chosen to be the extreme values of an m+1th order shifted Chebyshev polynomial.

$$x_j = \cos\frac{\pi(j-1)}{m+1} \quad ; \quad j = 1, 2,\ldots, m+2. \qquad (2)$$

The discretization end points are fixed as $\xi_1 = 0$ and $\xi_{m+2} = 1$.

The zeros of Legendre polynomial are calculated from the following recurrence relation:

$$(j-1)P_{j-1}(x) = (2j-3)xP_{j-2}(x) - (j-2)P_{j-3}(x)$$
$$; j = 2,\ldots, m+1 \qquad (3)$$

where $P_0(x) = 1$ and $P_{-1}(x) = 0$. In case of Legendre polynomial, 0 and 1 are taken to be the boundary points.

ORTHOGONAL COLLOCATION ON FINITE ELEMENTS

The orthogonal collocation method does not give fast converging results for large values of the parameters in case of stiff system of boundary value problems as mentioned by Carey and Finlayson (1975), Liu and Bhatia (1999), Arora et al. (2005). To overcome this problem orthogonal collocation method is conjectured with the finite element method to combine the features of both the methods.

Orthogonal collocation on finite elements was first proposed by Patterson and Cresswell (1971). It was further extended by Carey and Finlayson (1975) to solve effectiveness factor problem for large Thieles modulus. Ma and Guiochon (1991) have used orthogonal collocation on finite elements for calculation of chromatographic elution band profiles for different components. Arora et. al., (2009) has checked the asymptotic behavior of parabolic partial differential equations using finite element collocation method. Liu & Jacobsen (2004) have carried out the bifurcation analysis of heat integrated fixed bed reactor by reducing the order of distributed model using orthogonal collocation on finite elements.

In orthogonal collocation on finite elements, the whole domain is divided into small sub domains, called elements. The orthogonal collocation is applied within each element. In this process it is mandatory that the trial function and its first derivative should be continuous at the nodal points or the boundaries of the elements.

In orthogonal collocation on finite elements, The global variable x varies in the ℓ^{th} element, where $\ell = 1, 2, 3,\ldots, ne$. The node points are set at $x_1, x_2, x_3,\ldots, x_{ne+1}$.

The boundary points have been placed as $x_1 = 0$ and $x_{ne+1} = 1$. To apply the orthogonal collocation with in ℓ^{th} element, a new variable ξ is introduced in the ℓ^{th} element. The variable ξ is introduced in such a way that as x varies from x_ℓ to $x_{\ell+1}$, ξ varies from 0 to 1 in the ℓ^{th} element such that

$$\xi = \frac{x - x_\ell}{x_{\ell+1} - x_\ell}.$$

Orthogonal collocation is applied on the variable ξ within ℓ^{th} element, $\ell = 1,2,\ldots,ne$ .To avoid the problem of double calculation at the node points, the approximating function and its first derivative are taken to be continuous at the node points by using the principle of continuity.

$$y^{\ell}\Big|_{x_{\ell}+} = y^{\ell+1}\Big|_{x_{\ell+1}-} \qquad (4)$$

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$$\left.\frac{dy^{\ell}}{dx}\right|_{x_{\ell}+} = \left.\frac{dy^{\ell+1}}{dx}\right|_{x_{\ell+1}-} \quad (5)$$

These conditions are also called the continuity conditions. With the help of these conditions, the problem of double calculation arising in the application of orthogonal collocation on finite elements is overcome.

Solution of Boundary value Problems

In this section, both orthogonal collocation method (OCM) and method of orthogonal collocation on finite elements (OCFE) are used to solve boundary value problem numerically. The mathematical equation involving diffusion-dispersion phenomenon describing the pulp washing in packed beds of finite length with adsorption isotherm is described by Kukerja (1996) for washing zone is discussed below:

$$\frac{\partial c_1}{\partial t_1} + u_1\frac{\partial c_1}{\partial z_1} + \mu\frac{\partial n_1}{\partial t_1} = D_{L_1}\frac{\partial^2 c_1}{\partial z_1^2} \quad (6)$$

With adsorption isotherm

$$n_1 = kc_1 \quad (7)$$

equation (6) represent the mathematical models of diffusion-dispersion of washing, t_1 is the time of the displacement,z_1 is the distance from the initial point of the displacing fluid, $c_1 = c_1(z_1,t_1)$ is the solute concentration, the average interstitial velocity co-efficient is D_{L_1}, the average interstitial velocity of the fluid is $u_1$and the length the bed length is L_1.

