The Effect of Magnetic Field and Thermal Radiation on Natural Convection Flow of Viscous Incompressible Fluid from a Vertical Flat Plate through Porous Medium with Constant Suction

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Abstract: In this paper a two-dimensional natural convection flow of a viscous incompressible and electrically conducting fluid past a vertical permeable flat plate in the presence of uniform transverse magnetic thermal radiation is studied. The governing equations are reduced to ordinary differential equations by introducing appropriate coordinate transformations. Closed form solutions are obtained for the fluid velocity and the temperature. Further skin friction in terms of shear stress and rate of heat transfer in terms of Nusselt number are also derived. The effects of various parameters on the flow fields velocity, temperature, skin friction coefficient and Nusselt number have been discussed with the help of graphs and tables.

Key Words: Natural convection, Thermal radiation, Vertical flat plate, Nusselt number.

I. INTRODUCTION

The effects of radiation on Magneto Hydrodynamic flow and heat transfer issues have turned out to be modernly more critical. A few engineering forms occur at high temperatures and henceforth the information of thermal radiation heat transfer is fundamental for planning of legitimate supplies, for example, gas turbines, atomic power plants and diverse impetus gadgets for air ship, rockets and satellites. At the point when radiative heat move happens in the electrically conducting fluid, it is ionized because of the high working temperature. In perspective of these, numerous analysts have made commitments to the investigation of fluid flow with thermal radiation.

Viskanta [1] investigated the forced convection flow in a horizontal channel permeated by uniform vertical magnetic fluid taking radiation into account. He studied the effects of magnetic field and radiation on the temperature distribution and the rate of heat transfer in the flow. England and Emery [2] have studied the thermal radiation effects of optically thin gray-gas bounded by a stationary vertical plate. Free convection effects on the oscillatory flow past an infinite vertical plate with constant suction is analyzed by Soundalgekar [3]. Approximate solutions for the coupled non-linear equations are obtained for velocity and temperature field. He observed that there is reverse flow of the mean velocity profile of fluids with small prandtl number, in the boundary layer close to a plate which is being heated by the free convection currents. The mean skin-friction increases with more cooling and decreases with more heating of the plate. Also in the case of air the transient velocity is of non-reversed type for positive Grashof numbers and of reversed type for negative Grashof numbers. Instead of horizontal plate, if an infinite isothermal vertical plate is given an impulsive motion how the flow is affected by the free convection currents which exists due to temperature difference between the plate temperature and that of fluid far away from the plate. This was first studied by Soundalgekar [4] who presented an exact solution for free convection effects on the Stokes problem for an infinite vertical plate. MHD effects on impulsively started vertical plate with variable temperature in the presence of transverse magnetic field were considered by Soundalgekar et al [5]. The dimensionless governing equations were solved using Laplace-transform technique.

Soundalgekar and Takhar [6] first, studied the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using Cogly-Vincentine- Gilles equilibrium model. For the same gas Takhar et al [7] investigated the effects of radiation on the MHD free convection flow past a semi-infinite vertical plate while Das et al [8] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Hossain et al [9] studied the effect of radiation on free convection from porous vertical plates. Raptis and Peridikis [10] have studied the effects of radiation and free convection flow on moving plate. Azzam [11] presented radiation effects on the MHD mixed free-fixed convective flow past a semi-infinite moving vertical plate for high temperature differences. Muthucumaraswamy and Senthil kumar [12] analyzed the thermal radiation effects on moving infinite vertical plate in the presence of variable temperature. Muthucumaraswamy and Janakiraman[13] studied magnetic field and radiation effects on moving

