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Decompositions of (1, 2)*- ψ –Continuity in Bitopological Space

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Abstract: In this paper, we obtain decompositions of (1,2)*- ψ -continuity in bitopological spaces using (1,2)*- Ψ_p-continuity , (1,2)*- Ψ_α -continuity , (1,2)*- Ψ_t-continuity and (1,2)*- Ψ_{α^*}-continuity.

Keywords — (1,2)- Ψ_p-continuity, (1,2)*- Ψ_α –continuity, (1,2)*- Ψ_t-continuity, (1,2)*- Ψ_{α^*}-continuity*

I. INTRODUCTION

Various interesting problems arise when one considers continuity and generalized continuity. In recent years, the decomposition of continuity is one of the main interest for general topologists. In 1961, Levine [1] obtained a decomposition of continuity which was later improved by Rose [2]. Tong [3] decomposed continuity into α -continuity and A-continuity and showed that his decomposition is independent of Levine's. Ganster and Reilly [4] have improved Tong's decomposition result and provided a decomposition of A-continuity. Przemski [5] obtained some decomposition of continuity. Hatir et al. [6] also obtained a decomposition of continuity. Recently, Dontchev and Przemski [7] and Noiri et al. [8] obtained some more decompositions of continuity.

II. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

2.1 Definition

A subset A of a space X is called a (1,2)*- Ψ_p-closed set [9] if (1,2)*- pcl(A) ⊆ U whenever A ⊆ U and U is (1,2)*-gs-open in X. The complement of (1,2)*- Ψ_p-closed set is called (1,2)*- Ψ_p-open set.

2.2 Definition

A subset A of a space X is called:

(i) (1,2)*-t-set [10] if $\tau_{1,2}$-int(A) = $\tau_{1,2}$-int($\tau_{1,2}$-cl(A));

(ii) (1,2)*- α ∗ -set [10] if $\tau_{1,2}$-int(A) = $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)));

(iii) (1,2)*- χ *-set [11] if A = M ∩ N, where M is (1,2)*-gs-open and N is (1,2)*- α -closed in X;

(iv) (1,2)*- χ **-set [9] if A = M ∩ N, where M is (1,2)*- Ψ_α-open and N is (1,2)*-t-set in X;

(v) (1,2)*-gslc*-set [12] if A = M ∩ N, where M is (1,2)*-gs-open and N is $\tau_{1,2}$-closed in X.

The family of all (1,2)*- χ *-sets (resp. (1,2)*- χ **-sets, (1,2)*-gslc*- sets) in a space X is denoted by (1,2)*- χ * (X) (resp. (1,2)*- χ ** (X), (1,2)*-gslc* (X)).

2.3 Remark

In a bitopological space X, the followings hold.

(i) Every $\tau_{1,2}$-closed set is (1,2)*- ψ -closed but not conversely.

(ii) Every (1,2)*- ψ -closed set is (1,2)*- Ψ_α-closed but not conversely [13].

(iii) Every (1,2)*-α -closed set is (1,2)*- Ψ_α-closed but not conversely [13].

(iv) Every (1,2)*- Ψ_α-closed set is (1,2)*- Ψ_p-closed but not conversely [9].

(v) The concepts of (1,2)*- ω -closed sets and (1,2)*- Ψ_α-closed sets are independent [13].

(vi) The concepts of (1,2)*- α -closed sets and (1,2)*- ψ -closed sets are independent.

2.4 Remark [10]

(i) Every (1,2)*-t-set is an (1,2)*-α ∗ -set but not conversely.

(ii) An $\tau_{1,2}$-open set need not be an (1,2)*-α∗ -set .

(iii) The union of two (1,2)*-α ∗ -sets need not be an (1,2)*- α ∗ -set.

2.5 Definition

A function f : X → Y is said to be

(i) (1,2)*- α -continuous [14] if for each σ1,2-open set V of Y, f-1(V) is (1,2)*- α -open in X.

(ii) (1,2)*- ψ -continuous [15] if for each σ1,2-open set V of Y, f-1(V) is (1,2)*- ψ -open in X.

(iii) (1,2)*- Ψ_α- -continuous [13] (resp. (1,2)*- Ψ_p-continuous) if for, each $\sigma_{1,2}$-open set V of Y, f^{-1}(V) is (1,2)*- Ψ_α- -open (resp. (1,2)*- Ψ_p-open) set in X.

(iv) (1,2)*- χ *-continuous [12] if for each $\sigma_{1,2}$-open set V of Y, f^{-1}(V) ∈ (1,2)*- χ * (X).

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(v)(1,2)*- χ *-continuous [11] if for each $\sigma_{1,2}$-open set V of Y, $f^{-1}(V) \in$ (1,2)*- χ *(X).

(vi) (1,2)*- χ ** -continuous [9] if for each $\sigma_{1,2}$-open set V of Y, $f^{-1}(V) \in$ (1,2)*- χ ** (X).

Recently, the following decompositions have been established.

2.6 Theorem [14]

A function f : X → Y is (1,2)*-continuous if and only if it is both (1,2)*- ψ - continuous and (1,2)*- gslc#-continuous.

2.7 Theorem [14]

A function f : X → Y is (1,2)*- α -continuous if and only if it is both (1,2)*- Ψ_α-continuous and (1,2)*- χ *-continuous.

2.8 Theorem [14]

A function f : X → Y is (1,2)*- Ψ_{α^*}-continuous if and only if it is both (1,2)*- Ψ_p- -continuous and (1,2)*- χ ** -continuous.

