

Building of Mean-Variance Efficient Portfolio

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Abstract - An investor while selecting portfolio with the objective of minimisation of risk and high return faced various constraints like dividend, good return, earning per share, company and industrial diversification and also he wants sufficient liquidity etc. An attempt is made to incorporate various relevant factors for developing and testing of efficient portfolio model for construction of portfolio. Secondary data is used for the companies listed in Nifty 50 index. Regression analysis, granger causality test, portfolio optimisation is undertaken for the formation of portfolio and evaluated by Sharpe and Treynor ratio. It is found that as compared to market trend portfolio, capital gain bias portfolio and dividend gain bias portfolio, Proposed portfolio model performed better.

Key word: Portfolio, earning per share, dividend, diversification, liquidity.

I. INTRODUCTION

A portfolio is a grouping of financial assets such as stocks, bonds, commodities, currencies and cash equivalents, as well as their fund counterparts, including mutual, exchange-traded and closed funds. It can also consist of non publicly tradable securities, like real estate, art, and private investments. An Efficient Portfolio is a combination of investment which provides the greatest expected return for a given level of risk or the lowest possible risk at a given yield level.

Investors generally construct an investment portfolio in accordance with their risk tolerance and their investing objectives. An investor can also have multiple portfolios for various purposes. Portfolio selection is collection of risky assets combined with different weights to provide an acceptable trade-off between return and risk to an investor.

Portfolio selection consists of selecting a portfolio of assets or securities that provides the investor a given expected return and lowest possible risk. For any investor and/or stock market speculator, the foremost consideration is the return on investment and the associated risk. Money cannot be earned or wealth cannot be maximised without risk, thus, the risk proportionately increases with the expected profits for securities located on the efficient frontier.

Markowitz introduced Mean-Variance optimisation in 1952 for portfolio selection. He showed optimal portfolio for the investor lies on the Mean-Variance Efficient Frontier.

Scenarios change from time to time, now various constraints faced by investor while selecting portfolio are illiquidity, minimum capital requirement, short-selling, turnover, volume, volatility etc which has a great impact on construction of portfolio.

II. REVIEW OF LITERATURE

Haas(1972) on examining portfolio theory of Markowitz, Tobin, Sharpe, Lintner he found that assumptions of the models were in line with contemporary portfolio theory. A Single optimal portfolio exists consisting foreign and domestic investments and this portfolio with total wealth of the economy determines the desired stock of foreign assets at any point of time. To test the model he used Multiple regression equation by Koyck distributed lags of the different countries data. The portfolio approach empirically tested.

Lee and Chang (1995) fitted an instability portfolio selection model on eight Taiwan stock. They found instability in preference i.e. Investors speculate in high variance stock whereas U.S. investors speculate in low variance stock and short selling increases the risk of portfolio. Investors were prone to cater the next peak which affects frequencies of stock. It was single period model which does not incorporate taxes and transaction cost.

After introducing new Bounding Utility Theorem for optimum portfolio with preference for profitability and safety, **(Ballester, 1998)** concluded that the lesser the preference deviate from the average preference behaviour, the narrower are bounds for utility optimum on the efficient frontier.

Polson & Tew (2000) presented a technique for implementing large- scale optimal portfolio selection. They used high- frequency daily data of equity database i.e. Standard & Poor's index (S&P 500) for portfolio selection problem. Methodology were: (i) employed informative priors on the expected returns and variance- covariance matrices, (ii) daily data with upper and lower holding limits for individual securities, (iii) dynamic asset- allocation approach that was based on re-estimating and then

rebalancing the portfolio weights on a pre- specified time window. The key inputs to the optimization process were the predictive distributions of expected returns and the predictive variance- covariance matrix. It was found that their optimal portfolio outperformed the underlying benchmark.

Roon, Nijman & Werker (2001) applied regression based test for mean- variance spanning in the case of short sales constraints and transaction cost. Test was conducted on US investors to examine whether they can extend their efficient set by investing in emerging market with such frictions. It was found that diversification benefits when market friction were excluded but this disappears when investors face short sales constraint or small transaction cost. Seventeen indices were used from U.S.A., Canada etc. Multivariate regression was done.

Pellizon and Weber (2008) focused on issue of efficiency with illiquid wealth and cases of correlated returns. Markowitz's expected return, variance-covariance matrix of assets, Bayesian method of error estimation and GARCH (second order) were calculated. Majority of Italian households were found to have non- diversify and non-efficient portfolio. Thus, housing wealth pays a key role in determining efficiency of portfolio of the homeowners.

According to **Panageas and Westerfield (2009)** risk-neutral hedge funds managers put a constant proportion of funds in a mean- variance efficient portfolio and remaining in risk free asset. Even in the presence of option like contract, they act as constant relative risk aversion investors.

