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Factorial Labeling Of Some Middle Graphs

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Abstract. In this paper, we launch a new type of labeling said to be factorial labeling and apply to some middle graphs. A factorial labeling of a connected graph is bijection $f : V(G) \to \{1, 2, \dots, p\}$ such that the induced function $g_f : E(G) \to N$ defined as $g_f(uv) = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ then the edge labels are distinct. Graph which admits a factorial labeling is a factorial graph. We discuss this labeling condition satisfies to some middle graphs of path, cycle, star, fan, comb, ladder, sun let graphs and also find the chromatic number of some middle graphs. We introduce the factorial number is the minimum number of edge labels of factorial graphs.

Keywords-Factorial labeling, Factorial graph, Factorial number, Middle graphs, Chromatic number.

I. INTRODUCTION

Graph labeling is one of the fascinating areas of graph theory. Graph labeling was first introduced in the late 1960's. Assign the positive integers to vertices in a graph is known as graph labeling. A dynamic survey on graph labeling is systematic updated by Gallian[6] and it is published by Electronic Journal of Combinatorics. An enormous body of literature is available on different types of graph labeling grown around in the last four decades and more than 1000 research papers have been published. Graph labeling are using in many departments likely communication network, software testing, X-ray crystallography, etc., Basic definitions are being used in Harary[4] and G. Chartrand, P. Zhang[5] . J. Vernold Vivin and M. Vekatachalam[7] have proved on b-chromatic number of sun let graph and wheel graph families. Ali Ahmad, Misbah Arshad and Gabriela Iˇzar´ıkov´a[1] have proved irregular labelings of helm and sun graphs. Ladder graph investigated in A. K. Handa, Aloysius Godinho and T. Singh[2]. Middle graph has discussed in Devsi Bantva[3] and Weigen Yan[10]. Fan graph discussed in S.Roy[8]. Star graph investigated in S. David Laurence, K. M. Kathiresan[9]. We prove that the middle graph of some graphs like path, cycle, star, fan, comb, ladder, sun let graphs are admits the factorial graphs and also find the factorial number, chromatic number.

II. PRELIMINARIES

A. Definition[4]

A path if all the points (and thus necessarily all the lines) are distinct and by P_n a path with n points.

B. Definition[4]

If the walk is closed, then it is a cycle provided its n points are distinct and $n \geq 3$. We denote by C_n the graph consisting of a cycle with n points.

C. Definition[9]

A star graph with n vertices is a tree with one vertex having degree $n-1$ and other $n-1$ vertices having degree 1. A star graph is a complete bipartite graph with $n+1$ vertices. It is denoted by $K_{1,n}$.

D. Definition[8]

A Fan graph $F_n (n \geq 2)$ is defined as the graph $K_1 + P_n$, where K_1 is the singleton graph and P_n is the Path on n vertices.

E. Definition

A graph obtained by attaching a single pendant edge to each vertex of a path $P_n = v_1 v_2 v_3 \dots v_n$ is called a comb.

F. Definition[2]

The ladder $L_n (n \geq 2)$ is the product graph $P_2 \times P_n$ which contains $2n$ vertices and $3n-2$ edges.

G. Definition[7]

The n-sun let graph on $2n$ vertices is obtained by attaching n pendant edges to the cycle C_n and is denoted by S_n.

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H. Definition[3]

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The middle graph of G, denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y of $M(G)$ are adjacent in $M(G)$ in case one of the following holds : (i) x, y are in $E(G)$ and x, y are adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

I. Definition[5]

A coloring of a graph is an assigned color to its points so that two adjacent points have different colors and also non-adjacent vertices have either same color or different colors. The chromatic number $\chi(G)$ is defined as the minimum k for which G has an k-coloring.

II Factorial Labeling To Some Middle Graphs With Factorial Number And Chromatic Number

a. Definition

A factorial labeling of a connected graph G is a bijection $f : V(G) \rightarrow \{1, 2, \dots, p\}$ such that the induced function $g_f : E(G) \rightarrow N$ defined as $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$, then the edge labels are distinct. Any graph which admits a factorial labeling is called a factorial graph.

b. Definition

The minimum number of the edge label of the factorial graph is said to be a factorial number. It is denoted by $\omega(n)$.

c. Theorem

The middle graph $M(P_n)$ of a path graph $P_n, (n \geq 2)$ acknowledges a factorial graph.

