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(1,2)*-GPR-Closed Sets in Bitopological Spaces

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Abstract: In this paper, the concepts of (1,2)*-gpr-closed sets, (1,2)*-generalized pre-regular open sets, (1,2)*-gpr-continuous and (1,2)*-gpr-irresolute functions are introduced and some of their properties are investigated.

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I. INTRODUCTION

Levine [13] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Maki et.al. [15], Bhattacharya and Lahiri [2], Arya and Nour [1] and Dontchev [4] introduced and studied the notions of αg-closed sets, sg-closed sets, gs-closed sets and gsp-closed sets in topological spaces. The study of bitopological space was initiated by Kelly [10] in the year 1963. Recently Ravi, Lellis Thivagar, Ekici and Many others [9, 11, 12, 18 - 24] defined different weak forms of semi-open, preopen, regular open and α-open sets in bitopological spaces.

In this paper, we introduce the notions of (1,2)*-generalized pre-regular closed (briefly, (1,2)*-gpr-closed) sets and investigate their properties. By using the class of (1,2)*-gpr-closed sets, we study the properties of (1,2)*-gpr-open sets, (1,2)*-gpr-continuous and (1,2)*-gpr-irresolute functions. In most of the occasions our ideas are illustrated and substantiated by some suitable examples.

II. PRELIMINARIES

Throughout this paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) respectively, on which no separation axioms are assumed.

Definition 2.1

Let S be a subset of a bitopological space X. Then S is called $\tau_{1,2}$-open [18] if $S = A \cup B$, where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Definition 2.2

Let A be a subset of a bitopological space X. Then

(i) the $\tau_{1,2}$-closure of A [18], denoted by $\tau_{1,2}$-cl(A), is defined by
$\cap\{U: A \subseteq U$ and U is $\tau_{1,2}$-closed $\}$;

(ii) the $\tau_{1,2}$-interior of A [18], denoted by $\tau_{1,2}$-int(A), is defined by
$\cup\{U: U \subseteq A$ and U is $\tau_{1,2}$-open$\}$.

Example 2.3

Let $X = \{a, b, c\}$, $\tau_1 = \{\varphi, X, \{b\}\}$ and $\tau_2 = \{\varphi, X, \{c\}\}$. Then the sets in $\{\varphi, X, \{b\}, \{c\}, \{b, c\}\}$ are $\tau_{1,2}$-open and the sets in $\{\varphi, X, \{a\}, \{a, b\}, \{a, c\}\}$ are $\tau_{1,2}$-closed.

Remark 2.4 [18]

Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.

Now we recall some definitions and results, which are used in this paper.

Definition 2.5

A subset S of a bitopological space X is said to be

(i) (1,2)*-semi-open [19] if $S \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(S));

(ii) (1,2)*-preopen [19] if $S \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(S));

(iii) (1,2)*-α-open [19] if $S \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(S)));

(iv) regular (1,2)*-open [21] if $S = \tau_{1,2}$-int($\tau_{1,2}$-cl(S)).

The complements of the above mentioned open sets are called their respective closed sets.

The family of all (1,2)*-semi-open (resp. (1,2)*-preopen, (1,2)*-α-open, regular (1,2)*-open) sets of X will be denoted by (1,2)*-SO(X) (resp. (1,2)*-PO(X), (1,2)*-αO(X), (1,2)*-RO(X)).

The (1,2)*-α-closure (resp. (1,2)*-semi-closure, (1,2)*-pre-closure) of a subset S of X is, denoted by (1,2)*-αcl(S) (resp. (1,2)*-scl(S), (1,2)*-pcl(S)), defined as the intersection of all (1,2)*-α-closed (resp. (1,2)*-semi-closed, (1,2)*-preclosed) sets containing S. The (1,2)*-pre-interior of S is, denoted by (1,2)*-pint(S), defined as the union of all (1,2)*-preopen sets contained in S.

Definition 2.6

A subset S of a bitopological space X is said to be

(i) a (1,2)*-αg-closed [15] if (1,2)*-αcl(S) $\subseteq$ U whenever $S \subseteq U$ and $U \in$ (1,2)*-αO(X).