III. STATEMENT OF THE PROBLEM

The stock market is one of the most vital and dynamic sectors in the financial system making an important contribution to the economic development of a country. Investors are the backbone of the capital market and they are not alike. Institutional investors are capable of understanding the stock market activities and trends but the individual investor are lack of adequate awareness. Large amount of savings emanate from the households, and the small investor is still the only source of risk capital for upcoming enterprises, to undertake new industrial activities, the capital market cannot grow without their participation, directly or indirectly. High dependence on funds of foreign institutional investors will lead to a volatile

and high risk market which will make the small equity investor the only risk capital providers. As small investors find it difficult to participate directly in the capital market to a significant extent, SEBI encourages them to offer innovative products to suit the risk appetite of the small investors.

There are limitations of existing portfolio models by Markowitz, Sharpe, Fama and French regarding multiple

constraints in portfolio optimisation. Most of the existing models have emphasised optimality in terms of one or two key variables ignoring minimum performance of the portfolio for other various financial variables. The present study take into account of various (key variables in current scenario) constraints faced by investors.

IV. OBJECTIVE OF THE STUDY

1. To develop a model for the investors that optimises across multiple constraints while minimising the variance of the efficient portfolio.
2. To measure the performance of the proposed portfolio using Sharpe and Treynor ratio.

V. RESEARCH METHODOLOGY

(a) Source of data

Secondary data were obtained for monthly stock returns, beta, trading volume, turnover and impact cost (a measure of liquidity) for firms at the National Stock Exchange Nifty and annual accounting data such as book-to-market equity, market capitalisation, sales, net profit, dividend, earnings per share and price to earnings ratio, total assets from the Annual Reports of the selected companies.

The data were collected from the official website of National Stock Exchange Limited (www.nseindia.org), annual reports of companies and Centre for Monitoring Indian Economy (CMIE) database PROWESS. The measure of risk-free interest rate, 91 days T-bill rate was taken from the official website of Reserve Bank of India (www.rbi.org.in).

The variables namely Return, EPS, Beta, Dividend, Impact cost, Institutional Holding, Market capitalisation, Net Profit, Price to Book Value ratio (P/BV), Price to Earning ratio (P/E), Promoters' Holding, Sales, Turnover, Unsystematic risk and Volume were aggregated and averaged

NSE Nifty constitutes fifty companies at a particular time. For the present study, thirty companies which were consistent throughout the period of the study in the Nifty 50 index were selected. All the assets included in the sample were equity shares only. The companies are operating across eighteen industrial sectors.

List of the selected companies

S. No.	Security Symbol	Name of the security	Industry code	Industry classification
1	ACC	ACC Ltd.	I ₁	Cement and Cement Products
2	AMBUJACEM	Ambuja Cements Ltd.	I ₁	Cement and Cement Products
3	BHARTIARTL	Bharti Airtel Ltd.	I ₂	Telecommunication- Services

4	BHEL	Bharat Heavy Electricals Ltd.	I ₃	Electrical equipments
5	BPCL	Bharat Petroleum Corporation Ltd.	I ₄	Refineries
6	CIPLA	Cipla Ltd.	I ₆	Pharmaceuticals
7	GAIL	GAIL (India) Ltd.	I ₇	GAS
8	HCLTECH	HCL Technologies Ltd.	I ₈	Computers-Software
9	HDFC	Housing Development Finance Corporation Ltd.	I ₉	FINANCE-HOUSING
10	HDFCBANK	HDFC Bank Ltd.	I ₁₀	BANKS
11	HEROMOTOCO	Hero Motocorp Ltd.	I ₁₁	Automobiles- 2 and 3 wheelers
12	HINDALCO	Hindalco Industries Ltd.	I ₁₃	Aluminium
13	HINDUNILVR	Hindustan Unilever Ltd.	I ₁₈	Diversified
14	ICICIBANK	ICICI Bank Ltd.	I ₁₀	BANKS
15	INFY	Infosys Ltd.	I ₈	Computers-Software
16	ITC	ITC Ltd.	I ₁₄	CIGARETTES
17	LT	Larsen & Toubro Ltd.	I ₁₅	Engineering
18	M&M	Mahindra & Mahindra Ltd.	I ₁₂	Automobiles- 4 wheelers
19	MARUTI	Maruti Suzuki India Ltd.	I ₁₂	Automobiles- 4 wheelers
20	NTPC	NTPC Ltd.	I ₁₆	Power
21	ONGC	Oil Natural Gas Corporation Ltd.	I ₅	Oil Exploration/Production
22	POWERGRID	Power Grid Corporation of India Ltd.	I ₁₆	Power
23	RELIANCE	Reliance Industries Ltd.	I ₄	Refineries
24	SBIN	State Bank of India	I ₁₀	BANKS
25	SUNPHARMA	Sun Pharmaceutical	I ₆	Pharmaceuticals

		Industries Ltd.		
26	TATAMOTOR	Tata Motors Ltd.	I ₁₂	Automobiles- 4 wheelers
27	TATAPOWER	Tata Power Co. Ltd.	I ₁₆	Power
28	TATASTEEL	Tata Steel Ltd.	I ₁₇	Steel and Steel Products
29	TCS	Tata Consultancy Services Ltd.	I ₈	Computers-Software
30	WIPRO	Wipro Ltd.	I ₈	Computers-Software

(b) Period of the study

The study covered a period of ten years i.e. from 1st April 2007 to 31st March 2017.