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isothermal vertical plate. Ogulu et al [14] analyzed unsteady MHD free convective flow of a compressible fluid past moving a vertical plate in the presence of radiative heat transfer. Rajesh and Varma [15] investigated radiation effects on MHD flow past moving infinite vertical plate in the presence of heat generation. Ahmmed and Alam Sarkar [16] analyzed a two-dimensional natural convection flow of a viscous incompressible and electrically conducting fluid past a vertical impermeable flat plate. The radiation effect on the unsteady natural convection flow past an infinite vertical plate is presented by Rudra Kanta Das and Sankar Kumar Das [17]. Vijaya Kumar et al [18] was investigated the effects of radiation and heat source on unsteady two dimensional laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate. Elsayed M.A. Elbashbeshy et al [19] discussed numerically a problem of unsteady laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid in the presence of thermal radiation, internal heat generation or absorption and magnetic field over an exponentially stretching surface subjected to suction with an exponential temperature distribution. Alivene and Sreevani [20] studied the effects of hall current and thermal radiation on hydro magnetic non-Darcy mixed convective heat and mass transfer flow plat a stretching sheet in the presence of heat source. Mabood et al [21] examines the effects of heat transfer and thermal radiation in stagnation point flow towards a stretching surface.

In this paper we examine thermal radiation effect on two dimensional natural convection flow of a viscous incompressible and electrically conducting fluid past a vertical permeable flat plate in the presence of uniform transverse magnetic field. Also, for the solutions of the governing non-similar equations, a group of transformations is used to get a group of ordinary differential equations. Here we consider the low Prandtl number (Pr) to liquid metals.

II. FORMULATION OF THE PROBLEM

We consider steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid past a semi-infinite vertical permeable flat plate. The physical model of the problem is shown in figure 1. The x'-axis is taken along the plate in the vertical upward way direction and the y'-axis is taken normal to the plate.

We made the following assumptions:

1. The flow is steady and laminar.
2. A uniform magnetic field H_0 is applied in the transverse direction to the flow.
3. Viscous dissipation is neglected in the energy equation as the motion is due to free convection only.
4. Viscosity depending on temperature.
5. Thermal conductivity depending on temperature.
6. The bounding surface is infinite in length.

Hence all the variables are functions of y only.

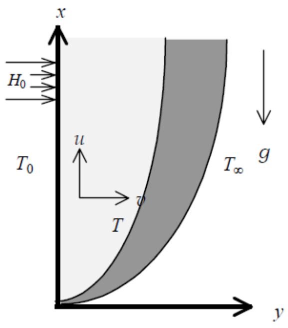

Figure 1: The flow configuration

Under, these assumptions, the steady flow are governed by the following partial differential equations:

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad (1)$$

$$v^*\frac{\partial u^*}{\partial y^*} = \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\upsilon}{K^*}u^* - \frac{\sigma H_0^2}{\rho}u^* + g\beta(T^* - T_\infty) \qquad (2)$$

$$v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q^*}{\partial y^*} \qquad (3)$$

With the boundary conditions

$$u^* = 0,\quad T^* = T_0 \quad \text{at } y^* = 0$$
$$u^* \to 0,\quad T^* \to T_\infty \quad \text{as } y^* \to \infty \qquad (4)$$

where u^* and v^* is the velocity components associated with the direction of increase of coordinates x^* and y^* measured along and normal to the vertical plate. ρ the fluid density, υ is the kinematic viscosity coefficient, K^* the porous medium permeability coefficient, σ is the conductivity of the fluid, H_0 is magnetic field coefficient, g is the acceleration due to gravity, β the coefficient of volume expansion due to temperature, T^* the fluid temperature, T_∞ is the temperature of the ambient fluid, k the thermal conductivity, c_p is the specific heat of constant pressure and q^* is the radiative heat flux.

The radiative heat flux is given by $\frac{\partial q^*}{\partial y^*} = -4\alpha^2(T^* - T_\infty)$ (5)

where α is the mean radiation absorption coefficient.