(ii) a (1,2)*-sg-closed [2] if (1,2)*-scl(S) $\subseteq$ U whenever $S \subseteq U$ and $U \in$ (1,2)*-SO(X).

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(iii) a (1,2)*-gs-closed [1] if (1,2)*-scl(S) ⊆ U whenever S ⊆ U and U ∈ (1,2)*-αO(X).

(iv) a (1,2)*-gp- closed [4] if (1,2)*-pcl(S) ⊆ U whenever S ⊆ U and U ∈ (1,2)*-αO(X).

(v) a (1,2)*-gsp-closed [4] if (1,2)*-βcl(S) ⊆ U whenever S ⊆ U and U ∈ (1,2)*-αO(X).

Result 2.7

Let A and B be subsets of a bitopological space X. Then

(i) A ⊆ (1,2)*-pcl(A) and A ⊆ (1,2)*-αcl(A).

(ii) A is (1,2)*-α-closed (resp. (1,2)*-preclosed) if and only if A = (1,2)*-αcl(A) (resp. (1,2)*-pcl(A)).

(iii) A ⊆ B ⇒ (1,2)*-pcl(A) ⊆ (1,2)*-pcl(B) and (1,2)*-pcl((1,2)*-pcl(A)) = (1,2)*-pcl(A).

III. BITOPOLOGICAL PROPERTIES OF (1,2)*-GPR-CLOSED SETS

Definition 3.1

A subset S of a bitopological space X is said to be (1,2)*-regular-α-generalized-closed (briefly, (1,2)*-rαg-closed) if (1,2)*-αcl(S) ⊆ U whenever S ⊆ U and U ∈ (1,2)*-RO(X).

The collection of all (1,2)*-rαg-closed sets of X will be denoted by (1,2)*-RαGC(X).

Definition 3.2

A subset S of a bitopological space X is said to be (1,2)*-generalized pre-regular closed (briefly, (1,2)*-gpr-closed) if (1,2)*-pcl(S) ⊆ U whenever S ⊆ U and U ∈ (1,2)*-RO(X).

The collection of all (1,2)*-gpr-closed sets of X will be denoted by (1,2)*-GPRC(X).

Example 3.3

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}, {a, b}}. Then the sets in {φ, X, {a}, {b}, {a, b}} are $\tau_{1,2}$-open. Clearly the sets in {{c}, {a, b}, {a, c}, {b, c}, φ, X} are (1,2)*-gpr-closed sets.

Theorem 3.4

Every (1,2)*-rαg-closed set is (1,2)*-gpr-closed but not conversely.

Proof

Let S ⊂ X be a (1,2)*-rαg-closed set. Obviously (1,2)*-pcl(S) ⊆ (1,2)*-αcl(S) ⊆ U. Hence S is (1,2)*-gpr-closed.

Example 3.5

Let X = {a, b, c, d, e}, τ_1 = {φ, X, {a, b}, {a, b, c, d}} and τ_2 = {φ, X, {c, d}}. Then the sets in {φ, X, {a, b}, {c, d}, {a, b, c, d}} are $\tau_{1,2}$-open. Clearly the set {c} is (1,2)*-gpr-closed set but not (1,2)*-rαg-closed.

Remark 3.6

The following examples show that the concepts of

(i) (1,2)*-sg-closed sets and (1,2)*-gpr-closed sets are independent.

(ii) (1,2)*-gs-closed sets and (1,2)*-gpr-closed sets are independent.

(iii) (1,2)*-gpr-closed sets and (1,2)*-gsp-closed sets are independent.

Example 3.7

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}}. Then the sets in {φ, X, {a}, {b}, {a, b}} are $\tau_{1,2}$-open. Clearly the set {a, b} is (1,2)*-gpr-closed set but not (1,2)*-sg-closed and the set {b} is (1,2)*-sg-closed set but not (1,2)*-gpr-closed.

Example 3.8

In Example 3.7, Clearly the set {a, b} is (1,2)*-gpr-closed set but not (1,2)*-gs-closed and the set {a} is (1,2)*-gs-closed set but not (1,2)*-gpr-closed.

