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Some Paranormed Type Sequence Spaces Over a Normed Linear Space

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Abstract: In this article we introduce the some double sequence spaces defined over a normed linear space *X*. A paranorm is assigned to introduce the sequence spaces. The main property is proved that the completeness property of the introduced sequence spaces. Some other properties are also examined like symmetricity, solidness of the introduced sequence spaces and some inclusion results.

Keywords. Double sequence, Completeness, Solid space, Symmetric space.

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I. INTRODUCTION

The initial works on double sequences are found in Bromwich [1]. The notion of regular convergence was introduced and studied by Hardy [2]. Following Hardy many works have done by Sarma [6,7], Tripathy and Sarma [8] and many others. The paranormed sequence spaces were introduced by Nakano [5].

Let $(X, ||.||)$ be a normed linear space. Throughout the article ${}_2w$, ${}_2\ell_\infty(X)$, ${}_2c(X)$, ${}_2c^R(X)$ denote the spaces of *all, bounded, convergent in Pringsheim's sense and regularly convergent* double sequence spaces respectively defined over X.

A double sequence will be denoted as $A = \langle a_{nk}\rangle$, a double infinite array of the elements a_{nk}, for $n, k \in N$.

A double sequence $\langle a_{nk}\rangle$ is said to converge in *Pringsheim's sense* if

$$\lim_{n,k\to\infty} a_{nk} = L \text{ exists},$$

where n and k tend to ∞ independent of each other.

A double sequence $\langle a_{nk}\rangle$ is said to *converge regularly* (Due to Hardy [3]) if it converges in the Pringsheim's sense and the following limits hold:

$$\lim_{n\to\infty} a_{nk} = L_k \text{, exist for each } k \in N,$$

and

$$\lim_{k\to\infty} a_{nk} = M_n \text{, exist for each } n \in N.$$

II. DEFINITIONS AND PRELIMINARIES

Sequences of fuzzy real numbers relative to the paranormed sequence spaces is studied by Choudhury and Tripathy [2]. The following definition due to Maddox [4].

A mapping $g: X \to R$ is said to be a paranorm if

$g(x) = 0$ if and only if $x = 0$,

$g(x+y) \le g(x) + g(y)$ for all $x, y \in X$ and if

$\lambda_n, \lambda_0 \in C$ with $\lambda_n \to \lambda_0 (n \to \infty)$ and if $x_n, a \in X$ with $x_n \to a (n \to \infty)$ in the sense that $g(x_n - a) \to 0 (n \to \infty)$, then $\lambda_n x_n \to \lambda_0 a (n \to \infty)$, in the sense that $g(\lambda_n x_n - \lambda_0 a) \to 0 (n \to \infty)$

Definition 2.1. A double sequence space E is said to be *symmetric* if $\langle a_{nk}\rangle \in E$ implies $\langle a_{\pi(n)\pi(k)}\rangle \in E$, where π is a permutations of N.

Definition 2.2. A double sequence space E is said to be *solid* if $\langle \alpha_{nk} a_{nk}\rangle \in E$ whenever $\langle a_{nk}\rangle \in E$ for all double sequences $\langle \alpha_{nk}\rangle$ of scalars with $|\alpha_{nk}| \le 1$ for all $n, k \in N$.

Definition 2.3. A double sequence space E is said to be a *sequence algebra* if $\langle a_{nk} b_{nk}\rangle \in E$ whenever $\langle a_{nk}\rangle, \langle b_{nk}\rangle \in E$.

We now introduce the following paranormed sequence spaces.

$${}_2\ell_\infty(p) = \left\{ \langle a_{nk} \rangle \in {}_2w : \sup_{n,k}(\| a_{nk} \|)^{p_{nk}} < \infty \right\}$$

$${}_2c(p) = \left\{ \langle a_{nk} \rangle \in {}_2w : \lim_{n,k}(\| a_{nk} - L \|)^{p_{nk}} = 0, \text{ for some } L \in X \right.$$

Also $\langle a_{nk} \rangle \in {}_2c^R(p)$ if $\langle a_{nk} \rangle \in {}_2c(p)$ and the rows and columns are also convergent under paranorm.

III. MAIN RESULTS

Theorem 3.1. *The classes $Z(p)$ where $Z = {}_2\ell_\infty, {}_2c, {}_2c^R$ are linear spaces.*

Theorem 3.2. *If $0 < \inf p_{nk} \le \sup p_{nk} < \infty$ then the sequence spaces $Z(p)$ where $Z = {}_2\ell_\infty, {}_2c^R$ are paranormed spaces, paranormed by*

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$$f(<a_{nk}>) = \sup_{n,k}(\| a_{nk} \|)^{\frac{p_{nk}}{H}}, \text{ where}$$

$H = \max(1, \sup p_{nk})$.