From the continuity equation (1) we consider the velocity normal to the plate is of the form

$$v = -V_0 \qquad (6)$$

Now we introduce the following transformations to the equations (2) and (3)

$$y = \frac{V_0 y^*}{v},\quad u = \frac{u^*}{U_0},\quad \theta = \frac{T^* - T_\infty}{T_0 - T_\infty},\quad Gr = \frac{g\beta(T_0 - T_\infty)\upsilon}{V_0^2 U_0}, \qquad (7)$$
$$\Pr = \frac{\mu C_p}{k},\quad M = \frac{\sigma H_0^2 \upsilon}{\rho V_0^2},\quad K = \frac{K^* V_0^2}{\upsilon^2},\quad N = \frac{4\alpha^2\upsilon^2}{kV_0^2}$$

where V_0 is the constant suction velocity, U_0 is the uniform velocity, *Gr* is Grashof number, *Pr* is the Prandtl number,

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M the Hartmann number, N is radiation parameter and K is the permeability parameter. We get the following equations

$$\frac{d^2u}{dy^2}+\frac{du}{dy}-M_1u+Gr\theta=0 \qquad (8)$$

$$\frac{d^2\theta}{dy^2}+\Pr\frac{d\theta}{dy}+N\theta=0 \qquad (9)$$

where $M_1=M+\dfrac{1}{K}$

With the boundary conditions

$$u=0,\quad \theta=1 \text{ at } y=0$$

$$u\to 0,\quad \theta\to 0 \text{ as } y\to\infty \qquad (10)$$

III. SOLUTION OF THE PROBLEM

The equations (8) and (9) with boundary conditions (10) are simply ordinary differential equations. We can find the solution of that equation (8) and (9) as the following form equations (11) and (12) respectively.

$$\theta=e^{-m_1y} \qquad (11)$$

$$u=C_3\left(e^{-m_1y}-e^{-m_2y}\right) \qquad (12)$$

The shearing stress and the Nusselt number are important physical parameters for this type of boundary layer flow which are defined and determined as follows:

Shearing stress

The skin friction at the plate, which is in the non-dimensional form is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=C_3(m_2-m_1) \qquad (13)$$

Nusselt Number

The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number is given by

$$Nu=\left(-\frac{\partial\theta}{\partial y}\right)_{y=0}=m_1 \qquad (14)$$

Where

$$m_1=\frac{\Pr+\sqrt{\Pr^2-4N}}{2},\quad M_1=M+\frac{1}{K},\quad m_2=\frac{1+\sqrt{1+4M_1}}{2},$$

$$C_1=0,\quad C_2=-C_3,\quad C_3=\frac{Gr}{M_1-m_1-m_1^2}$$

IV. RESULTS AND DISCUSSIONS

In this section we discuss the results obtained from the solution of the equations governing the MHD free convection flow of a viscous incompressible and electrically conducting fluid with uniform viscosity and uniform thermalconductivity, in the presence of thermal radiation and uniform transverse magnetic field along a permeable vertical flat plate. For the solutions of the governing non-similar equations, a group of transformations is used to get a group of ordinary differential equations. Here we consider the low Prandtl number (Pr) to liquid metals. We have pursued solutions for Pr equals 0.004 for sodium, 0.05 for lithium, 0.72 for air, and 0.92 for ammonia at 649^0 C. The variations in velocity and temperature profiles with y for different values of the Prandtl number are plotted in figures 2 and 3. Here for increasing values of Pr, the velocity profiles as well as temperature profiles decreases. The velocity profiles for Grashof number for cooled plate (Gr > 0) and heated plate (Gr < 0) are depicted in the figure 4. From this figure we see that the velocity profile increases with the increasing values of Grashof number Gr. These effects are significant near the surface of the plate. In the downstream region these profiles go to a limiting point.

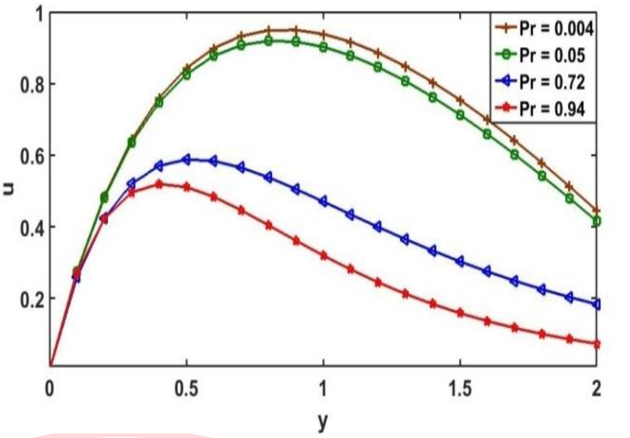

Figure 2: Velocity profiles for different values of Prandtl number Pr withM = 2, *N* = 0.5, K = 0.5, Gr = 5