Proof. Let $P_n, (n \geq 2)$ be the path graph. The length of a path graph is $n-1$ with vertices $u_1, u_2, \dots, u_n$. Let $e_i = u_i u_{i+1}$ for $1 \leq i \leq n-1$ be the edges of path $P_n, (n \geq 2)$. Let G be the middle graph of path P_n and it is denoted by $M(P_n)$. Here, $V(G) = V(M(P_n)) = \{u_i, v_j / 1 \leq i \leq n, 1 \leq j \leq n-1\}$ and $E(G) = E(M(P_n)) = \{v_j v_{j+1} / 1 \leq j \leq n-2\} \cup \{u_i v_j, v_j u_{i+1} / 1 \leq i, j \leq n-1\}$. Here we note that $|V(M(P_n))| = 2n-1$ and $|E(M(P_n))| = 3n-4$. The maximum degree is $\Delta = 4$ and the minimum degree is $\delta = 1$ of the middle graph $M(P_n)$. The mapping is defined $f : V(G) \rightarrow \{1, 2, \dots, 2n-1\}$ as follows. $f(u_i) = i$ for $1 \leq i \leq n$; $f(v_j) = n + j$ for $1 \leq j \leq n-1$. Then every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and for any edge $f(e_i) \neq f(e_j), i \neq j$. Thus the function f is a factorial labeling for a graph $M(P_n)$. That is, the middle graph $M(P_n)$ of a path graph P_n acknowledges a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M(P_n)\} = n+2$. The chromatic number $\chi(M(P_n))$ of a graph $M(P_n)$ is the minimum k. i.e. $\chi(M(P_n)) = 3$.

d. Example

The factorial labeling of middle graph $M(P_n)$ of a path graph P_n is shown figure 1.

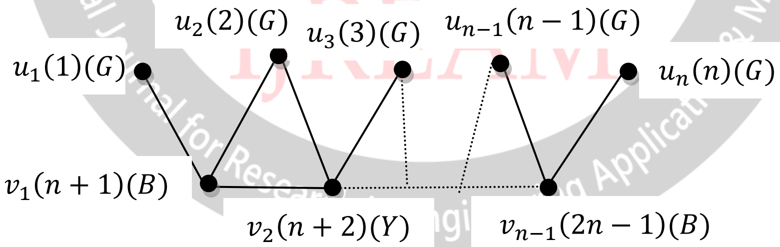

Figure 1. Factorial labeling for a graph $M(P_n)$ and $\omega(n) = n+2$ and also $\chi(M(P_n)) = 3$.

e. Theorem

The middle graph $M(C_n)$ of a cycle graph $C_n, (n \geq 3)$ admits a factorial graph.

Proof. Let $C_n, (n \geq 3)$ be the cycle of n vertices and the edges of cycle C_n is $e_i = u_i u_{i+1}$ for $1 \leq i \leq n-1$, $e_n = v_n v_1$. Let the graph G be the middle graph of cycle graph C_n is $M(C_n)$. Here we note that, $V(G) = V(M(C_n)) = \{u_i, v_j / 1 \leq i, j \leq n\}$ and $E(G) = E(M(C_n)) = \{u_i v_j / 1 \leq i, j \leq n, v_j v_{j+1}, v_j u_{i+1} / 1 \leq i, j \leq n-1, v_n v_1, v_n u_1\}$. Here, $|V(M(C_n))| = 2n$ and $|E(M(C_n))| = 3n$. The maximum degree is $\Delta = 4$ and the minimum degree is $\delta = 2$ of the middle graph $M(C_n)$. Define a labeling $f : V(M(C_n)) \rightarrow \{1, 2, \dots, 2n\}$ as follows. $f(u_i) = 2i - 1$ for $1 \leq i \leq n$; $f(v_j) = 2j$ for $1 \leq j \leq n$. Then for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$. i.e. The receive edge values are distinct. Then f admits factorial labeling. Hence $M(C_n)$ is a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M(C_n)\} = 3$. The

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chromatic number $\chi(M(C_n))$ of a graph $M(C_n)$ is the minimum k. i.e. $\chi\left(M(C_n)\right) = 3$.

f. Example

The factorial labeling of the middle graph $M(C_n)$ of a cycle graph C_n is shown figure 2.