Example 3.9

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}}. Then the sets in {φ, X, {a}, {b}, {a, b}} are $\tau_{1,2}$-open. Clearly the set {a, b} is (1,2)*-gpr-closed set but not (1,2)*-gsp-closed.

Example 3.10

In Example 3.5, Clearly the set {a, b} is (1,2)*-gsp-closed set but not (1,2)*-gpr-closed.

Theorem 3.11

Every (1,2)*-gp-closed set is (1,2)*-gpr-closed but not conversely.

Proof

It follows from the fact that (1,2)*-RO(X) ⊂ (1,2)*-αO(X).

Example 3.12

Let X = {a, b, c, d}, τ_1 = {φ, X, {c}, {d}, {c, d}, {a, d}, {a, c, d}} and τ_2 = {φ, X, {a}, {c}, {a, c}, {b, c}, {a, b, c}}. Then the sets in {φ, X, {a}, {c}, {d}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, c, d}, { b, c, d}} are $\tau_{1,2}$-open. Clearly the set {a, c, d} is (1,2)*-gpr-closed set but not (1,2)*-gp-closed.

Proposition 3.13

(i) Every (1,2)*-α-closed set is (1,2)*-αg-closed.

(ii) Every (1,2)*-αg-closed set is (1,2)*-gpr-closed.

Proof

(i) Let A be a (1,2)*-α-closed set. Then A = (1,2)*-αcl(A). Let A ⊆ U and U ∈ (1,2)*-αO(X). We have (1,2)*-αcl(A) ⊆ U whenever A ⊆ U and U ∈ (1,2)*-αO(X). Hence A is (1,2)*- αg-closed.

(ii) Let A ⊆ U and U ∈ (1,2)*-RO(X). Since (1,2)*-RO(X) ⊆ (1,2)*-αO(X) and A is (1,2)*-αg-closed, (1,2)*-αcl(A) ⊆ U whenever A ⊆ U and U ∈ (1,2)*-αO(X). Hence A is (1,2)*- gpr-closed.

Theorem 3.14

Let S be a regular (1,2)*-open subset of a bitopological space X. Then

(i) If S is (1,2)*-gpr-closed then S is (1,2)*-preclosed.

(ii) If S is (1,2)*-rαg-closed then S is (1,2)*-α-closed.

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Proof

(i) Let S be regular (1,2)*-open and (1,2)*-gpr-closed. Then (1,2)*-pcl(S) ⊂ S which implies that S is (1,2)*-preclosed.

(ii) Since S is regular (1,2)*-open and (1,2)*-rαg-closed, (1,2)*-αcl(S) ⊂ S. Thus S is (1,2)*-α-closed.

Remark 3.15

The intersection of two (1,2)*-gpr-closed sets need not be a (1,2)*-gpr-closed as shown in the following example.

Example 3.16

Let X = {a, b, c}, τ_1 = {φ, X, {b}, {c}, {b, c}} and τ_2 = {φ, X, {b}}. Then the sets in {φ, X, {b}, {c}, {b, c}} are $\tau_{1,2}$-open. Clearly the sets {a, b} and {b, c} are (1,2)*-gpr-closed but the set {b} is not (1,2)*-gpr-closed.

Remark 3.17

The union of (1,2)*-gpr-closed sets also need not be a (1,2)*-gpr-closed as shown in the following example.

Example 3.18

In Example 3.5, Clearly the sets {a} and {b} are (1,2)*-gpr-closed but the set {a, b} is not (1,2)*-gpr-closed.

Theorem 3.19

If S is a (1,2)*-gpr-closed set in X, then (1,2)*-pcl(S) \ S does not contain any nonempty regular (1,2)*-closed set.

Proof

Let F be a regular (1,2)*-closed set such that F ⊂ (1,2)*-pcl(S) \ S. Then F ⊂ (1,2)*-pcl(S) but F is not a subset of S. On the other hand, S ⊂ X \ F and X \ F is regular (1,2)*-open. Since S is (1,2)*-gpr-closed, (1,2)*-pcl(S) ⊂ X \ F and then F ⊂ X \ (1,2)*-pcl(S). Now, we have F ⊂ (1,2)*-pcl(S) ∩ [X \ (1,2)*-pcl(S)]. This is a contradiction. Thus F = φ.