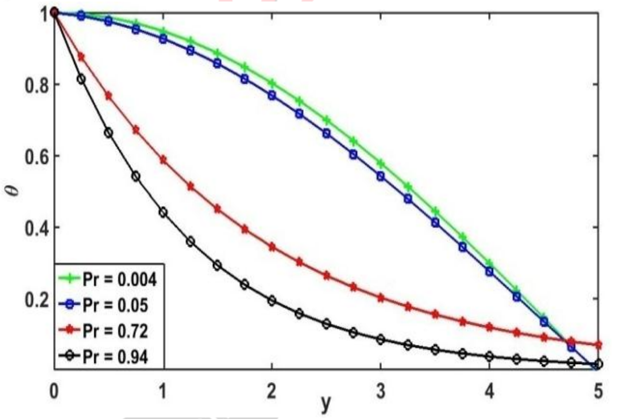

Figure 3: Temperature profiles for different values of *Pr* with *N* = 0.5

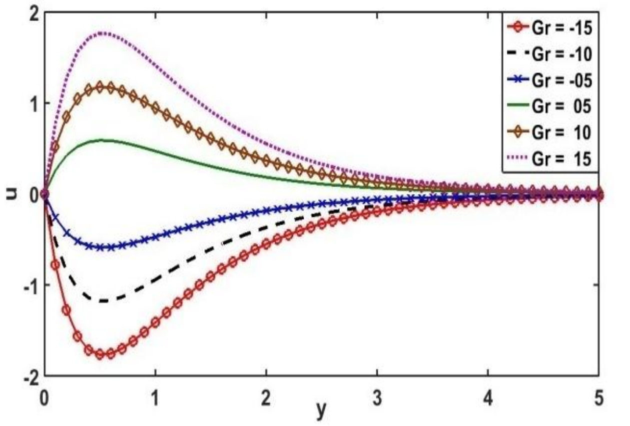

Figure 4: Velocity profiles for different values of Grashof number *Gr* with Pr = 0.72, *N* = 0.5, K = 0.5, M = 2

The influence of magnetic parameter on velocity profiles is plotted in figure 5. These velocity profiles increases with the increasing values of magnetic field parameter M. We see that for y = 0.7 these effect is significant and for larger values of y these profiles go to a limiting point.

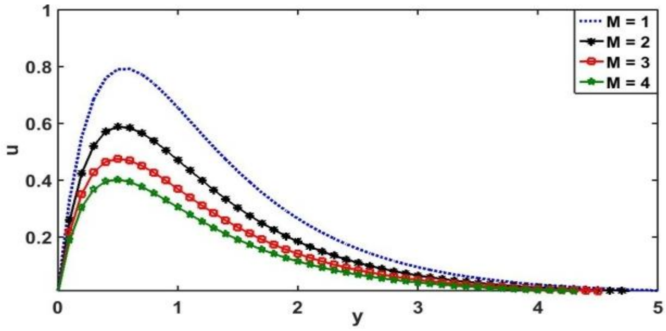

Figure 5: Velocity profiles for different values of magnetic parameter *M* with Pr = 0.72, *N* = 0.5, K = 0.5, Gr = 5