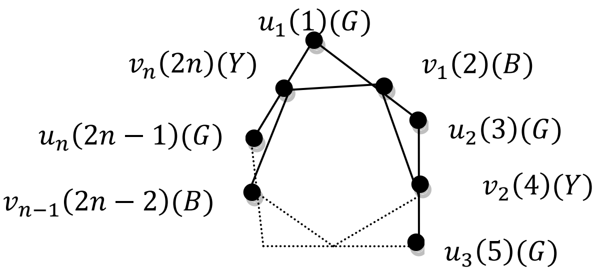

Figure 2. Factorial labeling for a graph $M(C_n)$ and $\omega(n) = 3$ and also $\chi\left(M(C_n)\right) = 3$.

g. Theorem

The middle graph $M(K_{1,n})$ of a star graph $K_{1,n}$ is a factorial graph.

Proof. Let $V\left(K_{1,n}\right) = \{u_i \,/\, 1 \le i \le n+1\}$ be the vertices of $K_{1,n}$. i.e. $K_{1,n}$ has $n+1$ vertices and n edges. Let G be the middle graph of star graph is denoted by $M\left(K_{1,n}\right)$. Here, $V\left(M\left(K_{1,n}\right)\right) = \{u_i \,/\, 1 \le i \le n+1\} \cup \{v_j \,/\, 1 \le j \le n\}$ and $E\left(M\left(K_{1,n}\right)\right) = \{u_1 v_j \,/\, 1 \le j \le n\} \cup \{u_i v_{j-1} \,/\, 2 \le i,j \le n+1\} \cup \{v_n v_1\} \cup \{v_j v_{j+k} \,/\, 1 \le j \le n-k\,; k = 1,2,\dots,n-1\}$. Here we note that, $\left|V\left(M\left(K_{1,n}\right)\right)\right| = 2n+1$ and $\left|E\left(M\left(K_{1,n}\right)\right)\right| = \frac{n(n+1)}{2} + n = \frac{n^2+3n}{2}$. The maximum degree of $M\left(K_{1,n}\right)$ is $\Delta = n+1$ and the minimum degree of $M\left(K_{1,n}\right)$ is $\delta = 1$. Define a mapping, $f : V\left(M\left(K_{1,n}\right)\right) \rightarrow \{1,2,\dots,2n+1\}$ as follows. $f(u_i) = i$ for $1 \le i \le n+1$; $f\left(v_j\right) = n+j+1$ for $1 \le j \le n$. Then for every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and that the edge values are distinct. i.e. $f(e_i) \ne f(e_j)$. Thus f is factorial labeling. Therefore $M(K_{1,n})$ is a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M\left(K_{1,n}\right)\} = n+3$. The chromatic number $\chi(M(K_{1,n}))$ of a graph $M(K_{1,n})$ is the minimum k. i.e. $\chi\left(M\left(K_{1,n}\right)\right) = n+1$.

h. Example

The factorial labeling of the middle graph $M(K_{1,n})$ of a star graph $K_{1,n}$ is shown figure 3.

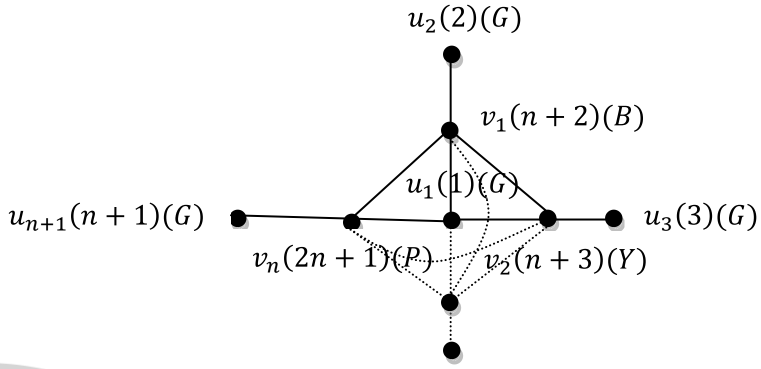

Figure 3. Factorial labeling to a graph $M(K_{1,n})$ and $\omega(n) = n+3$ and also $\chi\left(M\left(K_{1,n}\right)\right) = n+1$.

i. Theorem

The middle graph $M(F_n), n \ge 2$ of a fan graph F_n produces a factorial graph.