Remark 3.20

The converse of Theorem 3.19 need not be true as shown in the following example.

Example 3.21

In Example 3.5, we have the sets in {{a, b, e}, {c, d, e}, φ, X} are regular (1,2)*-closed. Clearly the set {a, b} is not (1,2)*-gpr-closed while (1,2)*-pcl({a, b}) \ {a, b} = {e}, which does not contain any nonempty regular (1,2)*-closed set.

Theorem 3.22

Let S be a (1,2)*-gpr-closed set in X. Then S is (1,2)*-preclosed if and only if (1,2)*-pcl(S) \ S is regular (1,2)*-closed.

Proof

Let S be (1,2)*-preclosed. Then (1,2)*-pcl(S) = S. Hence (1,2)*-pcl(S) \ S = φ is a regular (1,2)*-closed set. Conversely, suppose (1,2)*-pcl(S) \ S is regular (1,2)*-closed. As S is (1,2)*-gpr-closed, by Theorem 3.19, (1,2)*-pcl(S) \ S = φ, and therefore (1,2)*-pcl(S) = S. Hence S is (1,2)*-preclosed.

Definition 3.23

Let X be a bitopological space and S ⊂ X and x ∈ X. Then

(i) x is said to be a (1,2)*-pre-limit point of S if every (1,2)*-preopen set containing x contains a point of S different from x.

(ii) x is said to be a (1,2)*-α-limit point of S if every (1,2)*-α-open set containing x contains a point of S different from x.

Definition 3.24

Let X be a bitopological space and S ⊂ X and x ∈ X. Then

(i) The set of all (1,2)*-pre-limit points of S is said to be (1,2)*-pre-derived set and is denoted by (1,2)*-D_p(A).

(ii) The set of all (1,2)*-α-limit points of S is said to be (1,2)*-α-derived set and is denoted by (1,2)*-D(A).

Theorem 3.25

Let A and B be (1,2)*-gpr-closed sets in a bitopological space X. If

(i) (1,2)*-D(A) ⊆ (1,2)*-D_p(A) and

(ii) (1,2)*-D(B) ⊆ (1,2)*-D_p(B),

then A ∪ B is (1,2)*-gpr-closed.

Proof

For any set A ⊂ X, (1,2)*-D_p(A) ⊆ (1,2)*-D(A). By using assumption, we obtain (1,2)*-D(A) = (1,2)*-D_p(A) and similarly, (1,2)*-D(B) = (1,2)*-D_p(B). Let A ∪ B ⊆ U and U be regular (1,2)*-open. Since A and B are (1,2)*-gpr-closed, then (1,2)*-pcl(A) ⊆ U and (1,2)*-pcl(B) ⊆ U. Now (1,2)*-pcl(A) ∪ (1,2)*-pcl(B) ⊆ U and hence (1,2)*-pcl(A ∪ B) ⊆ U. Hence A ∪ B is (1,2)*-gpr-closed.

Theorem 3.26

If A is (1,2)*-gpr-closed and A ⊆ B ⊆ (1,2)*-pcl(A), then B is (1,2)*-gpr-closed.

Proof

Let B ⊆ U and U be regular (1,2)*-open. Then A ⊆ U and hence (1,2)*-pcl(A) ⊆ U (Since A is (1,2)*-gpr-closed set). By assumption, B ⊆ (1,2)*-pcl(A). Then (1,2)*-pcl(B) ⊆ (1,2)*-pcl(A) ⊆ U. Hence B is (1,2)*-gpr-closed.

Definition 3.27

A subset S of a bitopological space X. Then (1,2)*-gpr-closure of S, denoted by (1,2)*-gprcl(S), is the intersection of all (1,2)*-gpr-closed sets of X containing S.

Proposition 3.28

Let A and B be subsets of X. The followings hold.

(i) (1,2)*-gprcl(φ) = φ and (1,2)*-gprcl(X) = X.

