

Thermal Diffusion and Joule Heating Effects on MHD Radiating Fluid Embedded in Porous Medium

¹K. Sidda Reddy, ²P. Chandra Reddy, ³G.S.S. Raju

¹Department of Mathematics, JNTUA College of Engineering, Pulivendula, A.P., India.

²Department of Mathematics, Annamacharya Institute of Technology and Sciences (Autonomous),
Rajampet-516126, A.P., India.

Corresponding author Email: koppalasiddu@gmail.com , chandramsc01@gmail.com

Abstract - This paper reveals a numerical study on joule heating effect on MHD Newtonian fluid embedded in porous medium under the presence of thermal diffusion, variable temperature and also variable concentration. The governing equations pertinent to the fluid flow are solved by applying finite difference schemes. The variations in velocity, temperature and concentration are exhibited and discussed with the help of graphs. Also the numerical values for local skin friction and Nusselt number are recorded and analyzed. Increasing values of Soret number results in rising of the concentration, but it falls down under the influence of Schmidt number. Skin friction decreases for increasing values of Soret number.

Keywords: *MHD, Thermal radiation, Porous medium, Soret effect, Joule Heating, Viscous dissipation, Variable temperature.*

I. INTRODUCTION

In many branches of science, technology and also industries, the phenomenon of magneto hydrodynamic flow with heat transfer has been a great interested subject with its possible applications. Convection heat transfer from vertical surfaces embedded in porous media have many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation and enriched oil recovery, ground water purification and underground energy transportation. Barik et al. [1] considered thermal radiation effect on an unsteady MHD flow past inclined porous heated plate in the presence of chemical reaction and viscous-dissipation. Pattnaik et al. [2] reported on radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature. Rajput and Kumar [3] examined radiation and chemical effects on MHD flow through porous media past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation. Agarwalla and Ahmed [4] analyzed MHD mass transfer flow past an inclined plate with variable temperature and plate velocity embedded in a porous medium. Bhukta et al. [5] found numerical simulation of heat transfer effect on Oldroyd 8-constant fluid with wire coating analysis. Nayak [6] considered and studied chemical reaction effect on MHD viscoelastic fluid over a stretching sheet through porous medium. Kumaresan and Vijaya Kumar [7] presented an exact solution on unsteady MHD viscoelastic fluid flow

past an infinite vertical plate in the presence of thermal radiation. Ravikumar and Jayarami Reddy [8] discussed viscous dissipation and radiation effects on MHD convective flow past a vertical porous plate with injection. Chand and Thakur [9] examined the effects of rotation, radiation and Hall Current on MHD flow of a viscoelastic fluid past an infinite vertical porous plate through porous medium with heat absorption, chemical reaction and variable suction. Hossain and Alam [10] established the effects of thermal diffusion on viscoelastic fluid flow through a vertical at plate. Choudhury and Dhar [11] discussed the effects of MHD visco-elastic fluid flow past a moving plate with double diffusive convection in presence of heat generation. Chandra Reddy et al. [12, 13, 14] analyzed Soret and Dufour effects on MHD free convection flow of Rivlin-Ericksen fluid as well as visco elastic fluid past a semi infinite vertical plate flow past a moving vertical plate in the presence of radiation and thermal diffusion. Rout and Pattanayak [15] examined chemical reaction and radiation effects on MHD flow past an exponentially accelerated vertical plate in presence of heat source with variable temperature embedded in a porous medium. Zaib and Shafie [17] presented thermal diffusion and diffusion thermo effects on unsteady MHD free convection flow over a stretching surface considering Joule heating and viscous dissipation with thermal stratification, chemical reaction and Hall current. Muhammad et al. [18] examined viscous dissipation and Joule heating effects in MHD 3D flow with heat and mass fluxes.

Keeping in view the above studies we made an attempt to discuss thermal diffusion and Joule heating effects on MHD free convective heat absorbing viscous dissipative Newtonian fluid flow with variable temperature and concentration. We have extended the study of Kaprawi [16] by considering some more physical parameters. The novelty of this work is the consideration of Joule heating, Soret effect, porous medium, thermal radiation and Eckert number.