(c) Statistical tools used for analysis

Efficient Portfolio model has been developed by using LINGO software (LINGO 17.0), Multivariate Regression analysis, Granger causality test undertaken by using E-Views software (Eviews 10.0).

(d) Ratios for performance evaluation

Sharpe ratio is a measure of excess portfolio return over the risk-free relative to its standard deviation.

$$\text{Sharpe ratio} = (R_p - R_f) / \text{sp} \quad \text{(equation 1.1)}$$

where R_p= Portfolio return, R_f= Risk free rate of return and sp= standard deviation of the portfolio.

Treynor ratio is a measure of excess portfolio return over the risk-free relative to its beta.

$$\text{Treynor ratio} = (R_p - R_f) / \beta \quad \text{(equation 1.2)}$$

where R_p= Portfolio return, R_f= Risk free rate of return and beta= beta of the portfolio.

VI. DATA ANALYSIS AND INTERPRETATION

1.1 (a) Regression Analysis

An attempt was made to examine return as function of accounting and financial variables like dividend, earning per share, impact cost, institutional holding, market capitalisation, net profit, price to book value ratio, price to earnings ratio, promoters' holding, sales, turnover, unsystematic risk and volume with the help of multiple regression equation. Beta was not included in the analysis as it has been found to have a significant effect on return of security. To examine the importance of other factors, it was purposefully removed from the analysis.

The result is shown in the table 1.1

Table 1.1 Result of Regression Model 1

Dependent Variable: RATE_1
 Method: Least Squares
 Date: 05/22/18 Time: 17:43
 Sample: 1 30
 Included observations: 30

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.632937	1.892600	-1.391175	0.1832
DIVIDEND	0.000169	0.000108	1.571514	0.1356
EPS	0.017809	0.004362	4.082368	0.0009
IMPACT_COST	55.44594	26.04156	2.129133	0.0491
INST_H	-0.013822	0.016898	-0.817947	0.4254
MARK_CAP	5.90E-06	4.28E-06	1.380833	0.1863
NP	-0.000125	6.21E-05	-2.013532	0.0612
P_BV	0.029667	0.021677	1.368574	0.1900
P_E	0.007405	0.004494	1.647878	0.1189
PRO_H	-0.009831	0.010800	-0.910249	0.3762
SALES	7.82E-06	2.80E-06	2.792211	0.0130
TURNOVER	-9.01E-05	0.000136	-0.664575	0.5158
UNSYS_RISK	23.99884	12.81541	1.872655	0.0795
VOL	2.33E-09	1.15E-09	2.029658	0.0594
R-squared	0.755961	Mean dependent var	1.734213	
Adjusted R-squared	0.557679	S.D. dependent var	0.679377	
S.E. of regression	0.451835	Akaike info criterion	1.553725	
Sum squared resid	3.266476	Schwarz criterion	2.207617	
Log likelihood	-9.305872	Hannan-Quinn criter.	1.762911	
F-statistic	3.812557	Durbin-Watson stat	1.363986	
Prob(F-statistic)	0.006603			

From the above table, the coefficient column shows the explaining variance in the return. Impact cost has the highest coefficient 55.44 with p-value 0.0491 at 5% level of significance, which shows this is the most important factor for explaining return. Other significant variables are EPS, Sales (at 5% level of significance).

R-square value of 0.75 shows the success of regression model i.e. 75% in the dependent variable can be explained by these identified factors. p-value (F-statistic) 0.006603 reject the hypothesis of all the slope coefficient being equal to zero.