(ii) If A ⊆ B then (1,2)*-gprcl(A) ⊆ (1,2)*-gprcl(B).

(iii) A ⊆ (1,2)*-gprcl(A).

(iv) (1,2)*-gprcl(A ∪ B) ⊇ (1,2)*-gprcl(A) ∪ (1,2)*-gprcl(B).

(v) (1,2)*-gprcl (A ∩ B) ⊆ (1,2)*-gprcl(A) ∩ (1,2)*-gprcl(B).

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Remark 3.29

If $S \subseteq X$ is (1,2)*-gpr closed, then (1,2)*-gprcl(S) = S

The converse of this remark need not be true as shown in the following example.

Example 3.30

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b}, {a, b}} and τ_2 = {φ, X}. Then the sets in {φ, X, {a}, {b}, {a, b}} are $\tau_{1,2}$-open. Then (1,2)*-gprcl({b}) = {b} but the set {b} is not a (1,2)*-gpr-closed.

Proposition 3.31

Let S be a subset of X. Then $x \in$ (1,2)*-gprcl(S) if and only if $V \cap S \neq \varphi$ for every (1,2)*-gpr-open set V containing x.

Proof

Suppose that there exists a (1,2)*-gpr-open set V containing x such that $V \cap S = \varphi$. Then $S \subset X \setminus V$ and $X \setminus V$ is (1,2)*-gpr-closed. So (1,2)*-gprcl(S) $\subset X \setminus V$ implies $x \notin$ (1,2)*-gprcl(S). This is a contradiction. Conversely, suppose $x \notin$ (1,2)*-gprcl(S). Then there exists a (1,2)*-gpr-closed subset F containing S such that $x \notin F$. Then $x \in X \setminus F$ and $X \setminus F$ is (1,2)*-gpr-open and $(X \setminus F) \cap S = \varphi$. This is a contradiction.

IV. (1,2)*-GENERALIZED PRE-REGULAR OPEN SETS

Definition 4.1

A subset S of a bitopological space X is called (1,2)*-gpr-open if its complement is (1,2)*-gpr-closed.

Remark 4.2

For a subset S of a space X, (1,2)*-pcl($X \setminus A$) = $X \setminus$ (1,2)*-pint(A).

Theorem 4.3

Let S be a subset of a bitopological space X. Then S is a (1,2)*-gpr-open if and only if $F \subseteq$ (1,2)*-pint(S), where F is a regular (1,2)*-closed set and $F \subseteq S$.

Proof

Let S be a (1,2)*-gpr-open set. Let F be a regular (1,2)*-closed and $F \subseteq S$. Then $X \setminus S \subseteq X \setminus F$, $X \setminus F$ is regular (1,2)*-open and $X \setminus S$ is (1,2)*-gpr-closed. Since (1,2)*-pcl($X \setminus S$) $\subseteq X \setminus F$, Then $X \setminus$ (1,2)*-pint(S) $\subseteq X \setminus F$ and so $F \subseteq$ (1,2)*-pint(S).

Conversely, suppose F is a regular (1,2)*-closed set and $F \subseteq S$ with $F \subseteq$ (1,2)*-pint(S). Let $X \setminus S \subseteq U$ where U is regular (1,2)*-open. Then $X \setminus U \subseteq S$ and hence by assumption, $X \setminus U \subseteq$ (1,2)*-pint(S). We have $X \setminus$ (1,2)*-pint(S) $\subseteq U$. Since (1,2)*-pcl($X \setminus S$) $\subseteq U$, $X \setminus S$ is (1,2)*-gpr-closed and hence S is (1,2)*-gpr-open.

Theorem 4.4

If (1,2)*-pint(A) $\subseteq B \subseteq A$ and A is (1,2)*-gpr-open, then B is (1,2)*-gpr-open.

Proof

Since (1,2)*-pint(A) $\subseteq B \subseteq A$, then $X \setminus A \subseteq X \setminus B \subseteq X \setminus$ (1,2)*-pint(A), that is $X \setminus A \subseteq X \setminus B \subseteq$ (1,2)*-pcl($X \setminus A$). Then by Theorem 3.26, $X \setminus B$ is (1,2)*-gpr-closed. Hence B is (1,2)*-gpr-open.