At the bed $z_1 = 0$, the boundary condition is:

$$c_1 = c_s \; At\; z_1 = 0, \quad (8)$$

$$\frac{\partial c_1}{\partial z_1} = 0 \; At\; z_1 = L_1 \quad (9)$$

And at the bed $z_1 = 0$, the initial condition

$$c_1 = c_1\,(z_1,0) = c_1 = n_1(z_1,0)$$

For $0 < x_1 < \frac{L_1}{u_1}$ (10)

Conversion of the model into dimensionless form

Using the dimensionless variable change the relations in dimensionless form by introducing the following:

$$c = \frac{c_1 - c_s}{c_i - c_s};\; N_1 = \frac{n_1 - c_s}{c_i - c_s};\; Z_1 = \frac{z_1}{L_1};\; T_1 = \frac{u_1 t_1}{L_1};$$

The dimensionless time $T_1 = \frac{u_1 t_1}{L_1}$, dimensionless concentration of solute in liquor $c = \frac{c_1 - c_s}{c_i - c_s}$, dimensionless concentration of solute in fiber $N_1 = \frac{n_1 - c_s}{c_i - c_s}$, at the initial stage of the experiment is equal to the ratio of total fluid volume initial at the free value of the bed.

After dimensionless form, equation (6) becomes

$$\frac{\partial c}{\partial T_1} = \frac{1}{Pe}\frac{\partial^2 c}{\partial Z_1^2} - \frac{\partial c}{\partial Z_1} \quad (11)$$

While $Pe = \frac{u_1 L_1}{D_{L_1}}$ is the Peclet number.

The boundary conditions in the dimensionless form

$$Pec - \frac{\partial c}{\partial Z_1} = 0 \; at\; X_1 = 0 \; for\; T_1 > 0 \quad (12)$$

$$\frac{\partial c}{\partial Z_1} = 0 \; at\; X_1 = 1 \; for\; T_1 > 0 \quad (13)$$

While the initial condition in the dimensionless form

$$c = 1 \; at\; T_1 = 0 \; for\; 0 < Z_1 \leq 1 \quad (14)$$

equation (11) is solved $c = c(Z_1,T_1)$ with the initial condition and boundary conditions. With the help of this solution the exit solute concentration in dimensionless form:

$$c_e = c_e(T_1) = c(1,T_1).$$

A condition (11) emerges regarding the indistinguishable condition in a barrel of the dissemination of matter containing a weaken suspension of little particles which are at the same time setting in a gravitational field and experiencing Brownian movement.

Analytic solution of mathematical model

The arrangement of condition (11) in dimensionless shape for limit and beginning conditions is additionally by Latinen et.al. (1959) prompt pulse input of solvent to the bed by the following substitution.

The substitution of equation in dimensionless form:

$$c(Z_1,T_1) = u_1(Z_1,T_1)exp\left(\frac{Pe}{2}Z_1 - PeT_1\right) \quad (15)$$

Reduces equation (11) to the heat conduction equation in one-dimensional form:

$$\frac{\partial u_1}{\partial x} = \frac{1}{Pe}\frac{\partial^2 u_1}{\partial Z_1^2} \qquad 0 \leq Z_1 \leq 1 \quad (16)$$

The limit and starting conditions in dimensionless for are:

$$\frac{\partial u_1}{\partial Z_1} - \frac{Pe}{2}u_1 = 0 \; at\; Z_1 = 0 \; for\; T_1 > 0 \quad (17)$$

$$\frac{\partial u_1}{\partial Z_1} + \frac{Pe}{2}u_1 = 0 \; at\; Z_1 = 1 \; for\; T_1 > 0 \quad (18)$$

$$\text{And}\;\; u_1(Z_1,0) = exp\left(-\frac{Pe}{2}Z_1\right) \quad (19)$$

These boundary conditions correspond to “radiation” at the ends of a slab onto media at zero temperature. The general solution of equation (16) under the boundary condition for

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any arbitrary temperature distribution is given by Carslaw and Jaeger (1959). Result of the equation of the type can be gotten by use of the Laplace transform:

Taking the Laplace transform of u_1, one gets:

$$\overline{u_1} = \overline{u_1}(Z_1, s) = \int_0^\infty \exp(-sT_1) u(Z_1, T_1) dT_1 \quad (20)$$

Substituting the condition (19) in equation (16) one gets:

$$\frac{d^2\overline{u_1}}{dZ_1^2} - q^2\overline{u_1} = -(Pe)exp\left(-\frac{Pe}{2}Z_1\right) \quad (21)$$

Where $q = \sqrt{sPe}$ (22)

Equations (17) and (18) of boundary condition can be written as:

$$\frac{d\overline{u_1}}{dZ_1} - \frac{Pe}{2}\overline{u_1} = 0 \; at \; Z_1 = 0 \; for \; T_1 > 0 \quad (23)$$

$$\frac{d\overline{u_1}}{dZ_1} + \frac{Pe}{2}\overline{u_1} = 0 \; at \; Z_1 = 1 \; for \; T_1 > 0 \quad (24)$$

The solution of equation (21) is:

$$\overline{u_1} = \frac{Pe\ exp\left[\frac{-PeZ_1}{2}\right]}{\left[q+\frac{Pe}{2}\right]\left[q-\frac{Pe}{2}\right]} - \frac{Pe^2 \exp(-qZ_1)}{\left[q+\frac{Pe}{2}\right]^2\left[q-\frac{Pe}{2}\right]} * \left[\frac{1+\left[\left\{\left[q-\frac{Pe}{2}\right]/\left[q+\frac{Pe}{2}\right]\right\}\exp(-2q(1-Z_1))\right]}{1-\left[\left\{\left[q-\frac{Pe}{2}\right]/\left[q+\frac{Pe}{2}\right]\right\}\exp(-q)\right]^2}\right] \quad (25)$$