Table 1.2 shows the correlation matrix

	DIVIDEND	EPS	IMPACT_C...	INST_H	MARK_CAP	NP	P_BV	P_E	PRO_H	SALES	TURNOVER	UNSYS_RISK	VOL
DIVIDEND	1.000000	0.155956	-0.277676	-0.192108	0.370076	0.737091	0.172624	-0.160539	0.132508	0.207331	0.239648	-0.037063	0.166582
EPS	0.155956	1.000000	-0.653644	0.132627	0.301174	0.385492	-0.118509	-0.268509	-0.112166	0.358494	0.473973	0.110265	0.102750
IMPACT_C...	-0.277676	-0.653644	1.000000	-0.071129	-0.498139	-0.411300	-0.268991	-0.061725	0.154567	-0.291477	-0.660853	-0.102404	-0.288034
INST_H	-0.192108	0.132627	-0.071129	1.000000	0.552258	-0.191096	-0.135167	0.068321	-0.908256	-0.201170	0.475722	-0.212463	0.308407
MARK_CAP	0.370076	0.301174	-0.498139	0.552258	1.000000	0.505499	0.075980	0.029854	-0.610310	0.287044	0.766788	0.064059	0.401888
NP	0.737091	0.385492	-0.411300	-0.191096	0.505499	1.000000	-0.129716	-0.318807	0.138162	0.664417	0.525407	0.253632	0.373916
P_BV	0.172624	-0.118509	-0.268991	-0.135167	0.075980	-0.129716	1.000000	0.204112	0.081069	-0.193361	-0.106174	-0.164981	-0.210313
P_E	-0.160539	-0.268509	-0.061725	0.068321	0.029854	-0.318807	0.204112	1.000000	-0.061765	-0.168499	0.108978	-0.022318	0.100626
PRO_H	0.132508	-0.112166	0.154567	-0.908256	-0.610310	0.138162	0.081069	-0.061765	1.000000	0.127572	-0.483238	0.038510	-0.272554
SALES	0.207331	0.358494	-0.291477	-0.201170	0.287044	0.664417	-0.193361	-0.168499	0.127572	1.000000	0.387436	0.422950	0.299850
TURNOVER	0.239648	0.473973	-0.660853	0.475722	0.766788	0.525407	-0.106174	0.108978	-0.483238	0.387436	1.000000	0.217926	0.663248
UNSYS_RISK	-0.037063	0.110265	-0.102404	-0.212463	0.064059	0.253632	-0.164981	-0.022318	0.038510	0.422950	0.217926	1.000000	0.075855
VOL	0.166582	0.102750	-0.288034	0.308407	0.401888	0.373916	-0.210313	0.100626	-0.272554	0.299850	0.663248	0.075855	1.000000

From the correlation matrix, it is found that market capitalisation is highly positively correlated with turnover (0.7667), then dividend and net profit is highly positively

To improve the result of regression analysis, correlation analysis of all independent variables was analysed to drop some variables which are highly correlated.

correlated (0.737091). Market capitalisation is moderately positively correlated with Institutional holding. Net profit is moderately positively correlated with market capitalisation,

sales, turnover. Turnover and volume also moderately positively correlated with the correlation coefficient ranging between 0.5 and 1.

The high degree of correlation between independent variables shows multi collinearity problem. Therefore, some of the variables namely turnover, market capitalisation and net profit are removed from the regression model and regression equation is re-run with other variables excluding unsystematic risk.

Table 1.3 Result of Regression Model II

Dependent Variable: RATE1
 Method: Least Squares
 Date: 05/25/18 Time: 00:54
 Sample: 1 30
 Included observations: 30

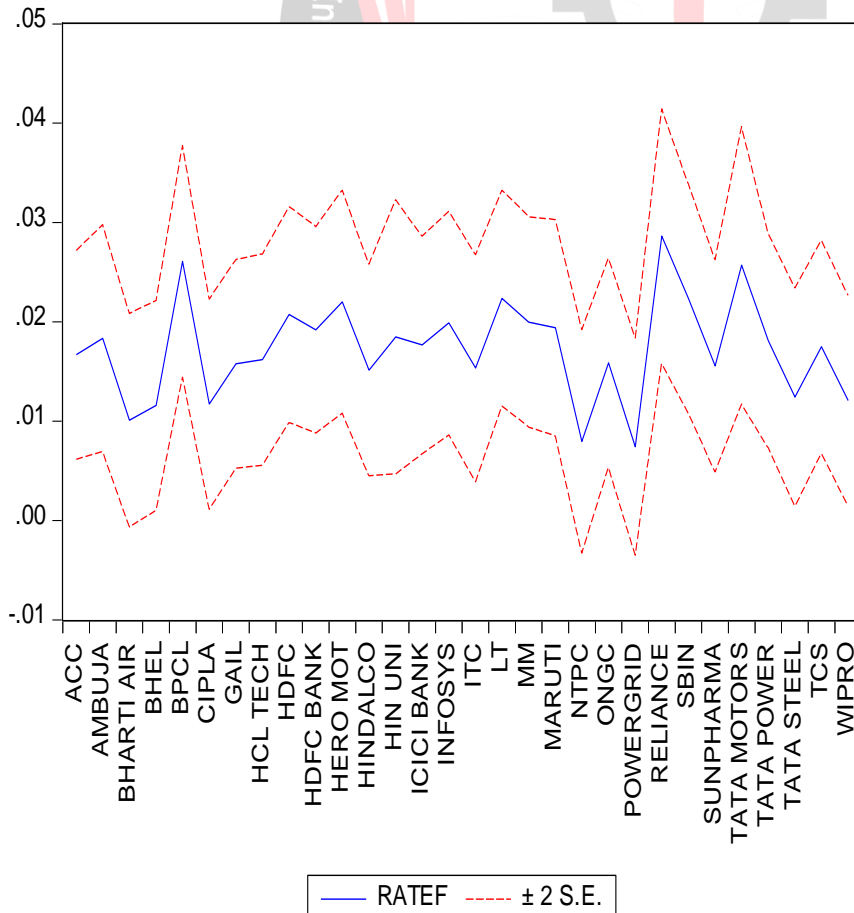
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.807482	1.792058	-1.566625	0.1329
DIVIDEND	-2.80E-05	6.01E-05	-0.465357	0.6467
EPS	0.018072	0.004675	3.865326	0.0010
IMPACT_COST	73.73770	22.91457	3.217939	0.0043
INST_H	-0.025424	0.014429	-1.761933	0.0934
P_BV	0.050299	0.021364	2.354429	0.0289
P_E	0.011473	0.004044	2.837107	0.0102
PRO_H	-0.024582	0.010142	-2.423709	0.0250
SALES	4.94E-06	2.19E-06	2.261411	0.0350
VOL	1.37E-09	1.07E-09	1.282056	0.2145