Proof. Let the fan graph F_n, $n \ge 2$ with $n+1$ vertices say $u_1, u_2, \dots, u_{n+1}$ and $2n-1$ edges. Let G be the middle graph of fan graph $M(F_n), n \ge 2$ with $3n$ vertices and $\frac{n^2+13n-12}{2}$ edges. i.e. $V\left(M(F_n)\right) = \{u_i \,/\, 1 \le i \le n+1\} \cup \{v_j \,/\, 1 \le j \le 2n-1\}$. i.e. $\left|V\left(M(F_n)\right)\right| = 3n$ and $\left|E\left(M(F_n)\right)\right| = \frac{n^2+13n-12}{2}$. The maximum and minimum degree of $M(F_n)$ are denoted as $\Delta = n+3$ and $\delta = 2$ respectively. The mapping, $f : V\left(M(F_n)\right) \rightarrow \{1,2,\dots,3n\}$ as follows. $f(u_i) = i$ for $1 \le i \le n+1$; $f\left(v_j\right) = n+j+1$ for $1 \le j \le 2n-1$. For every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and that the edge receive values are distinct. So, f produces a factorial labeling. Therefore $M(F_n), n \ge 2$ is a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M(F_n)\} = n+3$. The chromatic number $\chi(M(F_n))$ of a graph $M(F_n)$ is the minimum k. i.e. $\chi\left(M(F_n)\right) = n+1,\ n \ge 3$.

j. Example

The factorial labeling of the middle graph $M(F_n)$ of a fan graph F_n is shown figure 4.

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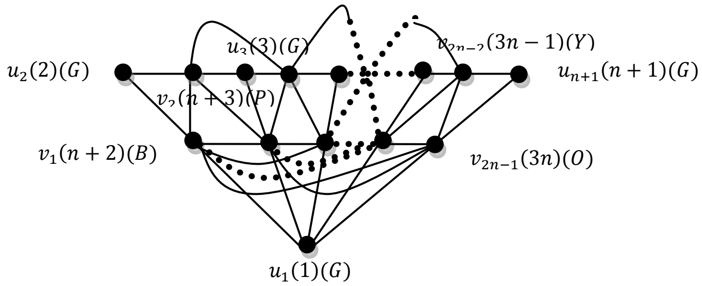

Figure 4. Factorial labeling to a graph $\mathbf{M(F_n)}$ and $\boldsymbol{\omega(n) = n+3}$ and also $\boldsymbol{\chi(M(F_n)) = n+1, n \geq 3}$.

k. Theorem

The middle graph $M(P_n \odot K_1), n \geq 2$ of the comb graph $P_n \odot K_1, n \geq 2$ is the factorial graph.

Proof. Let the comb graph obtained by attaching a single pendant edge to each vertex of a path graph. The comb graph $P_n \odot K_1$, $n \geq 2$ has $2n$ vertices and $2n-1$ edges. Let G be the middle graph of comb graph $M(P_n \odot K_1), n \geq 2$ with $4n-1$ vertices and $7n-6$, $n \geq 2$ edges. i.e. $V(M(P_n \odot K_1)) = \{u_i \,/\, 1 \leq i \leq 2n\} \cup \{v_j \,/\, 1 \leq j \leq 2n-1\}$ and $E(M(P_n \odot K_1)) = \{u_i v_j,\ u_i v_{2n-j} \,/\, 1 \leq i,j \leq n\} \cup \{v_j u_{i+1} \,/\, 1 \leq i,j \leq 2n-1\} \cup \{v_j v_{j+1},\ v_j v_{2n-j},\ v_j v_{2n-1-j} \,/\, 1 \leq j \leq n-1\}$. Here, $|V(M(P_n \odot K_1))| = 4n-1$ and $|E(M(P_n \odot K_1))| = 7n-6, n \geq 2$. The maximum and minimum degree of $M(P_n \odot K_1)$ are denoted as $\Delta = 6$ and $\delta = 1$ respectively. The labeling, $f : V(M(P_n \odot K_1)) \to \{1, 2, \ldots, 4n-1\}$ as follows. $f(u_i) = i$ for $1 \leq i \leq 2n$; $f(v_j) = 2n + j$ for $1 \leq j \leq 2n-1$. For every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ and the edge labels are distinct. Then, f admits a factorial labeling. Therefore $M(P_n \odot K_1), n \geq 2$ is a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M(P_n \odot K_1)\} = 2n+2$. The chromatic number $\chi(M(P_n \odot K_1))$ of a graph $M(P_n \odot K_1)$ is the minimum k. i.e. $\chi(M(P_n \odot K_1)) = 4$.

l. Example

The factorial labeling to middle graph $M(P_n \odot K_1)$ of the comb graph $P_n \odot K_1$ is shown figure 5.