The term of the denominator in curly brackets is of the form $(1-x^2)$ where 0<x<1. It can be solved by the binomial theorem. The term of the denominator in curly brackets must be equal to one for a satisfactory approximation at sufficiently large Pe and sufficient small time:

Taking $\left[q+\frac{Pe}{2}\right]\left[q-\frac{Pe}{2}\right] = Pe\left[s-\frac{Pe}{4}\right]$ and using method of partial fractions:

$$\overline{u_1} = \frac{exp\left[\frac{-PeZ_1}{2}\right]}{s-\frac{Pe}{4}} - \frac{\exp(-qZ_1)}{s-\frac{Pe}{4}} + \frac{Pe\ exp(-qZ_1)}{\left[q+\frac{Pe}{2}\right]^2} - \frac{Pe^2\ exp(-q(2-Z_1))}{\left[q+\frac{Pe}{2}\right]^3}$$

The inverse Laplace transforms of equation (25) is found by using the method of Carslaw and Jaeger (1959) as:

$$L^{-1}\left[\frac{Pe^2\ exp(-q(2-Z_1))}{\left[q+\frac{Pe}{2}\right]^3}\right] = \frac{Pe^2}{4}\frac{\partial}{\partial Pe} * L^{-1}\left[\frac{Pe\ exp(-qZ_1)}{\left[q+\frac{Pe}{2}\right]^2}\right] \quad (26)$$

This method accepts that it is reasonable to turn around the request of separating as for Pe and of upsetting the change. The inverse transform of the last term is accessible Carslaw and Jaeger (1959). The reversal could, obviously, likewise have been made through complex inversion integral.

$$c_1(Z_1,T_1) = 1 - \frac{1}{2}erfc\left[(Z_1-T_1)\sqrt{\frac{Pe}{4T_1}}\right] - \sqrt{\frac{PeT_1}{\pi}}exp\left[-(Z_1-T_1)^2\frac{Pe}{4T_1}\right] + \frac{1}{2}\Bigg[1 + Pe(Z_1+T_1)\exp(PeZ_1) * erfc\left[(Z_1+T_1)\sqrt{\frac{Pe}{4T_1}}\right] - 2\sqrt{\frac{PeT_1}{\pi}}\left[1+(2-Z_1+T_1)\frac{Pe}{4}\right] * exp\left[Pe-(2-Z_1+T_1)^2\frac{Pe}{4T_1}\right]\Bigg] + \frac{Pe}{2}\Bigg[2(2-Z_1+T_1)+T_1 + (2-Z_1+T_1)^2\frac{Pe}{2}\Bigg] * \exp(Pe)erfc\left((2-Z_1+T_1)\sqrt{\frac{Pe}{4T_1}}\right) \quad (27)$$

While erfc(z_1) is he complimentary error function defined by:

$$erfc(z_1) = 1 - \text{erf}(z_1) = \frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-y_1^2)dy_1 \quad (28)$$

The concentration distribution for the situation where the medium is semi-unendingly long, is given by the initial four (of the six) terms in condition (27). This outcome is affirmed by the free arrangement of Mason et al. (1924) for the Brownian movement gravitational setting issue in a semi-limited tube.

$$c_{is} = 1 - \frac{1}{2}erfc\left[(1-T_1)\sqrt{\frac{Pe}{4T_1}}\right] - \sqrt{\frac{PeT_1}{\pi}}\left[3+\frac{Pe}{2}(1+T_1)\right] * exp\left[-Pe(1-T_1)^2/4T_1\right] + \left[\frac{1}{2}+\frac{Pe}{2}(3+4T_1)+\frac{Pe^2}{4}(1+T_1)^2\right] * \exp(Pe)erfc\left[(1+T_1)\sqrt{\frac{Pe}{4T_1}}\right] \quad (29)$$

III. NUMERICAL SOLUTION

The above boundary value problem with suitable boundary and initial condition solved by OCM and OCFE collocation techniques. The complete analysis of description of mathematical model for different values of peclet numbers is presented both in Tabular and Graphical form.

From table(1), the behavior of concentration has been described for Peclet Number 0.01, 0.05, 0.1, 0.5 and 0.9. It is clear from tabulated values that OCFE gives better results for Pe=0.9 as comparison to Pe=0.01. From table (2) to table4 it is clear that as the value of Peclet number increase it goes in converging stage in less time. From, figure (1) it is clear that solution profiles for Pe=0.01, Pe=0.05, and 0.1 are almost same but difference between solution profiles has been seen from Pe=0. 5, but still it is

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not converging even at time 3. As the value of Peclet number increases, solution profile going to converging after time 2. But as the values of Peclet number goes on increasing behavior of concentration profiles becomes more converging, has been checked from figure(3). From figure (4), it has been easily seen that solution profiles goes on converging in very small time less than t=1 and behaves similar for large values of Peclet number.