II. FORMULATION OF THE PROBLEM

We have considered MHD free convective heat absorbing viscous dissipative Newtonian fluid with variable temperature and concentration under the influence of

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T(T^* - T_\infty) + g\beta_C(C^* - C_\infty) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\nu}{k} u^* \quad (1)$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = k_T \frac{\partial^2 T^*}{\partial y^{*2}} - Q^*(T^* - T_\infty) - \frac{\partial q_r^*}{\partial y^*} + \sigma B_0^2 u^{*2} + \mu \left(\frac{\partial u^*}{\partial y^*}\right)^2 \quad (2)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} + D_1 \frac{\partial^2 T^*}{\partial y^{*2}} \quad (3)$$

Corresponding initial and boundary conditions

$$\left. \begin{aligned} u^* = 0, T^* = T_\infty, C^* = C_\infty \quad \text{for all } y^*, t^* \leq 0 \\ t^* > 0: u^* = U_0 a^* t^*, \quad T^* = T_\infty + \left(\frac{T_s^* - T_\infty}{1 + At^*}\right), \quad C^* = C_\infty + \left(\frac{C_s^* - C_\infty}{1 + At^*}\right) \quad \text{at } y^* = 0 \\ u^* = 0, T^* = T_\infty, C^* = C_\infty \quad \text{as } y^* \rightarrow \infty \end{aligned} \right\} \quad (4)$$

Where $A = \frac{U_0^2}{\nu}$

The non-dimensional quantities are as follows:

$$u = \frac{u^*}{U_0}, \quad t = \frac{t^* U_0^2}{\nu}, \quad y = \frac{y^* U_0}{\nu}, \quad T = \frac{T^* - T_\infty}{T_s^* - T_\infty}, \quad C = \frac{C^* - C_\infty}{C_s^* - C_\infty},$$

$$a = \frac{a^* \nu}{U_0^2}, \quad \frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty)I^*, \quad \text{Pr} = \frac{\rho \nu C_p}{k_T}, \quad Q = \frac{Q^* \nu^2}{k_T U_0^2}, \quad K_p = \frac{k U_0^2}{\nu^2},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \quad G_r = \frac{\nu g \beta_T (T_s^* - T_\infty)}{U_0^3}, \quad G_m = \frac{\nu g \beta_C (C_s^* - C_\infty)}{U_0^3}, \quad S_c = \frac{\nu}{D},$$

$$E = \frac{U_0^2}{\rho C_p (T_s^* - T_\infty)}, \quad N_r = \frac{4\nu I^*}{\rho C_p U_0^2}, \quad S_0 = \frac{D_1 (T_s^* - T_\infty)}{\nu (C_s^* - C_\infty)}$$

When the above non-dimensional parameters are applied to the equations (1), (2) and (3) they decrease to the following forms

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r T + G_m C - M u - \frac{1}{K_p} u \quad (5)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\text{Pr}} Q T - N_r T + M E u^2 + \left(\frac{\partial u}{\partial y}\right)^2 \quad (6)$$

thermal diffusion and Joule heating effects. A magnetic field of consistent strength is applied vertical to the plate. Let x^* -axis is taken along with the plate in the vertically upward direction and the y^* -axis is taken perpendicular to the plate. At time $t^* \leq 0$, the plate is continued at the temperature higher than ambient temperature T_∞ and the fluid concentration C_∞ . At time $t^* > 0$, the plate is linearly accelerated with rising time in its own plane and the temperature reduces with temperature $T = 1 / (1 + At^*)$. Similarly the species concentration reduces with time t^* . Based on the above considerations the unsteady flow is governed by the following set of equations ("Rout and Pattanayak [15], Kaprawi [16]):

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 T}{\partial y^2} \quad (7)$$

The corresponding initial and boundary conditions are

$$\left. \begin{aligned} u = 0, \quad T = 0, \quad C = 0 \quad \text{for all } y, t \leq 0 \\ t > 0: \quad u = at, \quad T = \frac{1}{1+t}, \quad C = \frac{1}{1+t} \quad \text{at } y = 0 \\ u = 0, \quad T = 0, \quad C = 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (8)$$

III. SOLUTION OF THE PROBLEM

The linear partial differential equations (5)-(7) with the initial and boundary conditions (8) are to be solved. In general the exact solution is impossible for this type of set of equations and so we have solved the above equations by introducing finite-difference method. The comparable finite difference schemes pertaining to the equations (5)-(7) are as follows:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = G_r T_{i,j} + G_m C_{i,j} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} - M u_{i,j} - \frac{1}{K_p} u_{i,j} \quad (9)$$