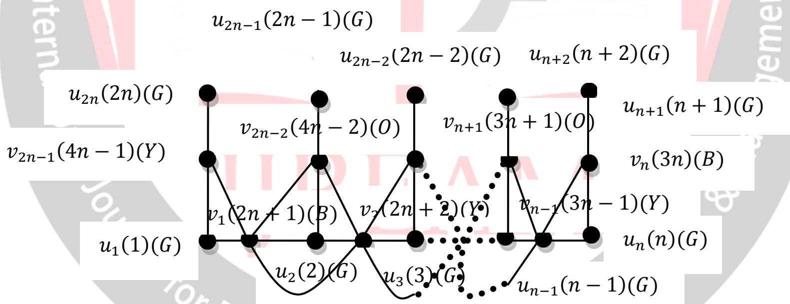

Figure 5. Factorial labeling to a graph $\boldsymbol{M(P_n \odot K_1)}$ and $\boldsymbol{\omega(n) = 2n+2}$ and also $\boldsymbol{\chi(M(P_n \odot K_1)) = 4}$.

m. Theorem

The middle graph $M(L_n), n \geq 2$ of the ladder graph L_n produces a factorial graph.

Proof. Let the ladder graph L_n, $n \geq 2$ is the product graph $P_2 \, X \, P_n$ with $2n$ vertices and $3n-2$ edges. Let G be the middle graph of ladder graph $M(L_n), n \geq 2$ with $5n-2$ vertices and $12(n-1)$ edges. i.e. $V(M(L_n)) = \{u_i \,/\, 1 \leq i \leq 2n\} \cup \{v_j \,/\, 1 \leq j \leq 3n-2\}$ and $E(M(L_n)) = \{u_i v_j \,/\, 1 \leq i,j \leq n+3\} \cup \{v_j u_{i+1} \,/\, 1 \leq i,j \leq n+1\} \cup \{v_j v_{j+1} \,/\, 1 \leq j \leq 3n-3\} \cup \{u_i v_{j+1} \,/\, n+2 \leq i,j \leq n+4\} \cup \{u_i v_{j+2} \,/\, n+3 \leq i,j \leq n+5\} \cup \{u_i v_{j+3} \,/\, n+4 \leq i,j \leq 2n\} \cup \{u_i v_{j+4} \,/\, 2n-1 \leq i,j \leq 2n\} \cup \{u_i v_{3n-2j}, v_j v_{3n-2j}, v_j v_{3n-2j-2} \,/\, 1 \leq i,j \leq n-1\} \cup \{v_j v_{j+2} \,/\, n+1 \leq j \leq n+7\}$. i.e. $|V(M(L_n))| = 5n-2$ and $|E(M(L_n))| = 12(n-1)$. The maximum and minimum degree of $M(L_n)$ are denoted as $\Delta = 6$ and $\delta = 2$ respectively. The function, $f : V(M(L_n)) \to \{1, 2, \ldots, 5n-2\}$ as follows. $f(u_i) = i$ for $1 \leq i \leq 2n$; $f(v_j) = 2n + j$ for $1 \leq j \leq 3n-2$. Then every edge $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$ receives the distinct values. So, that f produces a factorial labeling. Therefore $M(L_n), n \geq 2$ is a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M(L_n)\} = 2n+2$. The chromatic number $\chi(M(L_n))$ of a graph $M(L_n)$ is the minimum k. i.e. $\chi(M(L_n)) = 4$.

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n. Example

The factorial labeling of the middle graph $M(L_n)$ of the ladder graph L_n is shown figure 6.

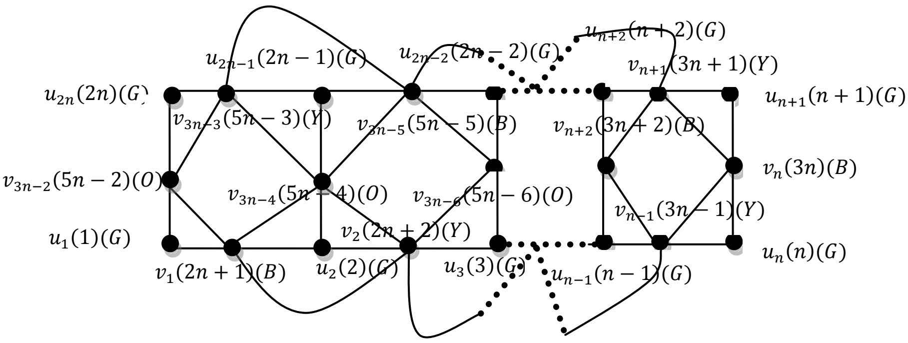

Figure 6. Factorial labeling to a graph $M(L_n)$ and $\omega(n) = 2n+2$ and also $\chi(M(L_n)) = 4$.