The performance of the solute concentration with respect to the dimensionless time and distance has been exposed in the 3-D graphs for different values of Peclet Number. The comparison of OCM and OCFE is done in terms of their relative errors. The comparison of OCM and OCFE in terms of relative error for Pe=5 is given in table (5) and shown graphically in figure(8). It can be seen easily that for Pe=5 OCM gives relative error of 45% at time t=0.5 but as time increases error goes on decreasing first but not approaches to zero. But on the other hand, for OCFE the relative error for Pe = 5 is just of 0.5% and rapidly goes on decreasing. For Pe=10 the comparison of OCM and OCFE in terms of relative error is given in table (6) and graphically presented in figure (9).

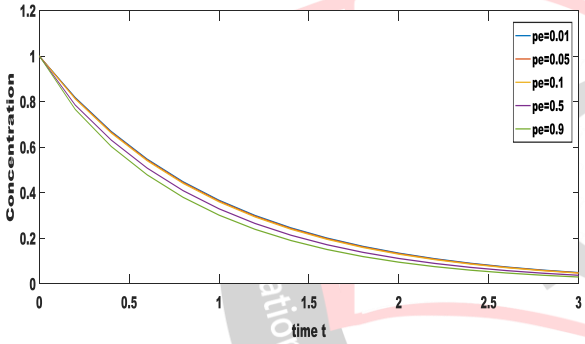

Figure 1: Behaviour of solution profiles at different Peclet numbers (Pe)

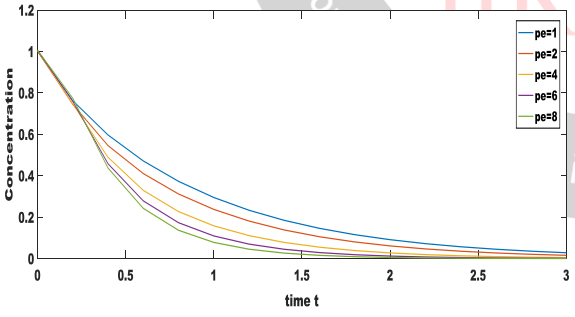

Figure 2: Behaviour of solution profiles at different Peclet numbers (Pe)

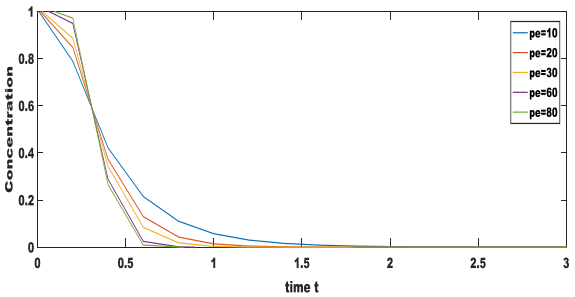

Figure 3: Behaviour of solution profiles at different Peclet numbers (Pe)

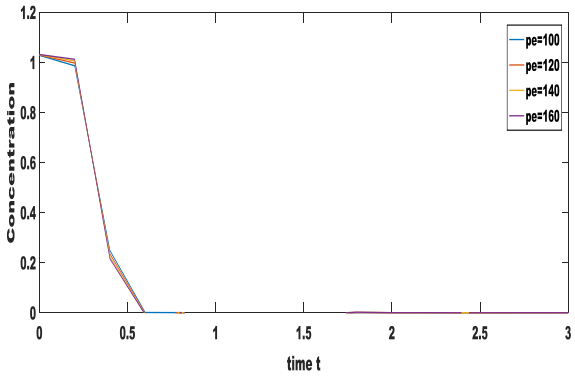

Figure 4: Behaviour of solution profiles at different Peclet numbers (Pe)