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = \frac{1}{Pr} \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\Delta y)^2} - \frac{Q}{Pr} T_{i,j} - N_r T_{i,j} + M E (u_{i,j})^2 + E \left(\frac{u_{i+1,j} - u_{i,j}}{(\Delta y)} \right)^2 \quad (10)$$

$$\frac{C_{i,j+1} - C_{i,j}}{\Delta t} = \frac{1}{Sc} \frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j}}{(\Delta y)^2} + S_0 \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\Delta y)^2} \quad (11)$$

Here, the suffix i corresponds to y and j to time. The mesh system is divided by taking $\Delta y = 0.1$. From the initial condition in (8), we have the following equivalent:

$$u(i, 0) = 0, \quad T(i, 0) = 0, \quad C(i, 0) = 0 \quad \text{for all } i \quad (12)$$

The boundary conditions from (8) are communicated in finite-difference form as follows

$$\left. \begin{aligned} u(0, j) = at, \quad T(0, j) = \frac{1}{1+t}, \quad C(0, j) = \frac{1}{1+t} \quad \text{for all } j \\ u(i_{\max}, j) = 0, \quad T(i_{\max}, j) = 0, \quad C(i_{\max}, j) = 0 \quad \text{for all } j \end{aligned} \right\} \quad (13)$$

The velocity at the end of time step viz, $u(i, j+1)$ ($i=1, 200$) is calculated from (9) in terms of temperature, velocity and concentration at points on the earlier time-step. After that $C(i, j+1)$ is calculated from (10) and then $C(i, j+1)$ is computed from (11). The method is repeated until $t = 0.5$ (i.e. $j = 500$). During computation Δt was selected as 0.001.

$$Sh = - \left(\frac{dC}{dy} \right)_{y=0}$$

IV. RESULTS AND DISCUSSION

To draw out the details of physics of the flow field, we have examined the influence of magnetic parameter (M), Prandtl number (Pr), porosity parameter (K_p), heat absorption (Q), radiation parameter (N_r), Schmidt number (Sc), Soret number (S_0), Eckert number (E) on the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number. Figures 1-5 illustrate the changes of velocity under the impact of different parameters related to the fluid flow. The influence of magnetic parameter on velocity can be seen in figure 1 and noticed that the velocity decreases with the increasing values of magnetic parameter. The transverse magnetic field normal to the flow acts in the opposite direction of the flow which has the effect to slow the motion of the fluid. The effect of porosity parameter on velocity is displayed in figure 2 which shows that for rising values of porosity parameter, the velocity also increases. Figure 3 shows the variation of the velocity on the Soret

Skin-friction:

The skin-friction in non-dimensional form is

$$\tau = - \left(\frac{du}{dy} \right)_{y=0}, \quad \text{where } \tau = \frac{\tau^1}{\rho U_0^2}$$

Rate of heat transfer

The dimensionless rate of heat transfer in terms of Nusselt number is

$$Nu = - \left(\frac{dT}{dy} \right)_{y=0}$$

Rate of mass transfer

The dimensionless rate of mass transfer in terms of Sherwood number is

number. It is evident that the velocity boundary layer increases with an increase in Soret number. The Schmidt number embodies the ratio of the momentum diffusivity to the species (mass) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass-transfer boundary layer. Figure 4 presents the velocity boundary-layer variations with the influence of Schmidt number. It is noticed that the velocity decreases with the increasing values of the Schmidt number. Figure 5 displays the effect of radiation parameter on velocity. It is noticed that the velocity decreases with the raising values of radiation parameter.

Figures 6-9 demonstrate the variations of the fluid temperature under the influence of various parameters. The influence of Prandtl number on temperature is presented in figure 6. It is clear that the surface temperature decreases with an increase in Prandtl number. This happens because reduced fluid velocity would mean that heat is not convected readily and hence surface temperature reduces. The influence of heat absorption on temperature is exhibited in figure 7. It reveals that the temperature decreases under the influence of heat absorption parameter. Figure 8 shows the influence of radiation parameter on temperature. It presents that the temperature reduces with raising values of radiation parameter. The effect of Eckert number on temperature is displayed in figure 9. The temperature increases with the increasing values of Eckert number. Figures 10 and 11 indicate the variations of the fluid concentration under the effects of different types of parameters. Figure 10 displays the effects of Schmidt number on the concentration. It is noticed that as the Schmidt number increases, there is a decreasing trend in the concentration field. Figure 11 depicts the variation of the concentration with the effect of Soret number. It shows that the concentration boundary layer thickness increases with an increase in Soret number.

