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A study on P – Fuzzy field USING P-FUZZY ALGEBRA and its Properties

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Abstract: In this paper, we provide definition for P –Fuzzy algebra on the Algebra A, Fuzzy field and P - fuzzy field. Then we study the connection of P – fuzzy subalgebras with P - fuzzy field. Also some Properties of P - fuzzy field are stated and proved.

Keywords: P – fuzzy subalgebras, Fuzzy Field and P - fuzzy field.

I. INTRODUCTION

In 1965, Lotfi. Zadeh introduced the notion of a fuzzy subset[6] of a set as a method for representing uncertainity. A fuzzy set A is defined as a map from A to the real unit interval I = [0, 1]. The set of all fuzzy sets on A is usually denoted by I^A. Applications of fuzzy sets and fuzzy logic were introduced by Mamdani in 1975. Early Greeks, such as Pythagoras and Euclid, relied on geometry to express all of their logical proofs. It was about 250CE when the Greek Diophantus began using Greek letters as numbers and other mathematical symbols. Finally, algebra began to take on its modern symbolic look when viete used letter for variables around 1600.

The concept of P-fuzzy subset[5,7} of A is used in Study of fuzzy algebras and its relation from a general view point by 'laslofilep' on Acta Mathematica Academiae Paedagogicae Nyiregyhaziensis, Tonus 14 (1998), 49-55.

In its trajectory of stupendous growth, it has also come to include the theory of fuzzy algebra and for the past decades, several researchers have been working on concepts like fuzzy semigroup, fuzzy groups, fuzzy rings, fuzzy ideals, fuzzy semirings and so on. The unit interval is replaced here by a partially ordered algebra[5] (P-Algebra) and this leads to partially ordered fuzzy algebra (P-fuzzy algebra). He unified and generalized the definition to the concept of fuzzy substructures, relations and compatibility also involve different membership sets and operations.

II. PRELIMINARIES

Definition 1.1. If $(P, \leq)$ is a partially ordered set[1,5] and X is a nonempty set, then any mapping $\mu: A \to P$ is a P-fuzzy subset of A or a P-fuzzy set on A, denoted by P^A

Definition 1.2.Let A be a non-empty set and $P = (P, *, 1, \leq)$ a (2, 0) type ordered algebra[5], i.e. Let

i) $(P, *)$ be a monoid, where 1 is the unity for $*$

ii) $(P, \leq)$ be a Partially ordered set aith 1 as the greatest element.

iii) $*$ be isotone in both variables.

P always denotes such a structure.

Definition 1.3. A P – fuzzy set $\mu \in P^A$ is called a P – fuzzy algebra[2,5] or fuzzy subalgebra on the algebra A, if

- For any n – ary($n \geq 1$) operation $f \in F$ $\mu(f(x_1 \ldots\ldots\ldots x_n)) \geq \mu(x_1) * \ldots\ldots\ldots\ldots\ldots * \mu(x_n)$ for all $x_1 \ldots\ldots\ldots x_n \in A$
- For any constant (nullary operation) C $\mu(c) \geq \mu(x)$ for all $x \in A$.

(Note: If A is a group then nullary operation is consequence of $n - ary$ operation)

Definition 1.4. Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group$G$ if

- $\mu(xy) \geq min(\mu(x), \mu(y))$ for every $x, y \in G$ and
- $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

Definition 1.5. A fuzzy subset μ of a ring R is called a Fuzzy subring of R if for all $x, y \in R$ then

- $\mu(x - y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy) \geq min(\mu(x), \mu(y))$ for all $x, y \in R$.

III. BASIC DEFINITIONS OF P – FUZZY SUBALGEBRA

Definition 2.1 A P - fuzzy subset $\mu \in P^A$ is called a P - Fuzzy group[1] of Gon the algebra A if for all $x, y \in G$ then

- $\mu(xy) \geq min(\mu(x), \mu(y))$ for every $x, y \in G$ and
- $\mu(x^{-1}) = \mu(x)$ for every $x \in G$.

Definition 2.2. A P - fuzzy subset $\mu \in P^A$ is called a P - Fuzzy subring[3] of R of the algebra A if for all $x, y \in R$ then

- $\mu(x - y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy) \geq min(\mu(x), \mu(y))$ for all $x, y \in R$

Theorem 2.1. A P – fuzzy subalgebra of a group G is a fuzzy subgroup of the group G iff $\mu(xy^{-1}) \geq \min\{\mu(x), \mu(y)\}$ for every $x,y \in G$.[1]

Theorem 2.2. Every P – Fuzzy subalgebra on the algebra A is P – fuzzy ring if for some $x, y \in A$ is a ring under

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addition if $\mu(x) < \mu(y)$ then $\mu(x-y) = \mu(x) = \mu(y-x)$.[3]

Proof

By the definition of P –Fuzzy algebra

$\mu(f(x_1 \ldots\ldots x_n)) \geq \mu(x_1) * \ldots\ldots * \mu(x_n)$ for all $x_1 \ldots\ldots x_n \in A$

Since the operation is addition and it is a ring.

Therefore there exists the inverse function subtraction

$$\Rightarrow f = \{+,-\}$$

Since $x, y \in A$.