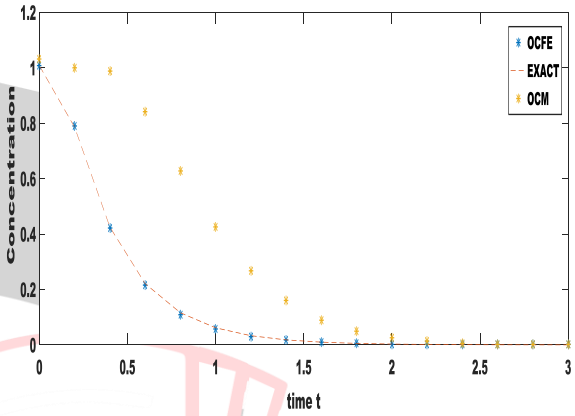

Figure 5: Comparison of solution profiles for OCM and OCFE at Pe=5

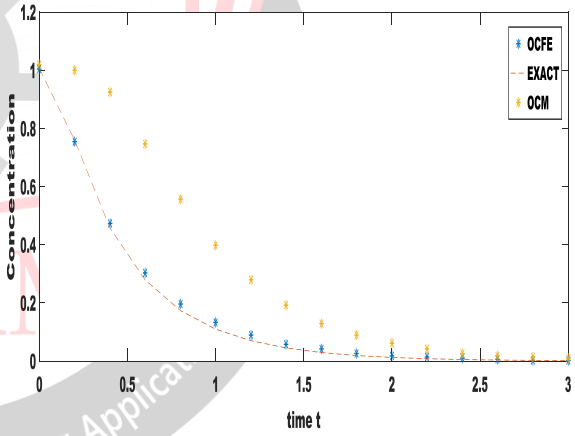

Figure 6: Comparison of solution profiles for OCM and OCFE at Pe=10

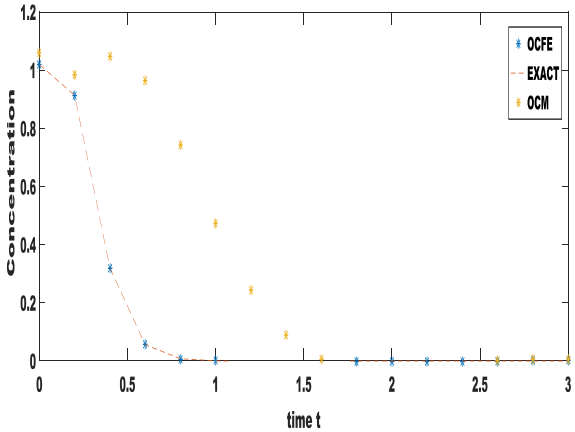

Figure 7: Comparison of solution profiles for OCM and OCFE at Pe=40

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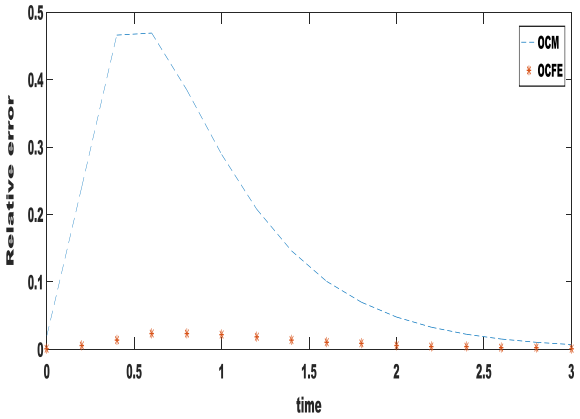