R-squared	0.630620	Mean dependent var	1.734213
Adjusted R-squared	0.464399	S.D. dependent var	0.679377
S.E. of regression	0.497200	Akaike info criterion	1.701555
Sum squared resid	4.944166	Schwarz criterion	2.168620
Log likelihood	-15.52332	Hannan-Quinn criter.	1.850973
F-statistic	3.793870	Durbin-Watson stat	1.509206
Prob(F-statistic)	0.006237		

From the above regression model, now eps, impact cost, price to book value ratio, price to earnings ratio, promoters' holding and sales are significant at 5% level.

The R-squared value shows that 63% variance in return can be explained by the set of variables, and most of the variables are significant. It shows the model is improved by the above set of variables excluding the insignificant highly correlated variables.

Figure 1.1 depicts the forecasted return of regression model II. Root mean error and mean absolute error give the relative measure to compare forecasts. Their inequality coefficient of 0.12 indicated a good fit of forecast model. Bias proportion is negligible implying the forecast mean is almost equal to actual mean. Variance proportion and covariance proportion indicate a difference in variance and co-variances of the actual values and forecasts.



Forecast: RATEF	
Actual: RATE	
Forecast sample: 1 30	
Included observations: 30	
Root Mean Squared Error	0.004410
Mean Absolute Error	0.003603
Mean Abs. Percent Error	26.80135
Theil Inequality Coef.	0.120360
Bias Proportion	0.000000
Variance Proportion	0.142129
Covariance Proportion	0.857871
Theil U2 Coefficient	0.399657
Symmetric MAPE	22.30873

Figure 1.1 Model Forecasted Return

1.1 (b) Granger causality Analysis

Granger causality test shows the causation of different variables on return.

Table 1.4 shows the granger causality test

Null Hypothesis	F-Statistics	Probability
DIVIDEND does not Granger cause RATE1	0.23935	0.7891
EPS does not Granger cause RATE1	0.65570	0.5285
IMPACT COST does not Granger cause RATE1	3.24088	0.0575
INST HOLDING does not Granger cause RATE1	0.10707	0.8989
MARK CAP does not Granger cause RATE1	2.16042	0.1381
NP does not Granger cause RATE1	1.43282	0.2592
P BV does not Granger cause RATE1	0.40641	0.6707
P E does not Granger cause RATE1	0.37263	0.6930
PRO H does not Granger cause RATE1	0.25568	0.7766
SALES does not Granger cause RATE1	0.49608	0.6153
TURNOVER does not Granger cause RATE1	4.06105	0.0309
UNSYS RISK does not Granger cause RATE1	2.02888	0.1544
VOLUME does not Granger cause RATE1	1.33587	0.2826

From the granger causality test result, it is clear that hypothesis cannot be rejected that dividend, eps, institutional holding, net profit, price to book value ratio, price to earnings ratio, promoters' holding, sales and volume do not cause return as the probability values are high. Therefore, as per the granger causality test these factors are important in portfolio selection modelling due to their explanatory power in return.

2.1 Model Formulations

The investor for the purpose of portfolio modelling is assumed to be a risk averse with indifference curve concave to origin and quadratic utility functions.

1. Markowitz's portfolio model is created focusing on variance, return and fund exhaustion. The constraints for this model are fund exhaustion and Q₃ (Quartile three) level of return.
2. For the Market trend portfolio, the average values of all the variables are targeted.
3. The Capital gain bias portfolio is for investor who aims at high level of capital gain i.e. gain from price movements, beta, earnings per share, net profit, price to book value ratio and price to earnings setting Q₃ (quartile three) level and is

satisfied with a Q₁ (quartile one) level of dividend. Also, other variables institutional holding, market capitalisation and sales were kept at their median values. Such kind of investor seems to be more of a speculator, requiring quick return and not regular income by trading in stock market.

4. Unlike from capital gain bias investor, Dividend gain bias investor desire very high level of regular income in the form of dividend. Here dividend and net profit were set at Q₃ (quartile three) level whereas return, beta, eps, impact cost, market capitalisation, price to book value ratio and price to earnings ratio targeted at Q₁ (quartile one) level and other constraints set at their median values.