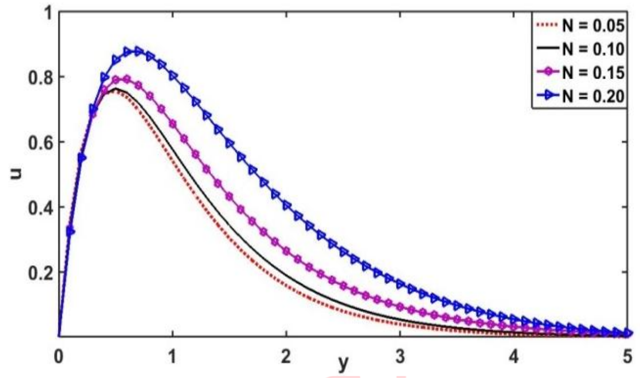

Figure 6: Velocity profiles for different values of radiation parameter N with M = 2, Pr = 0.72, K = 0.5, Gr = 5

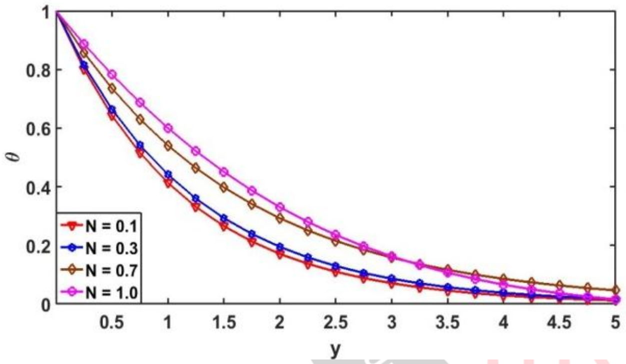

Figure 7: Temperature profiles for different values of *N* with *Pr*= 0.72

Figures 6 and 7 display that fluid velocity and temperature increase with an increase in radiation parameter N. This is due to the fact that the radiation provides an additional means to diffuse energy. The velocity profiles for the values of permeability of the porous medium K is shown in figure 8. Here we find that as the values of K increases, it leads to increase in velocity.

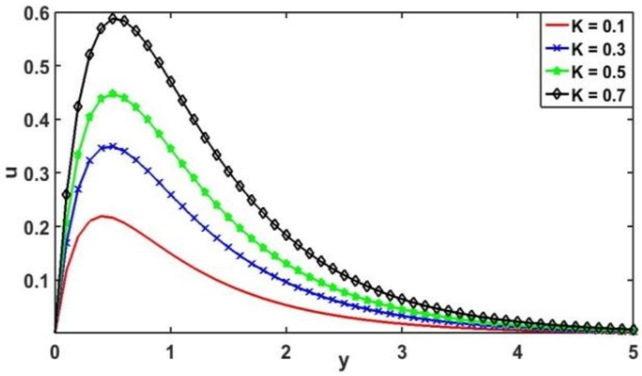

Figure 8: Velocity profiles for different values of permeability parameter K with M=2, Pr=0.72, N=0.1, Gr=5

Table 1: Skin-friction coefficient

M	*N*	*Pr*	*K*	*Gr*	*τ*
1	**0.1**	**0.72**	**0.5**	**5**	0.2687
2	0.1	0.72	0.5	5	0.2113
3	0.1	0.72	0.5	5	0.1790
1	**0.3**	0.72	0.5	5	-0.2662
1	**0.5**	0.72	0.5	5	-0.5277
1	0.1	**0.94**	0.5	5	0.0758
1	0.1	**1.00**	0.5	5	0.0578
1	0.1	0.72	**0.7**	5	0.3297
1	0.1	0.72	**1.0**	5	0.4107
1	0.1	0.72	0.5	**10**	0.5374
1	0.1	0.72	0.5	**15**	0.8061

We have calculated the skin friction and the rate of heat transfer in the equation (13) and (14). The numerical values of skin friction coefficient and rate of heat transfer are shown in tables 1 and 2. From table 1 it is noticed that for increasing values of magnetic parameter, Prandtl number and radiation parameter, the local skin friction decreases monotonically. Whereas reverse effect is observed in the case of permeability parameter and Grashof number. Table 2 depicts the effect of Prandtl number and radiation parameter on Nusselt number. Here it is found that Nusselt number decreases as radiation parameter N increases, whereas Nusselt number increases with increase of Prandtl number Pr.