Figure 8: Comparison of relative error for OCM and OCFE at Pe=5

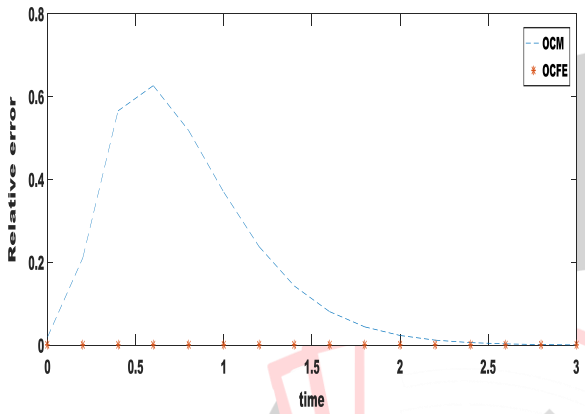

Figure 9: Comparison of relative error for OCM and OCFE at Pe=10

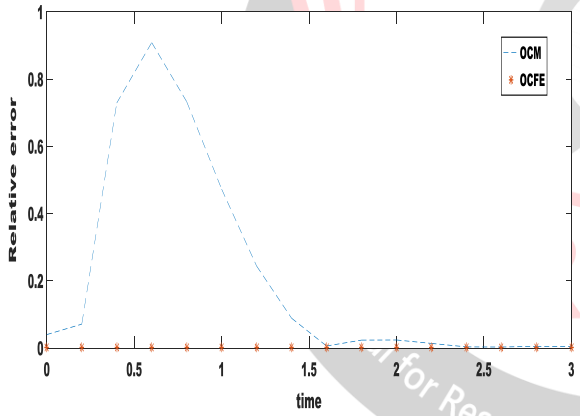

Figure 10: Comparison of relative error for OCM and OCFE at Pe=40

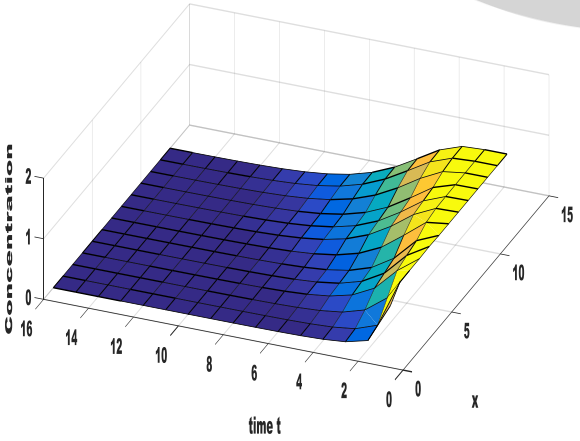

Figure 11: 3D behaviour of solute concentration profiles for Pe=10

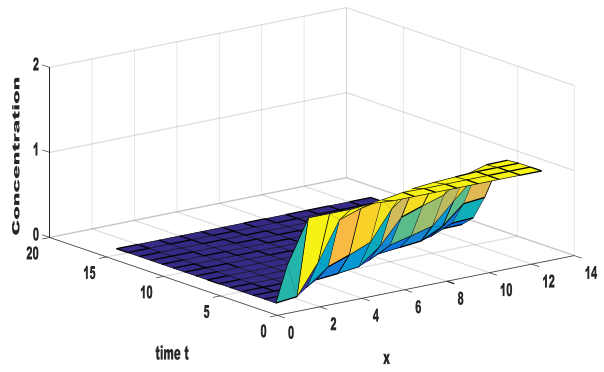

Figure 12: 3D behaviour of solute concentration profiles for Pe=40

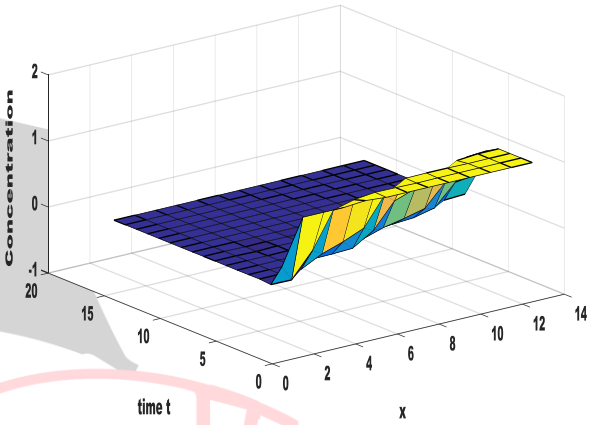

Figure 13: 3D behaviour of solute concentration profiles for Pe=80

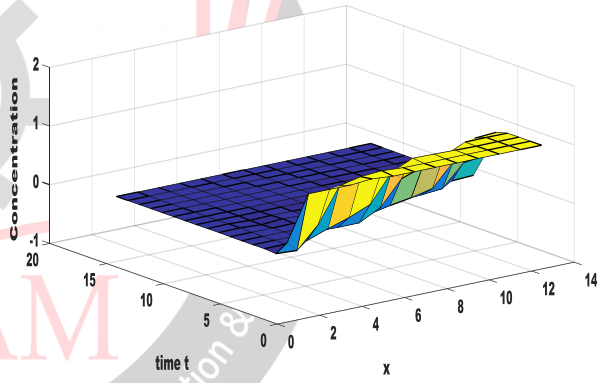

Figure 14: 3D behaviour of solute concentration profiles for Pe=120

Table 1: Exit solute concentration (c) for different values of peclet number (*pe*)

time t	*pe=0.01*	*pe=0.05*	*pe=0.1*	*pe=0.5*	*pe=0.9*
0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
2.00E-01	8.18E-01	8.15E-01	8.12E-01	7.85E-01	7.66E-01
4.00E-01	6.69E-01	6.66E-01	6.62E-01	6.31E-01	6.03E-01
6.00E-01	5.48E-01	5.45E-01	5.40E-01	5.08E-01	4.79E-01
8.00E-01	4.48E-01	4.45E-01	4.41E-01	4.09E-01	3.80E-01
1.00E+00	3.67E-01	3.64E-01	3.60E-01	3.29E-01	3.02E-01
1.20E+00	3.00E-01	2.97E-01	2.94E-01	2.65E-01	2.39E-01
1.40E+00	2.46E-01	2.43E-01	2.40E-01	2.13E-01	1.90E-01
1.60E+00	2.01E-01	1.99E-01	1.96E-01	1.72E-01	1.51E-01
1.80E+00	1.65E-01	1.62E-01	1.60E-01	1.38E-01	1.20E-01
2.00E+00	1.35E-01	1.33E-01	1.30E-01	1.11E-01	9.51E-02
2.20E+00	1.10E-01	1.08E-01	1.06E-01	8.96E-02	7.55E-02
2.40E+00	9.03E-02	8.87E-02	8.67E-02	7.21E-02	5.99E-02

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2.60E+00	7.39E-02	7.25E-02	7.07E-02	5.81E-02	4.76E-02
2.80E+00	6.05E-02	5.92E-02	5.77E-02	4.67E-02	3.78E-02
3.00E+00	4.95E-02	4.84E-02	4.71E-02	3.76E-02	3.00E-02