Figures 12-15 demonstrate the variations in Nusselt number (Nu) and skin friction (τ) under the effects of different parameters. Figure 12 demonstrates the effect of porosity parameter on skin friction. It shows that the skin friction reduces with raising values of porosity parameter. Figure 13 show the variations of magnetic parameter on skin friction. The skin friction increases with an increase in magnetic parameter. Figure 14 shows the effect of radiation parameter on Nusselt number. It is noticed that Nusselt number increases with an increase in radiation parameter. Figure 15 shows the effect of heat absorption parameter on Nusselt number. It is noted that Nusselt number increases with the increasing values of heat absorption parameter.

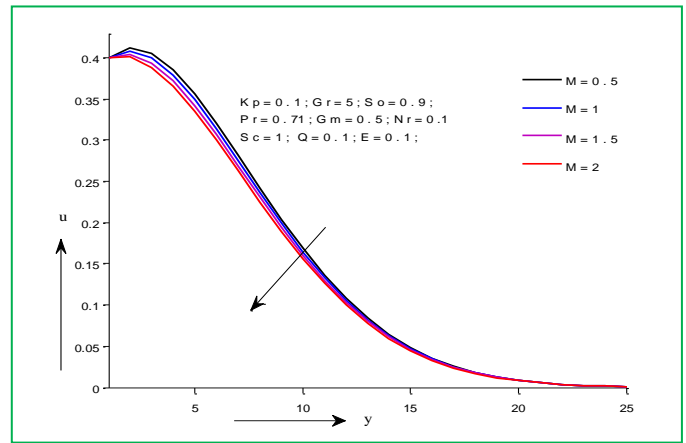


Figure 1: Effect of magnetic parameter (M) on velocity

