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Coloring and Global Domination in Vertex Switching of Graphs

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Abstract - Global dominating - χ- coloring number of a graph G is the maximum number of color classes which are global dominating sets of G, where the maximum is taken over all χ colorings of G. In this paper we obtain the global dominating - χ - coloring number for vertex switching of Pn, Cn, K1,n,n, Sn and Hn and also we obtain the global dominating - χ - coloring number degree splitting graph of some graphs.

Keywords: Global dominating set, Vertex Switching, Graph Coloring.

Subject Classifications: 05C15, 05C69.

I. INTRODUCTION

All graphs considered here are finite, undirected, simple graphs. Graph coloring and domination in graphs are two major areas within graph theory which have been extensively studied. Graph coloring deals with the fundamental problem of partitioning a set of objects into classes, according to certain rules. Time tabling, sequencing and scheduling problems in their many terms are basically of this nature. A proper coloring of a graph G = (V (G), E(G)) is an assignment of colors to the vertices of the graph, such that any two adjacent vertices have different colors. The chromatic number χ is the minimum number of colors needed in a proper coloring of a graph. A subset of vertices assigned to the same color is called a color class; every such class forms an independent set. Another fastest growing area within graph theory is the study of domination and related subset problems such as independent domination number , covering ,inverse domination. A complete analysis of the fundamentals of domination is given in Haynes et al.[4].

Given an undirected graph G = (V, E), a dominating set is a subset S ⊆ V of its vertices such that for all vertices v ∈ V , either v ∈ S or a neighbor u of v is in S. The domination number γ(G) is the minimum cardinality of a dominating set of G. A dominating set is called a global dominating set if it is a dominating set for a graph G and its complement G. The Global domination number γg(G) is the minimum cardinality of a Global dominating set of G.

II. PRELIMINARIES

The domcolor number dχ(G) is the maximum number of color classes which are dominating sets of G, where the maximum is taken over all k-colorings of G. and it is denoted by dχ(G).

Let G be a graph . Among all χ of G, a coloring with the maximum number of color classes that are global dominating sets in G is called a global dominating χ coloring of G [5]. The number of color classes that are global dominating sets in a global dominating χ coloring of G is defined to be the global dominating χ color number of G denoted by gdχ(G).

A vertex switching [6] of a graph G is the graph Gv which is obtained by taking a vertex u of G, removing all the edges incident to u and adding edges joining u to every other vertex which are not adjacent to u in G. We call u as the switching vertex

III. GDX - VERTEX SWITCHING OF SOME GRAPHS

Theorem 3.1. For a Cycle graph C_n, n ≥ 6, the global dominating - χ - coloring number on Vertex Switching of C_n is,

$gd_\chi(\hat{C}_n) = 2$

Proof. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be the vertices of cycle graph on n vertices. Assume that without loss of generality $\hat{C}_n$ is obtained from C_n by switching the vertex v_1. By the definition of vertex switching of C_n, let

$V(\hat{C}_n) = \{v_1, v_2, v_3, \ldots, v_n\}$ and

$E(\hat{C}_n) = \{e_i = v_iv_{i+1}, 2 \le i \le n-1\} \cup \{e_i = v_1v_i, 3 \le i \le n-1\}$

be the vertices and edges of vertex switching of Cycle C_n.

Case 1. n is even

Consider the following color classes.

$$V_1 = v_{2i} : 1 \le i \le [n/2]$$

$$V_2 = v_{2i+1} : 1 \le i \le [n/2]-1$$

$$V_3 = v_1$$

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Here v_1, v_2 and v_3 are the color classes of $\hat{C}_n$ in which each V_i (i = 1, 2) is a global dominating set of $\hat{C}_n$. Hence an easy verification shows that $gd_\chi(\hat{C}_n) = 2$.

Case 2. n is odd.

$$V_1 = v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$
$$V_2 = v_{2i+1} : 1 \le i \le \lfloor n/2 \rfloor$$
$$V_3 = v_1$$

From the above case 1 and case 2, color classes of $\hat{C}_n$ = 3 we observe that V_1 and V_2 are the global dominating set as well as global dominating χ coloring of $\hat{C}_n$,

thus $gd_\chi(\hat{C}_n) \ge 2$. Hence an easy verification shows that $gd_\chi(\hat{C}_n) = 2$.