5. The Proposed portfolio is created for minimising variance by keeping values of variable as per the result of multivariate regression and granger causality test. Returns, dividend and impact cost targeted at high Q₃ (Quartile three) level. Other significant variables namely earning per share, institutional holding, net profit, price to book value ratio, price to earnings ratio, promoters' holding, sales and volume are targeted at median level. The constraints of beta, market capitalisation, turnover and unsystematic risk were excluded from the set of variables as they failed significantly in explaining returns.

In all these portfolio the no short sales and funds exhaustion constraints were included, and also company diversification (upper bounds) and Industrial diversification constraints were set at fifteen percent and twenty percent respectively.

Efficient Portfolio Selection Model formulations: Analysis and Interpretation

The objective function is minimisation of variance.

1. Markowitz's Portfolio Selection Model

Mean-variance efficient portfolio created according to the Markowitz's Portfolio Selection Model diversified across sixteen companies and ten industrial sectors. The maximum investment are in HCL Technologies and ITC Ltd. whereas minimum investment in Hero Motocorp Ltd., Mahindra & Mahindra Ltd., NTPC Ltd. and State Bank of India. The average weight of a security is 6.25%.

Table 1.5 Markowitz's Portfolio Model: Targets and Achievements

Infeasibilities: 0.0000000		Model Class: QP	
Total solver iterations: 40			
Variables	Targets	Slack or Surplus	Dual Price
Variance	Minimise	0.2671886	-1.0000000
Full	1	0.000000000	-0.2883975
Returns	2.13	0.000000000	-0.1218770

All the constraints are achieved without any surpluses, return and variance of the portfolio are 2.13 and 0.267 respectively. The negative dual price -0.288 for fund exhaustion shows extent of increase in variance of the portfolio with one unit increase in the constraint. The return also displayed small negative shadow price.

2. Market trend portfolio

The market trend portfolio is diversified across eighteen companies and fourteen industrial sectors. Minimum investment is seen in Infosys Ltd, Larsen & Toubro Ltd, Maruti Suzuki India Ltd. and State Bank of India where as maximum investment is seen in HCL Technologies and Reliance Industries Ltd. The average weight of each security is 5.55%.

Table 1.6 Market trend Portfolio Model: Targets and Achievements

Variables		Targets	Slack or Surplus	Dual Price
Infeasibilities: 0.0000000 Model Class: QP Total solver iterations: 72				
Variance	Minimise		0.3260889	-1.00000
Full	1		0.00000	0.8650914
Return	1.73		0.014394	0.00000
EPS	44.07		0.00000	-0.0036
Beta	0.93		0.00000	-0.3534
Dividend	1626.28		0.00000	-0.134E-05
Impact cost	0.07		0.00000	-13.90140
Inst H	39.75		0.00000	-0.00116
Mark cap	57271.75		48907.60	0.00000
NP	5532.19		1317.93	0.00000
P BV	4.69		0.00000	-0.00053
P E	26.92		6.359541	0.00000
Pro H	44.79		0.0724	0.00000
Sales	45523.95		17137.75	0.00000
Turnover	3047.11		0.00000	-0.103E-05
Unsys risk	0.01		0.00000	-2.747
Vol	116474843.33		0.000001	0.00000

The targeted return 1.73 is achieved with surplus of 0.014394 i.e. 1.87 at 0.326 level of variance. The portfolio generates surplus market capitalisation, net profit, price to earnings ratio, promoters' holding and sales. The portfolio is sufficiently liquid with high impact cost of 0.07.

The dual price 0.8650914 for fund exhaustion implies that extent of reduction in portfolio variance with a unit increase in this constraint i.e. allowing borrowing for investment. The negative dual price of beta shows increase in portfolio variance with decrease in this measure of systematic risk. The negative dual price for earning per share, dividend, impact cost, institutional holding, price to book value ratio, turnover and systematic risk indicate the adverse impact on the variance of the portfolio with increase in these constraints.

3. Capital Gain Bias Portfolio

The Capital gain bias portfolio diversified across fifteen companies and eleven industrial sectors. Maximum investment of 15 percent was witnessed in Larsen & Toubro Ltd. (X17), Reliance Industries Ltd. (X23) and State Bank of India (X24). Heavy investment was seen in engineering, refineries and banking sectors whereas marginal investment was observed in housing finance,

automobiles- 4 wheelers and cigarette sectors. The average weight of each security was 6.67%.