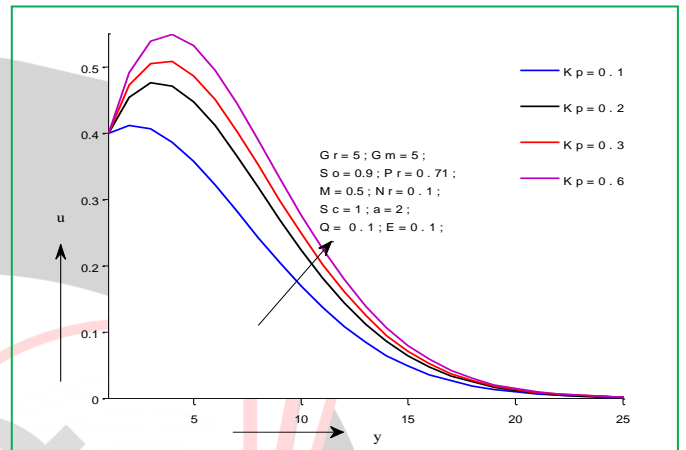


Figure 2: Effect of porosity parameter (Kp) on velocity

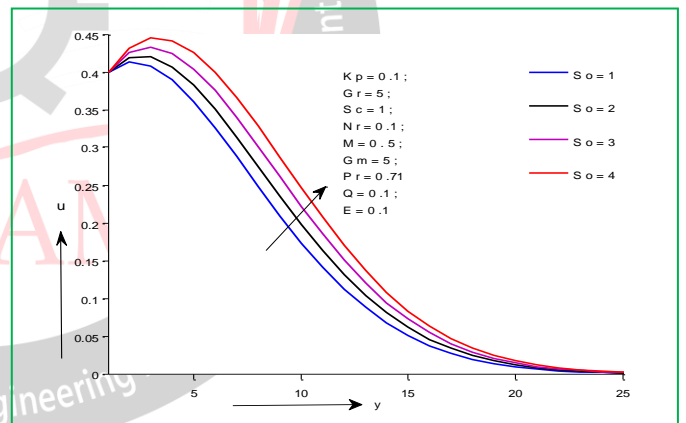


Figure 3: Effect of Soret number (S_0) on velocity

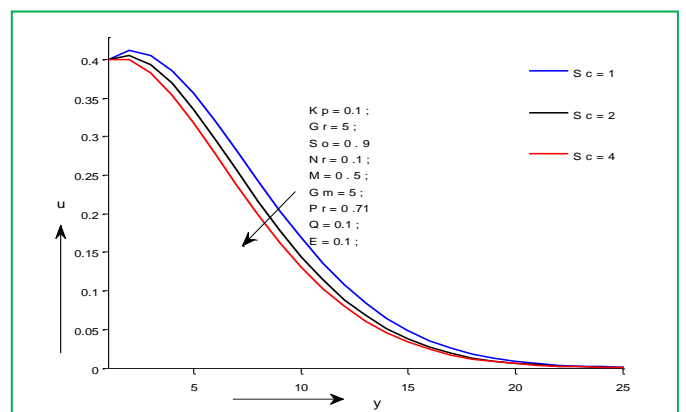


Figure 4: Effect of Schmidt number (Sc) on velocity

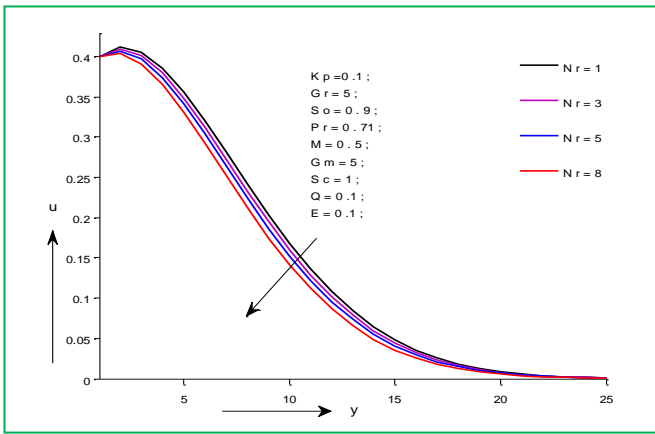


Figure 5: Effect of Radiation parameter (Nr) on velocity

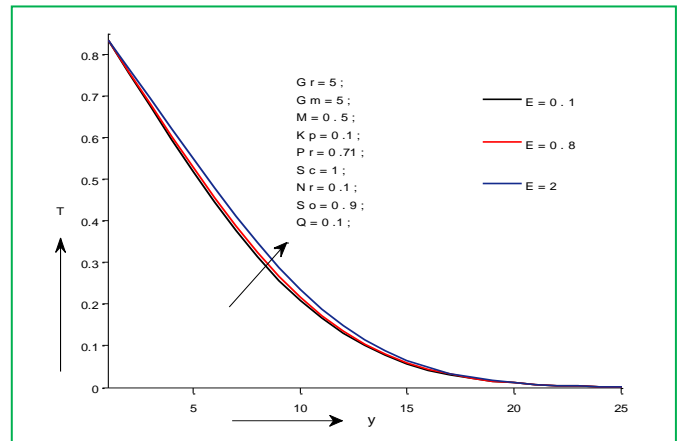


Figure 9: Effect of Eckert number (E) on temperature

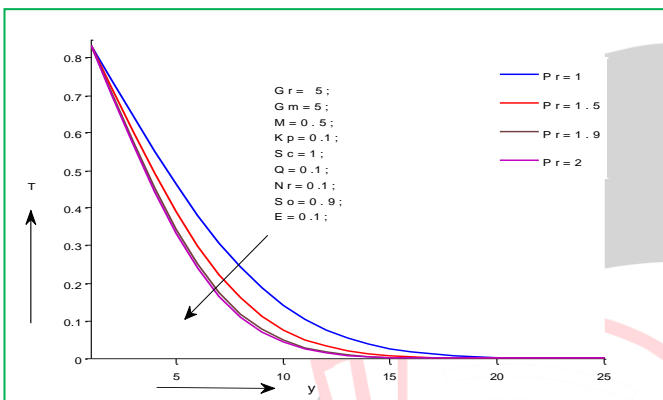


Figure 6: Effect of Prandtl number (Pr) on temperature

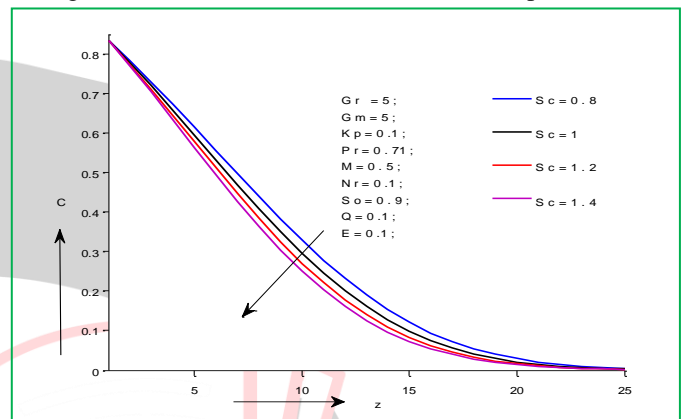


Figure 10: Effect of Schmidt number (Sc) on concentration

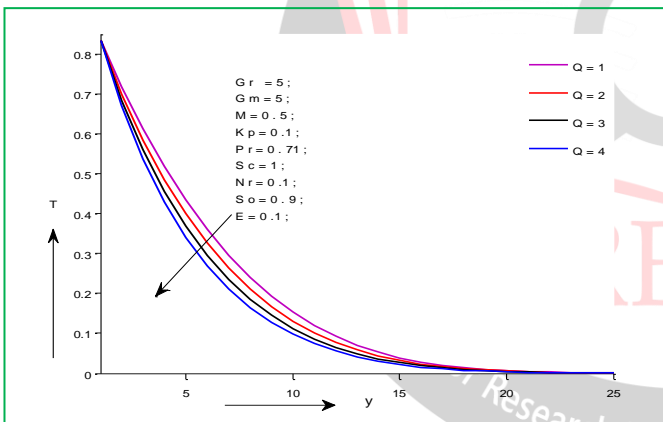


Figure 7: Effect of heat absorption parameter (Q) on temperature

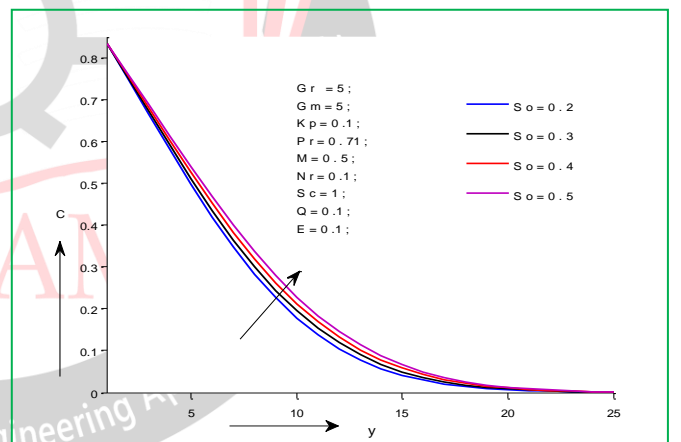