Theorem 3.2. For a Double star graph $K_{1,n,n}$, $n \ge 3$, the global dominating - χ - coloring number on Vertex Switching of $K_{1,n,n}$ is,

$$gd_\chi(\hat{K}_{1,n,n}) = 1$$

Proof. By switching the root vertex v, the graph $\hat{K}_{1,n,n}$ is obtained from $K_{1,n,n}$. By the definition of vertex switching, let

$V(\hat{K}_{1,n,n}) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{u_1, u_2, u_3, \ldots, u_n\} \cup \{v\}$ and

$E(\hat{K}_{1,n,n}) = \{e_i = vu_i, 1 \le i \le n\} \cup \{e_i = v_iu_i, 1 \le i \le n\}$

be the vertices and edges of vertex switching of $K_{1,n,n}$.

Consider the following color classes.

$$V_1 = v, v_i : 1 \le i \le n$$
$$V_2 = u_i : 1 \le i \le n$$

Here V_1 is the global dominating set of $\hat{K}_{1,n,n}$ but no V_2 thus $gd_\chi(\hat{K}_{1,n,n}) \ge 1$, which implies that $gd_\chi(\hat{K}_{1,n,n}) \le 1$. Hence $gd_\chi(\hat{K}_{1,n,n}) = 1$.

Theorem 3.3. For a Wheel graph W_n, $n \ge 8$, the global dominating - χ - coloring number of Vertex Switching of W_n is,

$$gd_\chi(\hat{W}_n) = 1$$

Proof. Let us assume that, the graph is obtained by switching the vertex v_0 from W_n. By the definition of vertex switching, let

$V(\hat{W}_n) = \{v_0, v_1, v_2, v_3, \ldots, v_n\}$ and

$E(\hat{W}_n) = \{e_i = v_iv_{i+1}, 2 \le i \le n-1\} \cup \{e_i = v_2v_i, 2 \le i \le n\} \cup \{e_i = v_1v_i, 3 \le i \le n-1\}$

be the vertices and edges of vertex switching of wheel graph.

Case 1. n is even

Consider the following color classes

$$V_1 = v_0, v_1$$
$$V_2 = v_{2i} : 1 \le i \le \lceil n/2 \rceil$$
$$V_3 = v_{2i+1} : 1 \le i \le \lfloor n/2 \rfloor - 1$$

Here v_1, v_2 and v_3 are the color classes of dominating set of $\hat{W}_n$, in which each V_i (i = 2) is a global dominating set of $\hat{W}_n$.

Case 2. n is odd.

$$V_1 = v_0, v_1$$
$$V_2 = v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$
$$V_3 = v_{2i+1} : 1 \le i \le \lfloor n/2 \rfloor$$

From the above case 1 and case 2, color classes of $\hat{W}_n$ = 3 we observe that V_2 is the global dominating set as well as global dominating χ coloring of $\hat{W}_n$,

thus $gd_\chi(\hat{W}_n) \ge 1$. Hence an easy verification shows that $gd_\chi(\hat{W}_n) = 1$

Theorem 3.4. For a Path P_n, $n \ge 6$, the global dominating - χ - coloring number on Vertex Switching of P_n is,

$$gd_\chi(\hat{P}_n) = 1$$

Proof. By the definition of vertex switching of P_n, let

$V(\hat{P}_n) = \{v_1, v_2, v_3, \ldots, v_n\}$ and

$E(\hat{P}_n) = \{e_i = v_1v_i, 3 \le i \le n\} \cup \{e_i = v_iv_{i+1}, 2 \le i \le n-1\}$

Be the vertices and edges of vertex switching of P_n. Here we are switching the vertex v_1. Consider the following color classes.

Case 1. n is even

Consider the following color classes

$$V_1 = v_1$$
$$V_2 = v_{2i} : 1 \le i \le \lceil n/2 \rceil$$
$$V_3 = v_{2i+1} : 1 \le i \le \lfloor n/2 \rfloor - 1$$

Here v_1, v_2 and v_3 are the color classes of $\hat{P}_n$ in which V_i (i = 2) is a global dominating set of $\hat{P}_n$.