Table 2: Exit solute concentration (c) for different values of peclet number (*pe*)

time t	*pe=1*	*pe=2*	*pe=4*	*pe=6*	*pe=8*
0.00E+00	1.00E+00	1.00E+00	1.00E+00	1.01E+00	1.01E+00
2.00E-01	7.62E-01	7.45E-01	7.47E-01	7.59E-01	7.74E-01
4.00E-01	5.97E-01	5.45E-01	4.89E-01	4.58E-01	4.37E-01
6.00E-01	4.72E-01	4.11E-01	3.30E-01	2.79E-01	2.43E-01
8.00E-01	3.73E-01	3.12E-01	2.27E-01	1.73E-01	1.37E-01
1.00E+00	2.95E-01	2.38E-01	1.58E-01	1.10E-01	7.82E-02
1.20E+00	2.33E-01	1.81E-01	1.11E-01	7.00E-02	4.53E-02
1.40E+00	1.85E-01	1.38E-01	7.81E-02	4.50E-02	2.65E-02
1.60E+00	1.46E-01	1.05E-01	5.50E-02	2.91E-02	1.56E-02
1.80E+00	1.16E-01	8.05E-02	3.88E-02	1.88E-02	9.23E-03
2.00E+00	9.14E-02	6.14E-02	2.74E-02	1.22E-02	5.47E-03
2.20E+00	7.23E-02	4.68E-02	1.93E-02	7.93E-03	3.25E-03
2.40E+00	5.72E-02	3.57E-02	1.37E-02	5.15E-03	1.94E-03
2.60E+00	4.52E-02	2.72E-02	9.64E-03	3.35E-03	1.15E-03
2.80E+00	3.58E-02	2.08E-02	6.81E-03	2.18E-03	6.88E-04
3.00E+00	2.83E-02	1.59E-02	4.81E-03	1.42E-03	4.11E-04

Table 3: Exit solute concentration (c) for different values of peclet number (*pe*)

time t	*pe=10*	*pe=20*	*pe=30*	*pe=60*	*pe=80*
0.00E+00	1.01E+00	1.01E+00	1.02E+00	1.02E+00	1.03E+00
2.00E-01	7.89E-01	8.47E-01	8.86E-01	9.49E-01	9.71E-01
4.00E-01	4.21E-01	3.73E-01	3.43E-01	2.88E-01	2.65E-01
6.00E-01	2.15E-01	1.30E-01	8.43E-02	2.47E-02	9.14E-03
8.00E-01	1.10E-01	4.28E-02	1.82E-02	1.48E-03	4.54E-04
1.00E+00	5.71E-02	1.39E-02	3.54E-03	-1.79E-03	-2.93E-03
1.20E+00	3.00E-02	4.50E-03	4.05E-04	-2.59E-03	-3.50E-03
1.40E+00	1.59E-02	1.46E-03	-9.99E-05	-1.70E-03	-2.41E-03
1.60E+00	8.52E-03	4.72E-04	-9.01E-05	-6.25E-04	-1.03E-03
1.80E+00	4.59E-03	1.54E-04	-3.96E-05	-7.60E-05	3.89E-05
2.00E+00	2.48E-03	5.03E-05	-1.39E-05	2.26E-05	1.20E-04
2.20E+00	1.34E-03	1.66E-05	-4.37E-06	4.93E-06	1.03E-05
2.40E+00	7.31E-04	5.47E-06	-1.30E-06	-1.63E-06	-1.02E-05
2.60E+00	3.98E-04	1.82E-06	-3.66E-07	2.78E-07	-2.42E-07
2.80E+00	2.17E-04	6.09E-07	-9.70E-08	8.69E-07	3.83E-06
3.00E+00	1.18E-04	2.04E-07	-2.41E-08	2.97E-07	1.07E-06

Table 4: Exit solute concentration (c) for different values of peclet number (*pe*)

time t	*pe=100*	*pe=120*	*pe=140*	*pe=160*
0.00E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00
2.00E-01	9.86E-01	9.97E-01	1.01E+00	1.01E+00
4.00E-01	2.48E-01	2.35E-01	2.25E-01	2.16E-01
6.00E-01	9.18E-04	-3.62E-03	-6.16E-03	-7.52E-03
8.00E-01	4.47E-04	4.81E-04	3.72E-04	9.58E-05
1.00E+00	-3.82E-03	-4.50E-03	-4.97E-03	-5.26E-03
1.20E+00	-3.71E-03	-3.49E-03	-3.06E-03	-2.57E-03
1.40E+00	-2.71E-03	-2.65E-03	-2.32E-03	-1.76E-03
1.60E+00	-1.64E-03	-2.52E-03	-3.67E-03	-5.03E-03
1.80E+00	2.89E-04	7.18E-04	1.35E-03	2.18E-03
2.00E+00	2.30E-04	2.83E-04	2.25E-04	2.55E-05
2.20E+00	1.16E-05	3.10E-05	9.74E-05	2.34E-04
2.40E+00	-2.48E-05	-4.50E-05	-7.73E-05	-1.31E-04
2.60E+00	-5.39E-06	-1.96E-05	-4.29E-05	-7.13E-05
2.80E+00	1.06E-05	2.34E-05	4.30E-05	6.85E-05
3.00E+00	2.87E-06	5.99E-06	1.03E-05	1.56E-05