Theorem 4.5

If $S \subseteq X$ is (1,2)*-gpr-closed, then (1,2)*-pcl(S) $\setminus$ S is (1,2)*-gpr-open.

Proof

Let S be a (1,2)*-gpr-closed. Let F be a regular (1,2)*-closed set such that $F \subseteq$ (1,2)*-pcl(S) $\setminus$ S. Then by Theorem 3.19, F = φ. Hence $F \subseteq$ (1,2)*-pint((1,2)*-pcl(S)) $\setminus$ S. By Theorem 4.3, this shows that (1,2)*-pcl(S) $\setminus$ S is (1,2)*-gpr-open.

Remark 4.6

The converse of Theorem 4.5 need not be true as shown in the following example.

Example 4.7

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {a, b}} and τ_2 = {φ, X, {b}}. Then the sets in {φ, X, {a}, {b}, {a, b}} are $\tau_{1,2}$-open. Let S = {b}. Then (1,2)*-pcl(S) $\setminus$ S = {c} which is (1,2)*-gpr-open but S is not (1,2)*-gpr-closed.

Lemma 4.8

Let X be a bitopological space and $x \in X$. Then $X \setminus \{x\}$ is either regular (1,2)*-open or (1,2)*-gpr-closed.

Proof

If $X \setminus \{x\}$ is not regular (1,2)*-open, then the only regular (1,2)*-open set containing $X \setminus \{x\}$ is X. We have (1,2)*-pcl($X \setminus \{x\}$) $\subseteq$ X and hence $X \setminus \{x\}$ is (1,2)*-gpr-closed.

Theorem 4.9

If (1,2)*-PO(X) = (1,2)*-PC(X), then (1,2)*-GPRC(X) = $\wp$(X) where (1,2)*-PC(X) is the collection of (1,2)*-preclosed sets of X and $\wp$(X) is the power set of X.

Proof

Suppose that $A \subseteq X$, $A \subseteq F$ and F is regular (1,2)*-open in X. Since $F \in$ (1,2)*-RO(X) $\subseteq$ (1,2)*-PO(X), then by assumption F is also (1,2)*-preclosed. Hence (1,2)*-pcl(A) $\subseteq$ F and so A is (1,2)*-gpr-closed. Hence (1,2)*-GPRC(X) = $\wp$(X).

Lemma 4.10 [19]

For a subset A of a bitopological space X, (1,2)*-pcl(A) = $A \cup \tau_{1,2}$-cl($\tau_{1,2}$-int(A)).

Theorem 4.11

If A is (1,2)*-α-closed and (1,2)*-gpr-closed, then it is (1,2)*-rαg-closed.

Proof

Suppose $A \subseteq F$ where F is a regular (1,2)*-open. Since A is (1,2)*-gpr-closed, (1,2)*-pcl(A) $\subseteq$ F. Since every (1,2)*-α-closed set is (1,2)*-preclosed, $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ A implies, by Lemma 4.10, (1,2)*-pcl(A) = A. Since A is (1,2)*-α-closed. we have (1,2)*-αcl(A) = (1,2)*-pcl(A) = $A \subseteq F$. Thus, A is (1,2)*-rαg-closed

Theorem 4.12

Let X be a bitopological space and $A, B \subset X$. If B is (1,2)*-gpr-open and $A \supseteq$ (1,2)*-pint(B), then $A \cap B$ is (1,2)*-gpr-open.

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Proof

Since B is (1,2)*-gpr-open and A $\supseteq$ (1,2)*-pint(B), we have (1,2)*-pint(B) $\subseteq$ A $\cap$ B $\subseteq$ B. Hence, by Theorem 4.4, A $\cap$ B is (1,2)*-gpr-open.

Theorem 4.13

Let (1,2)*-PO(X) be closed under finite intersection. If A and B are (1,2)*-gpr-open sets, then A $\cap$ B is (1,2)*-gpr-open.