$\mu(f(xy)) \geq \mu(x) * \mu(y)$

- $\mu(x-y) \geq \min(\mu(x), \mu(y)) \geq \mu(x)(since\ \mu(x) < \mu(y)) \quad ---(1)$

Similarly

- $\mu(y-x) \geq \min(\mu(x), \mu(y)) \geq \mu(x)(since\ \mu(x) < \mu(y)) \quad ---(2)$

Also under the operation .

$$\mu(f(xy)) \geq \min(\mu(x), \mu(y)) \geq \mu(x)(since\ \mu(x) < \mu(y)) \quad --(3)$$

From result (1), (2) & (3)

$\Rightarrow If\ \mu(x) < \mu(y)$ then $\mu(x-y) = \mu(x) = \mu(y-x)$

IV. PROPERTIES OF P - FUZZY FIELD

Definition 3.1 A P - fuzzy subset $\mu \in P^A$ is called a P - Fuzzy Field of F of the algebra A if for all $x, y \in$ F then

- $\mu(x-y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy^{-1}) \geq min\ (\mu(x), \mu(y))$

Definition 3.2 Let A be a P – Fuzzy subset of the P –Fuzzy field F, for $0 \leq t \leq 1$. Let $A_t = \{x \in F / A(x) \geq t\}$ then $A_{\neq} = A_t$ when t=A(1).

Theorem 3.1.If A is a P – Fuzzy subset of F and $s, t \in Im\ (A)$, the image of A, then $s \leq t$ if and only if $A_s \supseteq A_t$ and s = t if and only if $A_s = A_t$.

Theorem 3.2. If S is a P – Fuzzy subfield of F then for all $x \in$ F, $x \neq 0$, $S(0) \geq S(1) \geq S(x) = S(-x) = S(x^{-1})$.

Proof:

If S is a P – Fuzzy subfield of F then by definition of P - Fuzzy Field, F is

Function S from F into [0, 1]

- $\mu(x-y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy^{-1}) \geq min\ (\mu(x), \mu(y))$for all $x, y \in F$

Since P – Fuzzy subfield consists of identity element under addition $S(0)$

and identity element under multiplication $S(1)$.

$\Rightarrow S(0) \geq S(1) \geq S(x)$

By the property of P – Fuzzy subgroup and P – Fuzzy subring there exist inverse element under addition $S(-x)$ and multiplication $S(x^{-1})$.

- $S(0) \geq S(1) \geq S(x) = S(-x) = S(x^{-1})$.

Theorem 3.3 Let A be a P – Fuzzy subset of P – Fuzzy Field F for all $t \in Im\ (A)$, then A is P – Fuzzy Field of F. Conversely, if A is a P – Fuzzy Field of F, then for all t such that $0 \leq t \leq A(1), A_t$ is a P – Fuzzy subfield of F.

Theorem 3.4 Let S be a P – Fuzzy subset of P – Fuzzy Field F such that $|S|\ (Cardinality\ of\ S) \geq 2$. Then S is a P – Fuzzy subfield of F if and only if χ_S, the characteristic function of S, is a P – Fuzzy subfield of F.

Proof: If K be a P – Fuzzy subfield of F i.e., F is an extension field of K then the field extension is denoted by F/K and S(F/K) denote the set of all P – Fuzzy subfield of A of F such that $A_{\neq} \supseteq K\ and\ A_{\neq}$is a P – Fuzzy subfield of F.

Theorem 3.5 Let $F_1 \subset F_2 \subset F_3 \subset \ldots \subset F_i$ be a strictly ascending chain of P – Fuzzy subfield of F such that $\cup F_i$ =F. Define the P – Fuzzy subset S of F by $S(x) = t_i$, if $x \in F_i/F_{i-1}$ where $t_i > t_{i+1}$ for $i = 1, 2, \ldots$ and $F_0 = \emptyset$. Then S is a P–Fuzzy subfield of F.

Proof: Let $x, y \in F$, then $x - y \in F_i/F_{i-1}$ for some i.

Hence either $x \notin F_{i-1}$ or $y \notin F_{i-1}$ thus $S(x-y) = t_i \geq \min\{S(x), S(y)\}$

By the property of P –Fuzzy subgroup[1]

$$S(xy^{-1}) = t_i \geq \min\{S(x), S(y)\}\ for\ y \neq 0.$$

Theorem 3.6 Let $F = F_1 \supset F_2 \supset F_3 \supset \cdots \supset F_i \supset \cdots$ be a strictly descending chain of P – Fuzzy subfield of F such that $\cup F_i$ =F. Define the P – Fuzzy subset S of F by $S(x) = t_{i-1}$, if $x \in F_{i-1}/F_i$ where $t_{i-1} < t_i < 1$ for $i = 1, 2, \ldots$ and $S(x) = 1$ if $x \in \cap F_i$ Then S is a P–Fuzzy subfield of F.

Proof: Let $x, y \in F$, then $x - y \in F_{i-1}/F_i$ for some i.

Hence either $x \notin F_{i-1}$ or $y \notin F_{i-1}$ thus $S(x-y) = t_i \geq \min\{S(x), S(y)\}$

By the property of P –Fuzzy subgroup

$$S(xy^{-1}) = t_i \geq \min\{S(x), S(y)\}\ for\ y \neq 0.$$

Theorem 3.7 If F is a P – Fuzzy finite field, then every Fuzzy subfield of F is finite valued.

V. CONCLUSION

In this paper we have defined P-Fuzzy Field using P-Fuzzy Algebra. Further we have introduced new theorems on P-Fuzzy Field and results on, ascending and descending chain

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on P-Fuzzy Field is proved. The future work of P-Fuzzy field can be extended to P-Fuzzy Vector space. This concept of P-Fuzzy field can be applied in Database to scrutinize data.

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