Table 5: Comparison of OCM and OCFE for pe=5

time t	*EXACT*	*OCM*	*ERROR*	*OCFE*	*ERROR*
0.00E+00	1.00E+00	1.02E+00	0.02	1.00E+00	0
2.00E-01	7.59E-01	1.00E+00	0.241	7.53E-01	0.006
4.00E-01	4.58E-01	9.24E-01	0.466	4.72E-01	0.014
6.00E-01	2.79E-01	7.48E-01	0.469	3.02E-01	0.023
8.00E-01	1.73E-01	5.58E-01	0.385	1.97E-01	0.024
1.00E+00	1.10E-01	3.99E-01	0.289	1.31E-01	0.021
1.20E+00	7.00E-02	2.78E-01	0.208	8.78E-02	0.0178
1.40E+00	4.50E-02	1.91E-01	0.146	5.91E-02	0.0141
1.60E+00	2.91E-02	1.30E-01	0.1009	3.99E-02	0.0108
1.80E+00	1.88E-02	8.86E-02	0.0698	2.70E-02	0.0082
2.00E+00	1.22E-02	6.01E-02	0.0479	1.83E-02	0.0061
2.20E+00	7.93E-03	4.07E-02	0.0328	1.24E-02	0.0045
2.40E+00	5.15E-03	2.75E-02	0.0224	8.39E-03	0.0032
2.60E+00	3.35E-03	1.86E-02	0.0152	5.69E-03	0.0023
2.80E+00	2.18E-03	1.26E-02	0.0104	3.86E-03	0.0017
3.00E+00	1.42E-03	8.52E-03	0.0071	2.61E-03	0.0012

Table 6: Comparison of OCM and OCFE for pe=10

time t	*EXACT*	*OCM*	*ERROR*	*OCFE*	*ERROR*
0.00E+00	1.01E+00	1.03E+00	2.00E-02	1.01E+00	0
2.00E-01	7.89E-01	1.00E+00	2.11E-01	7.89E-01	0
4.00E-01	4.21E-01	9.87E-01	5.66E-01	4.21E-01	0
6.00E-01	2.13E-01	8.40E-01	6.27E-01	2.15E-01	2.00E-03
8.00E-01	1.09E-01	6.28E-01	5.19E-01	1.10E-01	1.00E-03
1.00E+00	5.62E-02	4.26E-01	3.70E-01	5.71E-02	9.00E-04
1.20E+00	2.94E-02	2.68E-01	2.39E-01	3.00E-02	6.00E-04
1.40E+00	1.55E-02	1.59E-01	1.44E-01	1.59E-02	4.00E-04
1.60E+00	8.27E-03	8.98E-02	8.15E-02	8.52E-03	2.50E-04
1.80E+00	4.43E-03	4.91E-02	4.47E-02	4.59E-03	1.60E-04
2.00E+00	2.38E-03	2.63E-02	2.39E-02	2.48E-03	1.00E-04
2.20E+00	1.29E-03	1.39E-02	1.26E-02	1.34E-03	5.00E-05
2.40E+00	6.96E-04	7.38E-03	6.68E-03	7.31E-04	3.50E-05
2.60E+00	3.77E-04	3.96E-03	3.58E-03	3.98E-04	2.10E-05

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2.80E+00	2.05E-04	2.17E-03	1.97E-03	2.17E-04	1.20E-05
3.00E+00	1.11E-04	1.21E-03	1.10E-03	1.18E-04	7.00E-06

Table 7: Comparison of OCM and OCFE for pe=40

time t	*EXACT*	*OCM*	*ERROR*	*OCFE*	*ERROR*
0.00E+00	1.02E+00	1.06E+00	4.00E-02	1.02E+00	0
2.00E-01	9.13E-01	9.84E-01	7.10E-02	9.13E-01	0
4.00E-01	3.21E-01	1.05E+00	7.29E-01	3.21E-01	0
6.00E-01	5.61E-02	9.65E-01	9.09E-01	5.61E-02	0
8.00E-01	7.91E-03	7.41E-01	7.33E-01	7.91E-03	0
1.00E+00	3.34E-04	4.73E-01	4.73E-01	3.34E-04	0
1.20E+00	-9.24E-04	2.43E-01	2.44E-01	-9.24E-04	0
1.40E+00	-6.83E-04	8.80E-02	8.87E-02	-6.83E-04	0
1.60E+00	-2.85E-04	5.75E-03	6.04E-03	-2.85E-04	0
1.80E+00	-8.22E-05	-2.36E-02	2.35E-02	-8.22E-05	0
2.00E+00	-1.83E-05	-2.39E-02	2.39E-02	-1.83E-05	0
2.20E+00	-3.86E-06	-1.37E-02	1.37E-02	-3.86E-06	0
2.40E+00	-8.58E-07	-3.35E-03	3.35E-03	-8.58E-07	0
2.60E+00	-1.27E-07	2.93E-03	2.93E-03	-1.27E-07	0
2.80E+00	3.25E-08	5.05E-03	5.05E-03	3.25E-08	0
3.00E+00	2.63E-08	4.50E-03	4.50E-03	2.63E-08	0

IV. CONCLUSION

Comparison of both the numerical technique OCM and OCFE is done for different values of peclet number. It has been concluded that solution profiles rapidly converges for large values of peclet number as comparison to smaller one. Comparison has been presented both in tabular and graphical form. This has been concluded that relative error for OCFE is very less as comparison with OCM. Thus OCFE is time friendly and more efficient numerical technique as comparison to OCM.

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