Figure 11: Effect of Soret number (S_0) on concentration

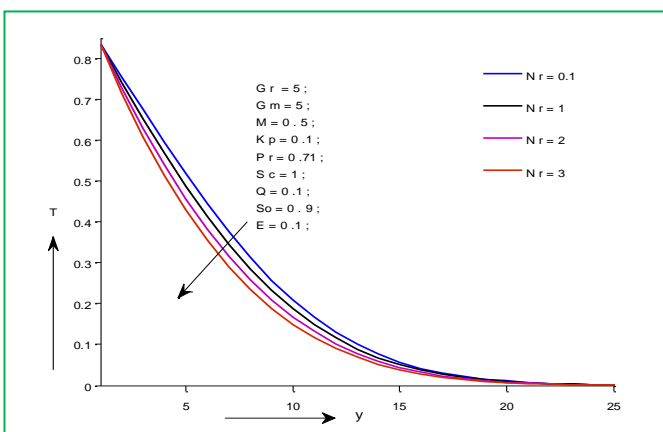


Figure 8: Effect of radiation parameter (Nr) on temperature

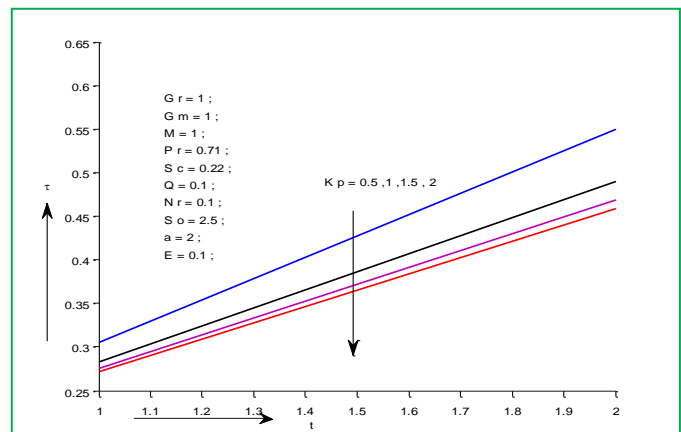


Figure 12: Effect of porosity parameter (Kp) on skin friction

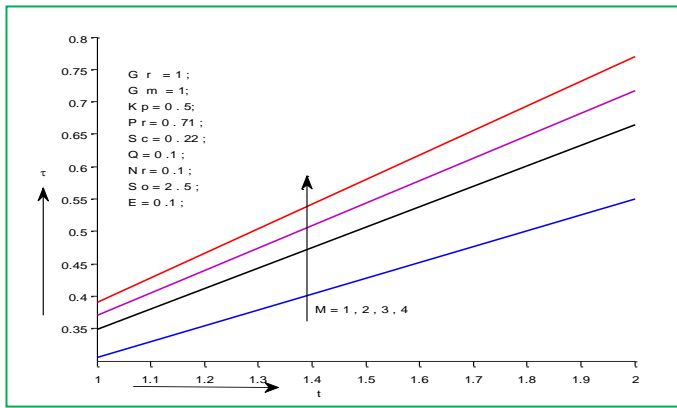


Figure 13: Effect of magnetic parameter (M) on skin friction

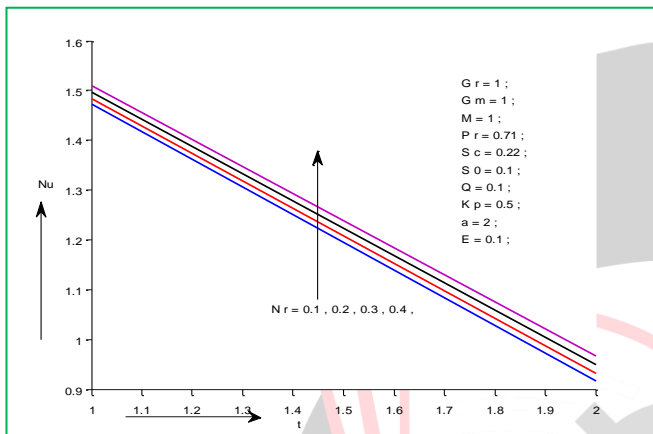


Figure 14: Effect of radiation parameter (Nr) on Nusselt number

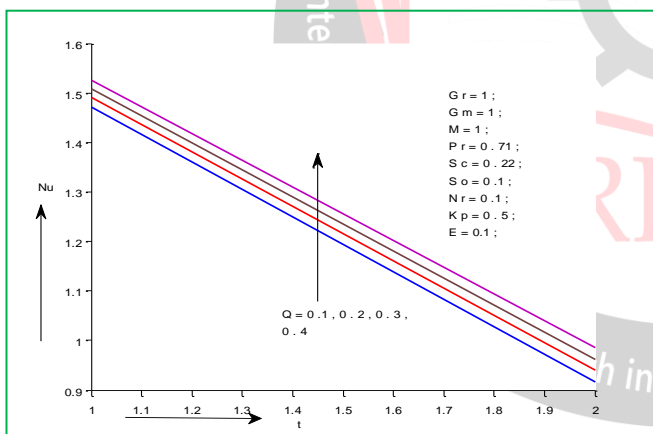


Figure 15: Effect of heat absorption parameter (Q) on Nusselt number

V. CONCLUSIONS

The variations in the velocity, temperature and concentration with the effects of various parameters examined in the problem are studied through graphs. Some of the above parameters on Skin friction and Nusselt number are observed.

- With the increasing values of porosity parameter and Soret number, the fluid velocity increases, but in case of magnetic parameter, radiation parameter and Schmidt number a reverse trend is found.

- For increasing values of Prandtl number, heat absorption parameter and radiation parameter, the temperature of the fluid reduces, but in the case of Eckert number it increases.
- When there is an increase in the values of Soret number, it results in rising of the concentration, but it falls down under the influence of Schmidt number.
- Skin friction decreases for increasing values of porosity parameter but increases for the case of magnetic parameter.
- With the increasing values of heat absorption parameter and radiation parameter, Nusselt number increases.

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