Proof

Let X \ (A $\cap$ B) = (X \ A) $\cup$ (X \ B) $\subseteq$ F where F is regular (1,2)*-open. This implies X \ A $\subseteq$ F and X \ B $\subseteq$ F. Since A and B are (1,2)*-gpr-open, (1,2)*-pcl(X \ A) $\subseteq$ F and (1,2)*-pcl(X \ B) $\subseteq$ F. So (1,2)*-pcl(X \ (A $\cap$ B)) $\subseteq$ F. Therefore A $\cap$ B is (1,2)*-gpr-open.

Definition 4.14

For any subset A $\subset$ X, (1,2)*-gpr-int(A) is defined as the union of all (1,2)*-gpr open sets contained in A.

Proposition 4.15

For a subset A of a bitopological space X, X \ (1,2)*-gpr-int(A) = (1,2)*-gpr-cl(X \ A).

Proof

Let x $\in$ X \ (1,2)*-gpr-int(A). Then x $\notin$ (1,2)*-gpr-int(A), i.e. every (1,2)*-gpr-open set B containing x is not a subset of A. Then B intersects X \ A and so x $\in$ (1,2)*-gpr-cl(X \ A). Hence X \ (1,2)*-gpr-int(A) $\subseteq$ (1,2)*-gpr-cl(X \ A).

Conversely, let x $\in$ (1,2)*-gpr-cl(X \ A). Then every (1,2)*-gpr-open set B containing x intersects X \ A which implies that the (1,2)*-gpr-open set B containing x is not a subset of A, i.e. x $\notin$ (1,2)*-gpr-int(A) implies x $\in$ X \ (1,2)*-gpr-int(A) and so (1,2)*-gpr-cl(X \ A) $\subseteq$ X \ (1,2)*-gpr-int(A).

V. (1,2)*-GPR-CONTINUOUS AND (1,2)*-GPR-IRRESOLUTE FUNCTIONS

Definition 5.1

(i) A function f : X $\rightarrow$ Y is said to be (1,2)*-αg-continuous if the inverse image of $\sigma_{1,2}$-closed set in Y is (1,2)*-αg-closed in X.

(ii) A function f : X $\rightarrow$ Y is said to be (1,2)*-M-preclosed if f(A) is (1,2)*-preclosed in Y for every (1,2)*-preclosed set A in X.

(iii) A function f : X $\rightarrow$ Y is said to be (1,2)*-continuous if the inverse image of $\sigma_{1,2}$-closed set in Y is $\tau_{1,2}$-closed in X.

Definition 5.2

A function f : X $\rightarrow$ Y is said to be (1,2)*-gpr-continuous (resp. (1,2)*-rαg-continuous) if $f^{-1}(V)$ is (1,2)*-gpr-closed (resp. (1,2)*-rαg-closed) in X for every $\sigma_{1,2}$-closed set in Y.

Definition 5.3

A function f : X $\rightarrow$ Y is said to be (1,2)*-gpr-irresolute if $f^{-1}(V)$ is (1,2)*-gpr-closed in X for every (1,2)*-gpr-closed set in Y.

Remark 5.4

Every (1,2)*-gpr-irresolute function is (1,2)*-gpr-continuous.

The converse of this remark need not be true as shown in the following example.

Example 5.5

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}}. Y = {a, b, c}, σ_1 = {φ, Y, {a}} and σ_2 = {φ, Y, {a, b}}. Then the function f : X $\rightarrow$ Y defined as f(a) = b, f(b) = a and f(c) = c is (1,2)*-gpr-continuous but not (1,2)*-gpr-irresolute.

Theorem 5.6

(i) If f : X $\rightarrow$ Y is (1,2)*-αg-continuous, then f is (1,2)*-gpr-continuous.

(ii) If f : X $\rightarrow$ Y is (1,2)*-rαg-continuous, then f is (1,2)*-gpr-continuous.

Proof

(i) It follows from Proposition 3.13 (ii).

(ii) It follows from Theorem 3.4.

Remark 5.7

The converse of Theorem 5.6 need not be true as shown in the following example.