Case 2. n is odd

$$V_1 = v_1$$
$$V_2 = v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$
$$V_3 = v_{2i+1} : 1 \le i \le \lfloor n/2 \rfloor$$

From the above case 1 and case 2, color classes of $\hat{P}_n$ =3. we observe that V2 is the global dominating set as well as global dominating χ coloring of $\hat{P}_n$, thus $gd\chi(\hat{P}_n) \ge 1$. Hence an easy verification shows that $gd\chi(\hat{P}_n) = 1$.

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Theorem 3.5. For a Helm graph H_n, n ≥ 6, the global dominating - χ - coloring number on Vertex Switching of H_n is,

(i) $gd_\chi(\widehat{H}_n) = 2$ When switching a root vertex

(ii) $gd_\chi(\widehat{H}_n) = 1$ When switching a pendent vertex

Proof. By the definition of vertex switching,

$V(\widehat{H}_n) = \{v, v_1, v_2, v_3, \ldots, v_n\} \cup \{u_1, u_2, u_3, \ldots, u_n\}$

be the vertices of vertex switching of helm graph.

Case 1. n is even

$$V_1 = v, v_{2i+1} : 0 \le i \le \lfloor n/2 \rfloor - 1$$

$$V_3 = u_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$

$$V_2 = u_{2i+1} : 0 \le i \le \lfloor n/2 \rfloor$$

$$v_{2i} : 1 \le i \le [n/2]$$

Case 2. n is odd

$$V_1 = v, v_{2i+1} : 0 \le i \le \lfloor n/2 \rfloor - 1$$

$$V_3 = u_n, u_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$

$$V_2 = u_{2i+1} : 0 \le i \le \lfloor n/2 \rfloor$$

$$v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$

It is clear that the chromatic color class of $\widehat{H}_n$ is 3. By the definition of gdχ V1, V2 are the global dominating set as well as global dominating χ coloring of $\widehat{H}_n$, thus $gd_\chi(\widehat{H}_n) = 2$.

1. When switching a pendant vertex

Case 1. n is even

Consider the following color classes

$$V_1 = v, u_i : 2 \le i \le n$$

$$V_2 = v_{2i+1} : 0 \le i \le \lceil n/2 \rceil - 1$$

$$V_3 = v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$

$$V_4 = u_1$$

Case 2. n is odd

$$V_1 = v, u_i : 2 \le i \le n$$

$$V_2 = v_{2i+1} : 0 \le i \le \lfloor n/2 \rfloor - 1$$

$$V_3 = v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$

$$V_4 = v_n, V_5 = u_1$$

It is clear that the chromatic color class of $\widehat{H}_n$ is 4 and 5. By the definition of gdχ, V2 is the global dominating set as well as global dominating χ coloring of $\widehat{H}_n$, thus $gd\chi(\widehat{H}_n) = 1$.

Theorem 3.6. For a Path P_n, n ≥ 6, the global dominating - χ - coloring number of degree splitting graph of P_n is,

$$gd_\chi(DsP_n) = 3$$

Proof. By the definition of degree splitting graph,

$V(DsP_n) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{u, w\}$

be the vertices of degree splitting graph of path. Consider the following color class

$$V_1 = v_{2i+1} : 0 \le i \le \lceil n/2 \rceil - 1$$

$$V_2 = v_{2i} : 1 \le i \le \lfloor n/2 \rfloor$$

$$V_3 = u, w$$

Here v1, v2 and v3 are the color classes of Ds(Pn). In which each color classes v1 v2 and v3 satisfies the definition of global dominating set and global dominating χ coloring . Hence Vi (i = 1, 2, 3) is a global dominating χ coloring of Ds(Pn). An easy check shows that gdχDs(Pn)) = 3.

IV. CONCLUSION

In this paper we obtained the global dominating - χ - coloring number for vertex switching of Pn, Cn, K1,n,n, Sn and Hn and also we obtain the global dominating - χ - coloring number degree splitting graph of some graphs. This paper can further be extended by identifying the global dominating - χ - coloring number for some new graph families.

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