Table 1.7 Capital Gain Bias Portfolio: Targets and Achievements

Variables		Targets	Slack or Surplus	Dual Price
Infeasibilities: 0.0000000 Model Class: QP Total solver iterations: 18				
Variance	Minimise		0.5068253	-1.0000000
Full	1		0.0000000	1.511967
Return	2.13		0.0000000	-0.2023466
EPS	60		0.0000000	-0.004443926
Beta	1.13		0.0000000	-1.613620
Dividend	519.8		955.8649	0.0000000
Impact cost	0.063		0.8326401E-03	0.0000000
Inst H	28.87		12.24849	0.0000000
Mark cap	37379.34		95298.74	0.0000000
NP	6797.07		1108.558	0.0000000
P BV	5.24		0.0000000	-0.01918655
P E	25.84		0.0000000	0.6897000E-03
Pro H	29.07		9.958452	0.0000000
Sales	34210.22		44217.96	0.0000000
Turnover	1496.46		3015.986	0.0000000
Unsys var	0		0.01524441	0.0000000
Vol	44849863		0.1372107E+09	0.0000000

High expected return of 2.13 percent at a high variance of 0.507 was achieved. This was the highest variance than all other portfolio model formulations. Interestingly, dividend was achieved with surplus of 955.8649. Heavy surplus could be seen in market capitalisation, net profit, institutional holding, sales, promoters holding, turnover.

The dual price of fund exhaustion constraint depicted the extent of reduction in portfolio variance by allowing borrowing/lending of funds for portfolio formation. The negative dual price of eps, beta and price to book value ratio pointed increase in portfolio variance if the value of these constraints increased. A positive but negligible dual price for price to earnings ratio indicates the marginal improvement means reduction of portfolio variance with unit increase in this constraint.

4. Dividend Gain Bias Portfolio

The portfolio diversified across thirteen companies and nine industrial sectors. Maximum investment of fourteen percent was observed in HCL Technologies Ltd. (X8). Heavy investments were seen in computers, automobiles- 4 wheelers, power and refineries sectors whereas oil exploration, automobiles- 2 and 3 wheelers and banks sectors witnessed marginal investment. The average weight of each security was found to be 7.69%.

Table 1.8 Dividend Gain Bias Portfolio: Targets and Achievements

Infeasibilities: 0.0000000		Model Class: QP	
Total solver iterations: 17			
Variables	Targets	Slack or Surplus	Dual Price
Variance	Minimise	0.3400225	-1.0000000
Full	1	0.0000000	-1.033144
Return	1.34	0.1742773	0.0000000
EPS	15.16	27.13421	0.0000000
Beta	0.71	0.1911731	0.0000000
Dividend	2035.84	0.0000000	-0.5670618E-05
Impact cost	0.063	0.005255657	0.0000000
Inst H	35.67	0.0000000	-0.006738382
Mark cap	22132.94	68192.08	0.0000000
NP	6797.07	1440.267	0.0000000
P BV	2.07	0.8455527	0.0000000
P E	16.59	0.0000000	0.04113666
Pro H	51.57	0.0000000	-0.001536162
Sales	34210.22	34179.31	0.0000000
Turnover	2334.29	731.9835	0.0000000
Unsys var	0	0.00948112	0.0000000
Vol	84301263	0.2685842E+08	0.0000000

Expected return of 1.34 was achieved with surplus of 0.174 at variance of 0.340 level. Surplus could be seen in most of the variables except dividend, institutional holding, price to earnings ratio and promoters holding.

The negative and negligible dual price of dividend, institutional holding, promoters holding indicate increase in the portfolio variance if the value of these constraints increased. A positive but negligible dual price for price to earnings ratio indicates the marginal improvement i.e. further reduction of portfolio variance with unit increase in this constraint. This portfolio was able to witness lesser level of variance providing high dividend and good capital gain returns.

5. Proposed portfolio

The portfolio is diversified across fifteen companies and twelve industrial sectors. The average weight of each security is 6.67% and that of each sector is 8.33%. BHEL and HCL Technologies Ltd are the highest investment securities whereas Sun Pharmaceuticals Industries Ltd. is the lowest investment security.

Table 1.9 Proposed Portfolio Model: Targets and Achievements

Infeasibilities: 0.0000000		Model Class: QP	
Total solver iterations: 15			
Variables	Targets	Slack or Surplus	Dual Price
Variance	Minimise	0.3572967	-1.00000
Full	1	0.000000	1.48993
Return	2.13	0.00000	-0.3318099
EPS	37.04	5.9164	0.00000
Dividend	2035.84	0.00000	-0.234E-04
Impact cost	0.07	0.00000	-10.4846

Inst H	35.67	0.00000	-0.0127
NP	3600.01	2494.529	0.00000
P BV	3.31	3.241682	0.00000
P E	20.59	0.00000	0.00840
Pro H	51.57	0.00000	-0.0105
Sales	34210.22	32352.21	0.00000
Vol	84301263.00	2246428	0.000000

The proposed portfolio yielded return 2.13 at variance of 0.357. It fulfils the minimum median requirement for all the constraints such as fund exhaustion, dividend, impact cost, institutional holding, price to earnings ratio and promoters' holding and generate surplus for earning per share, net profit, price to book value, sales and volume. The positive dual price of fund exhaustion indicate the reduction in portfolio variance by allowing borrowed funds.