Example 5.8

Let X = {a, b, c, d}, τ_1 = {φ, X, {a}, {a, c}, {a, c, d}} and τ_2 = {φ, X, {a, d}, {b, d}, {a, b, d}}. Then the sets in {φ, X, {a}, {a, c}, {a, d}, {b, d}, {a, b, d}, {a, c, d}} are $\tau_{1,2}$-open. Let Y = {a, b, c, d}, σ_1 = {φ, Y, {a}, {d}, {a, d}} and σ_2 = {φ, Y, {a, b, c}, {a, c, d}}. Then the sets in {φ, X, {a}, {d}, {a, d}, {a, b, c}, {a, c, d}} are $\sigma_{1,2}$-open. Then the identity function f : X $\rightarrow$ Y is (1,2)*-gpr-continuous.

(a) Here f is not (1,2)*-rαg-continuous, since f^{-1}({b}) = {b} is not (1,2)*-rαg-closed.

(b) Here f is not (1,2)*-αg-continuous, since f^{-1}({b}) = {b} is not (1,2)*-αg-closed.

Theorem 5.9

Let f : X $\rightarrow$ Y and g : Y $\rightarrow$ Z be any two functions.

(i) g o f is (1,2)*-gpr-continuous if g is (1,2)*-gpr-continuous and f is (1,2)*-gpr-irresolute.

(ii) g o f is (1,2)*-gpr-irresolute if g is (1,2)*-gpr-irresolute and f is (1,2)*-gpr-irresolute.

(iii) g o f is (1,2)*-gpr-continuous if g is (1,2)*-continuous and f is (1,2)*-gpr-continuous.

Proof

(i) Let A be $\eta_{1,2}$-closed set in Z. Since g is (1,2)*-gpr-continuous, $g^{-1}(A)$ is (1,2)*-gpr-closed set in Y. Since f is (1,2)*-gpr-irresolute, $f^{-1}(g^{-1}(A))$ is (1,2)*-gpr-closed set in X. Since $(g \text{ o } f)^{-1}(A) = f^{-1}(g^{-1}(A))$, g o f is (1,2)*-gpr-continuous.

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(ii) Let A be (1,2)*-gpr-closed set in Z. Since g is (1,2)*-gpr-irresolute, $g^{-1}(A)$ is (1,2)*-gpr-closed set in Y. Since f is (1,2)*-gpr-irresolute, $f^{-1}(g^{-1}(A))$ is (1,2)*-gpr-closed set in X. Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, g o f is (1,2)*-gpr-irresolute.

(iii) Let A be $\eta_{1,2}$-closed set in Z. Since g is (1,2)*-continuous, $g^{-1}(A)$ is $\sigma_{1,2}$-closed set in Y. Since f is (1,2)*-gpr-continuous, $f^{-1}(g^{-1}(A))$ is (1,2)*-gpr-closed set in X. Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, g o f is (1,2)*-gpr-continuous.

Theorem 5.10

If $f : X \to Y$ is (1,2)*-gpr-continuous, then f((1,2)*-gpr-cl(A)) $\subseteq$ (1,2)*-αcl(f(A)) for every subset A of X.

Proof

Let A be a subset of X. Since (1,2)*-αcl(f(A)) is (1,2)*-α-closed in Y, then f^{-1}((1,2)*- αcl(f(A))) is (1,2)*-gpr-closed. Since A $\subseteq f^{-1}(f(A)) \subseteq f^{-1}$((1,2)*-αcl(f(A))), then (1,2)*-gpr-cl(A) $\subseteq f^{-1}$((1,2)*-αcl(f(A))) and hence f((1,2)*-gpr-cl(A)) $\subseteq$ (1,2)*-αcl(f(A)).

Conclusion:

We have the following diagram:

$$\begin{array}{ccc}
(1,2)^*\text{-}\alpha\text{-closed} & \rightarrow & (1,2)^*\text{-}\alpha g\text{-closed} \\
 & & \downarrow \\
(1,2)^*\text{-}r\alpha g\text{-closed} & \rightarrow & (1,2)^*\text{-gpr-closed} \\
 & & \uparrow \\
 & & (1,2)^*\text{-gp-closed}
\end{array}$$

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