Table 2: Nusselt number

N	Pr	*Nu*
0.05	**0.72**	0.6421
0.1	0.72	0.5320
0.2	0.72	0.3600
0.1	**0.94**	0.8177
0.1	**1.00**	0.8873

In the absence of porous medium (K= 0) and radiation effect (N= 0), the velocity profile and temperature profiles that we obtained is similar to that of Ahmmed and Alam Sarker [16]. The effects of different Gr and Pr are significant near the surface of the plate (figures 9 and 10).

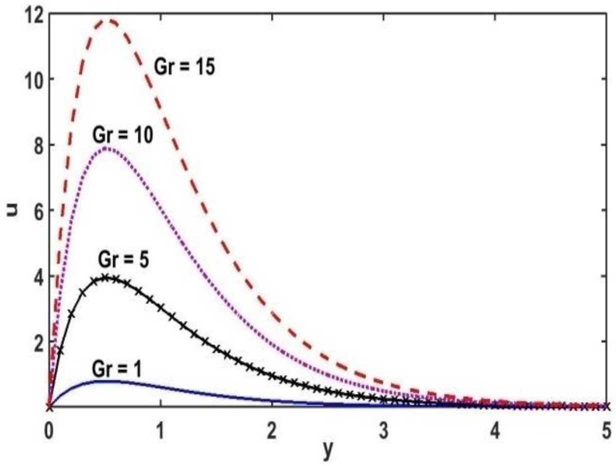

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Figure 9: Velocity profiles for different values of Grashof number *Gr* with Pr = 0.72, M = 1.5, *N* = 0, K = 0

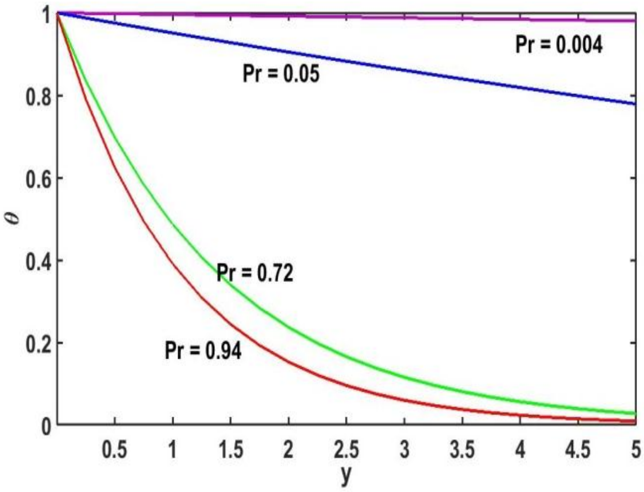

Figure 10: Temperature profiles for different values of *Pr* with *N* = 0

V. CONCLUSIONS

In this paper, the problem of magnetohydrodynamic free convection flow a longa vertical flat plate with thermal radiation is investigated. The local non-similarity equations governing the flow for the case of uniform viscosity and thermal conductivity are developed. The numerical computations were carried out only for the case of assisting flow for the fluids having low Prandtl number appropriate for liquid metals (Pr 0.92 for ammonia, 0.72 for air, 0.05 for lithium and 0.004 for sodium at 649^{0}C).The results thus we obtained for skin friction and the rate of heat transfer coefficient are shown in tabular form in the case of distinct properties of the liquid metals. The velocity profiles and the thermal conductivity profiles are given graphically. Finally, followings may be concluded from the throughout present investigations:

- Profiles for the velocity as well as the thermal conductivity decrease due to the increasing values of the Prandtl number, Pr.
- The velocity and temperature profile increases with the increase of the thermal radiation parameter.
- The velocity profile decreases with the increase of the magnetic parameter.
- The velocity profile increases with the increase of the Grashof number.
- The velocity profile increases with the increase of the permeability parameter.
- For increasing values of Prandtl number, magnetic parameter and radiation parameter, the local skin friction decreases monotonically.
- The skin friction increase at the increasing values of the permeability parameter and Grashof number.
- Nusselt number decreases as radiation parameter N increases, whereas Nusselt number increases with increase of Prandtl number Pr.

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