III. ON (1,2)*- Ψ_{α^*} -SETS AND (1,2)*- Ψ_{α^*}- SETS

3.1 Definition

A subset S of a space X is called:

(i) an (1,2)*- Ψ_t-set if $S = M \cap N$, where M is (1,2)*- ψ -open in X and N is a (1,2)*-t-set in X.

(ii) an (1,2)*- Ψ_{α^*}-set if $S = M \cap N$, where M is (1,2)*- ψ -open in X and N is a (1,2)*- α*-set in X.

The family of all (1,2)*- Ψ_t-sets (resp. (1,2)*- Ψ_{α^*} -sets) in a space X is denoted by (1,2)*- Ψ_t (X) (resp. (1,2)*- Ψ_{α^*}(X)).

3.2 Proposition

Let S be a subset of X. Then

(i) if S is a (1,2)*-t-set, then S ∈ (1,2)*- Ψ_t (X).

(ii) if S is an (1,2)*-α* -set, then S ∈ (1,2)*- Ψ_{α^*} (X).

(iii) if S is an (1,2)*- ψ -open set in X, then S ∈ (1,2)*- Ψ_t (X) and S ∈ (1,2)*- Ψ_{α^*}(X).

Proof

The proof is obvious.

3.3 Proposition

In a space X, every (1,2)*- Ψ_t-set is an (1,2)*- Ψ_{α^*} -set but not conversely.

3.4 Example

Let X = {a, b, c} with $\tau_1 = \{\varphi, \{a, b\}, X\}$ and $\tau_2 = \{\varphi, X\}$. Then {a, c} is (1,2)*- Ψ_{α^*}-set but it is not an (1,2)*- Ψ_t -set.

3.5 Remark

The following examples show that

(i) the converse of Proposition 3.2 need not be true.

(ii) the concepts of (1,2)*- Ψ_t -sets and (1,2)*- Ψ_p -open sets are independent.

(iii) the concepts of (1,2)*- Ψ_{α^*} -sets and (1,2)*- Ψ_α-open sets are independent.

3.6 Example

In Example 3.4, the set {a} is (1,2)*- Ψ_t-set but not a (1,2)*-t-set and the set {a, b} is an (1,2)*- Ψ_{α^*}-set but not an (1,2)*- α *-set.

3.7 Example

In Example 3.4, {c} is both an (1,2)*- Ψ_t-set and (1,2)*- Ψ_{α^*}-set but it is not an (1,2)*- ψ -open set.

3.8 Example

In Example 4.3.4, the set {c} is an (1,2)*- Ψ_t-set but not a (1,2)*- Ψ_p- open set whereas the set {b, c} is a (1,2)*- Ψ_p-open set but not an (1,2)*- Ψ_t-set.

3.9 Example

Let X = {a, b, c} with $\tau_1 = \{\varphi, \{a\}, X\}$ and $\tau_2 = \{\varphi, \{a, c\}, X\}$. Then {b, c} is an (1,2)*- Ψ_{α^*} -set but not an (1,2)*- Ψ_α-open set whereas the set {a, b} is an (1,2)*- Ψ_α -open set but not an (1,2)*- Ψ_{α^*}-set.

3.10 Example

In Example 4.3.9, the set {a} is (1,2)*- Ψ_{α^*}-set and (1,2)*- Ψ_t -set but it is not an (1,2)*- ψ -closed.

3.11 Remark

From the above discussions, we have the following diagram of implications. where A → B (resp. A B) represents A implies B but not conversely (resp. A and B are independent of each other).

Diagram 1

$$
\begin{tikzcd}
\tau_{1,2}\text{-closed} \arrow[r] \arrow[dd] & (1,2)^*\text{-}\ \psi\text{-closed} \arrow[r] \arrow[d] & (1,2)^*\text{-}\ \Psi_t\text{-set} \arrow[dd, leftrightarrow, "-" marking] \\
 & (1,2)^*\text{-}\ \Psi_{\alpha^*}\text{-set} \arrow[d, leftrightarrow, "-" marking] & \\
(1,2)^*\text{-}\ \alpha\text{-closed} \arrow[r] & (1,2)^*\text{-}\ \Psi_\alpha\text{-closed} \arrow[r] & (1,2)^*\text{-}\ \Psi_p\text{-closed}
\end{tikzcd}
$$

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3.12 Remark

(i) The union of two (1,2)*- Ψ_t-sets need not be an (1,2)*- Ψ_t -set.

(ii) The union of two (1,2)*- Ψ_{α^*} -sets need not be an (1,2)*- Ψ_{α^*}-set.

In Example 3.9, {a} and {b} are (1,2)*- Ψ_t-sets but {a} ∪ {b} = {a, b} is not an (1,2)*- Ψ_t-set.

In Example 3.9, {a} and {b} are (1,2)*- Ψ_{α^*} -sets but {a} ∪ {b} ={a, b} is not an (1,2)*- Ψ_{α^*}-set.

3.13 Remark

(i) The intersection of any numbers of (1,2)*- Ψ_t-sets belongs to (1,2)*- Ψ_t(X).

(ii) The intersection of any numbers of (1,2)*- Ψ_{α^*}-sets belongs to (1,2)*- Ψ_{α^*} (X).

3.14 Lemma

(i) A subset S of X is (1,2)*- ψ -open if and only if F ⊂ $\tau_{1,2}$-int(S) whenever F ⊂ S and F is (1,2)*-gs-closed in X.

(ii) A subset S of X is (1,2)*- Ψ_α- -open if and only if F ⊂ (1,2)*- α int(S) whenever F ⊂ S and F is (1,2)*-gs-closed in X.

(iii) A subset S of X is (1,2)*- Ψ_p -open if and only if F ⊂ (1,2)*- pint(S) whenever F ⊂ S and F is (1,2)*-gs-closed in X.

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