Table 1.10 exhibits the weight obtained by securities in each of the portfolio selection model.

Table 1.10 Weights of securities

Securities	Markowitz's portfolio	Market trend portfolio	Capital gain bias portfolio	Dividend gain bias portfolio	Proposed portfolio
X1	0.00	0.00	0.00	0.00	0.02
X2	0.00	0.06	0.00	0.00	0.00
X3	0.00	0.00	0.00	0.08	0.00
X4	0.00	0.04	0.04	0.00	0.00
X5	0.11	0.00	0.01	0.03	0.15
X6	0.04	0.05	0.00	0.00	0.00
X7	0.00	0.06	0.00	0.08	0.14
X8	0.15	0.15	0.12	0.14	0.15
X9	0.00	0.00	0.01	0.00	0.04
X10	0.00	0.00	0.00	0.00	0.00
X11	0.01	0.05	0.00	0.02	0.04
X12	0.00	0.05	0.06	0.00	0.00
X13	0.14	0.04	0.08	0.00	0.01
X14	0.00	0.03	0.05	0.03	0.04
X15	0.00	0.02	0.00	0.05	0.00
X16	0.15	0.04	0.01	0.00	0.04
X17	0.06	0.02	0.15	0.00	0.00
X18	0.01	0.08	0.04	0.12	0.00
X19	0.04	0.02	0.01	0.00	0.03
X20	0.01	0.00	0.00	0.07	0.00
X21	0.00	0.00	0.00	0.10	0.08
X22	0.06	0.04	0.00	0.12	0.00
X23	0.11	0.15	0.15	0.12	0.07
X24	0.01	0.02	0.15	0.00	0.03
X25	0.06	0.00	0.00	0.00	0.01
X26	0.00	0.00	0.04	0.00	0.00
X27	0.00	0.04	0.00	0.00	0.00
X28	0.00	0.00	0.08	0.04	0.00
X29	0.02	0.00	0.00	0.00	0.00
X30	0.02	0.00	0.00	0.00	0.05

All three portfolio are investing heavily in X8 (HCL Technologies Ltd.). All the three portfolio invest in X13

(Hindustan Unilever Ltd.), X16 (ITC Ltd.), X19 (Maruti Suzuki India Ltd.), X23 (Reliance Industries Ltd.) and X24 (State Bank of India). Some securities X3, X10, X26 and X28 are not part of any portfolio model formulation.

3.1 Performance evaluation of portfolios

The performance of the portfolio namely Markowitz's portfolio, market trend portfolio, capital gain bias portfolio, dividend gain bias portfolio and proposed portfolio are ranked by the popular evaluation measure Sharpe (1966) and Treynor (1965) ratio. Portfolios are then arranged in descending order of their Sharpe and Treynor ratio.

Table 1.11 Ranking of Portfolios as per Sharpe's Ratio

Portfolio	Variance (Risk)	Std. Dev	Return	Sharpe ratio
Markowitz's	0.267	0.517	2.13	1.21
Proposed	0.357	0.597	2.13	1.05
Capital gain bias	0.507	0.712	2.13	0.88
Market trend	0.326	0.571	1.74	0.42
Dividend gain bias	0.340	0.583	1.51	0.02

The Sharpe ratio of Markowitz's portfolio is the highest followed by proposed portfolio, capital gain bias, market trend portfolio and lastly dividend gain bias portfolio.

Table 1.12 Ranking of Portfolios as per Treynor's Ratio

Portfolio	Variance (Risk)	Beta	Return	Treynor ratio
Markowitz's	0.267	0.782	2.13	0.80
Proposed	0.357	0.832	2.13	0.75
Capital gain bias	0.507	1.13	2.13	0.56
Market trend	0.326	0.881	1.74	0.27
Dividend gain bias	0.340	0.90	1.51	0.01

Here also, according to Treynor ratio, Markowitz's portfolio is the ranked First, then Proposed portfolio ranked second and third position captured by capital gain bias portfolio, fourth market trend portfolio and last position by dividend gain bias portfolio.

So, in both the ratios, proposed portfolio performed better than all other portfolios except Markowitz's portfolio.

VII. SUMMARY AND CONCLUSION

An attempt was made to ease the portfolio selection decision for the investors with their desires and limitations. Multiple goals and constraints provided the direction for development and testing of a mean-variance efficient portfolio. Investors assumed to be risk averse. Markowitz's portfolio selection model using only return showed high performance but the portfolio created keeping current scenario variables that is Proposed portfolio model showed better performance than other portfolio like capital gain

bias, dividend gain bias and market trend portfolio. Hence, in current scenario other than only returns variables like dividend, earning per share, impact cost institutional holding, net profit, price to book value ratio, price to earnings ratio, promoters' holding, sales and volume have significant role in development of efficient portfolio. So, investors should pay attention to these important variables (constraints) while taking the portfolio selection decision.

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