Proof. Clearly $f(\theta) = 0$. For $A =< a_{nk} >$ and $B =< b_{nk} >$ we have $f(-A) = f(A)$ and $f(A + B) \le f(A) + f(B)$.

For a scalar λ, $f(\lambda A) = \sup_{n,k}(\| \lambda a_{nk} \|)^{\frac{p_{nk}}{H}} \le \max(1, |\lambda|). f(A) \to 0$ as $A \to \theta$.

Similarly $\lambda \to 0$ implies $f(\lambda A) \to 0$.

Also $\lambda \to 0$ and $A \to \theta$ implies $f(\lambda A) \to 0$.

Hence f is a paranorm.

Proposition 3.3. *The space $_2\ell_\infty(p)$ is solid but the spaces $_2c(p)$ and $_2c^R(p)$ are not solid.*

Proof. Let $< \alpha_{nk} >$ be a scalar sequence with $|\alpha_{nk}| \le 1$. Then solidity of the space $_2\ell_\infty(p)$ follows from the inequality

$$(\| \alpha_{nk} a_{nk} \|)^{p_{nk}} \le (\| a_{nk} \|)^{p_{nk}}$$

Consider the following example to show $_2c(p)$ and $_2c^R(p)$ are not solid.

Example 3.1. Let $X = C$, the field of complex numbers; $q(x) = |x|$, $p_{nk} = 3$, for all $n, k \in N$ and $<a_{nk}>$ be defined by

$$a_{nk} = 2, \text{ for all } n, k \in N.$$

Let $\alpha_{nk} = (-1)^{n+k}$. Then $<a_{nk}> \in {}_2c(p)$ and $_2c^R(p)$ but $< \alpha_{nk} a_{nk} > \notin {}_2c(p)$ or $_2c^R(p)$.

Proposition 3.4. *The space $_2\ell_\infty(p)$ is symmetric but the spaces $_2c(p)$ and $_2c^R(p)$ are not symmetric.*

Proof. The property of boundedness of a sequence is not altered by rearrangement so the symmetricity of the space $_2\ell_\infty(p)$ is obvious. For the spaces $_2c(p)$ and $_2c^R(p)$, consider the following example:

Example 3.2. Let $X = C$, the field of complex numbers; $q(x) = |x|$ and consider the sequence $<a_{nk}>$ is defined by

$$a_{nk} = \begin{cases} 0, & \text{when } n = 1 \text{ or } k = 1, \\ 1, & \text{otherwise.} \end{cases}$$

Let $p_{nk} = 1$, then $<a_{nk}> \in {}_2c(p)$ and $_2c^R(p)$. Let $<b_{nk}>$ be a rearrangement of $<a_{nk}>$ defined by

$$b_{nn} = 0, \text{ for all } n \in N,$$
$$b_{nk} = 1, \text{ otherwise.}$$

Then $<b_{nk}> \notin {}_2c(p)$ or $_2c^R(p)$. Hence $_2c(p)$ and $_2c^R(p)$ are not symmetric.

Theorem 3.5. *The spaces $_2c(p)$ and $_2c^R(p)$ are sequence algebras.*

Proof. Consider the sequence space $_2c(p)$. Let $<a_{nk}>$, $<a_{nk}> \in {}_2c(p)$. Then

$$\lim_{n,k}(\| a_{nk} - L \|)^{p_{nk}} = 0 \text{ and}$$

$$\lim_{n,k}(\| b_{nk} - J \|)^{p_{nk}} = 0$$

It can be easily shown that $\lim_{n,k}(\| a_{nk} b_{nk} - LJ \|)^{p_{nk}} = 0$

This shows that $<a_{nk} b_{nk}> \in {}_2c(p)$. Hence $_2c(p)$ is a sequence algebra. Similarly $_2c^R(p)$ is also a sequence algebra.

Theorem 3.6. *The spaces $_2c(p)$ and $_2c^R(p)$ are proper subset of $_2\ell_\infty(p)$.*

IV. Conclusion

In this paper we have introduced some new type of sequence spaces defined over a normed linear space. A paranorm is assigned to define the sequence spaces. We have proved properties like completeness, Sequence Algebra, symmetricity etc for the introduced sequence spaces.

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