o. Theorem

The middle graph $M(S_n), n \geq 3$ of the sun let graph $S_n, n \geq 3$ acknowledges the factorial graph.

Proof. Let the sun let graph S_n, $n \geq 3$ having the both vertices and edges are $2n$. Let G be the middle graph of sun let graph $M(S_n), n \geq 3$ with $4n$ vertices and $7n$ edges. The vertex and edge sets are $V(M(S_n)) = \{u_i,\ v_j\ /\ 1 \leq i,\ j \leq 2n\}$ and $E(M(S_n)) = \{u_i v_j\ /\ 1 \leq i,j \leq 2n\} \cup \{v_j u_{i+1}\ /\ 1 \leq i,j \leq n-1\} \cup \{v_j v_{j+1}\ /\ 1 \leq j \leq n-1\} \cup \{u_i v_{n+j+1}, v_j v_{n+j+1}\ /\ 1 \leq i,j \leq n-1\} \cup \{v_j v_{n+j+2}\ /\ 1 \leq j \leq n-2\} \cup \{v_j v_{j+2}\ /\ j = n-1, n\} \cup \{v_n u_1\} \cup \{v_n v_1\} \cup \{v_n v_{n+1}\} \cup \{u_n v_{n+1}\}$. i.e. $|V(M(S_n))| = 4n$ and $|E(M(S_n))| = 7n$. The maximum and minimum degree of $M(S_n)$ are denoted as $\Delta = 6$ and $\delta = 1$ respectively. The labeling function is defined as, $f : V(M(S_n)) \rightarrow \{1, 2, \ldots, 4n\}$. $f(u_i) = i$ for $1 \leq i \leq 2n$; $f(v_j) = 2n + j$ for $1 \leq j \leq 2n$. All edge values are distinct and then based on the following condition $e = uv = \frac{[f(u)+f(v)]!}{[f(u)]![f(v)]!}$. Hence, f acknowledges a factorial labeling and it is also a factorial graph. The minimum number of the edge label of the factorial graph. i.e. Factorial number $\omega(n) = \min\{edge\ label\ of\ M(S_n)\} = 2n+2$. The chromatic number $\chi(M(S_n))$ of a graph $M(S_n)$ is the minimum k. i.e. $\chi(M(S_n)) = 4$.

p. Example

The factorial labeling of the middle graph $M(S_n)$ of the sun let graph S_n is shown figure 7.

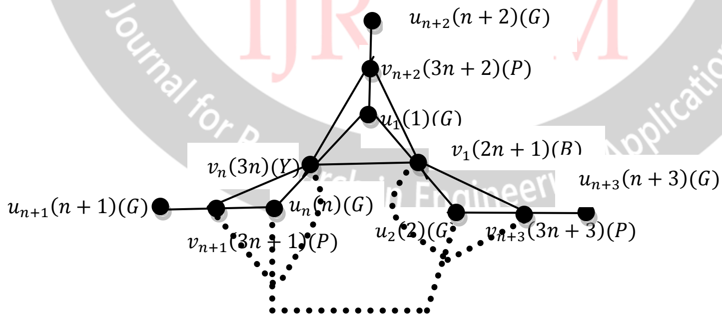

Figure 7. Factorial labeling to a graph $M(S_n)$ and $\omega(n) = 2n+2$ and also $\chi(M(S_n)) = 4$.

III. Acknowledgment

The author is thankful to the referee for the valuable comments which led to the substantial improvement in the paper.

IV. Conclusion

In this paper, we launched the new labeling said factorial labeling is defined and the graph which satisfied the factorial labeling conditions called the factorial graph. Factorial graph of middle graph of some special classes of graphs namely path, cycle, star, fan, comb, ladder and sun let graphs are discussed and also found the factorial number, chromatic number of middle graphs of some special classes of graphs likely path, cycle, star, fan, comb, ladder and sun let graphs of factorial graphs.

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