International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-04, July 2018

Mathematical model of two layered Renal blood flow, Newtonian and non -Newtonian in case of Diabetes

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Abstract - In the present paper we have reviewed the mathematical model for the renal blood flow along the capillaries in case of renal disease Diabetes. Blood is treated as homogeneous mixture, having two phase one red blood cell and other plasma and is explained by Fahreaus-Lindqvist effect. In thin blood capillaries blood flows in blood vessels in two layers. One layer made up of only plasma is attached with wall of the vessels is considered to be Newtonian. The second core layer where ratio of blood cells is too high in comparison to plasma, may be supposed to be non-Newtonian power law. we collected clinical data in case of Diabetes. The overall presentation is in tensorial form and solution technique is analytic as well as numerical. Thus, a linear graph is obtained numerically between pressure drop and the hematocrit.

Keywords —Diabetes, Glomerular capillary, Hematocrit, Nephron, Newtonian, plasma ,Pressure drop, power law, ,red blood cells , Renalcirculation.

I. INTRODUCTION

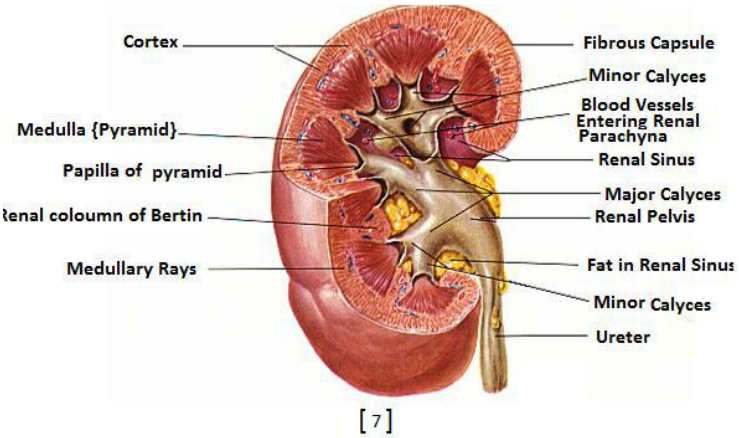
The whole paper is organized in four parts, In first pa structure, functioning and important parts are discussed. second part Blood flow as per diabetic person is discuss along with structure and function of renal capillaries. third part Problem is discussed as two-phase model wi one-layer Newtonian and other non-Newtonian, fourth pa mathematical modelling is done and results are obtained a discussed.

II. PART 1

STRUCTURE OF KIDNEY

The kidneys, pair of bean-shaped structures Filter and puri blood, it is located just below and posterior to the liver in t peritoneal cavity. These organs use up about 25 percent the oxygen absorbed through the lungs to perform th function. With Oxygen kidney cells efficiently manufactu chemical energy in the form of ATP (through aerob respiration). The filtrate released from the kidneys called urine.

The kidneys are surrounded by three layers, Figure 1. T outermost layer is a tough connective tissue layer call the renal fascia. The second layer is called the perirenal capsule, which helps anchor the kidneys in place. The thi and innermost layer is the renal capsule. Internally, t kidney has three regions—an outer cortex, a medulla in t middle, and the renal pelvis in the region called the hilum the kidney. The hilum is the concave part of the bean-shape where blood vessels and nerves enter and exit the kidney; The renal cortex is granular due to the presence of nephrons—the functional unit of the kidney. The medulla consists of multiple pyramidal tissue masses, called the renal pyramids [1][2][3]. In between the pyramids are spaces called renal columns through which the blood vessels pass. The tips of the pyramids, called renal papillae, point toward the renal pelvis. There are, on average, eight renal pyramids in each kidney. The renal pyramids and adjoining cortical region are called the lobes of the kidney. The renal pelvis primes to the ureter on the outside of the kidney. On the inside of the kidney, the renal pelvis divided out into two or three extensions called the major calyces, which further branch into the minor calyces. The ureters are urine-bearing tubes leave the kidney and empty into the urinary bladder. [4][5][6]



[7]

Figure-1

 DOI : 10.18231/2454-9150.2018.0451

FUNCTION

The three major functions of Kidney are maintaining fluid and acid–base balance, removal of nitrogenous waste products and synthesis of hormones, such as renin, erythropoietin, and active vitamin D_3 (calcitriol). Nephron is the functional unit of the kidney, which consists of a renal corpuscle, renal tubule, the proximal tubule (the part of tubule nearest to glomerulus), the loop of Henle, the distal tubule, and the collecting duct. The renal corpuscle consists of the glomerulus (tuft of capillaries) enclosed by Bowman's capsule. Each human kidney contains about one million nephrons. Proximal tubule is split into Proximal convoluted tubule and Proximal straight tubule. Straight portion go toward medulla, away from surface of kidney. The loop of of Henel which participate in counter current growth of urine concentration includes the proximal straight tubule, thin limb and thick rising limb. Connecting tubules connect the next segment, the small distal convoluted tubule, to the collecting duct system. Plasma is filtered in the glomerulus to form protein-free ultrafiltrate. most of this ultrafiltrate (60%) is reabsorbed in the proximal tubule. Several nephrons trench into a cortical collecting duct, which passes into an outer medullary collecting duct. In the inward medulla, inner medullary collecting ducts bond to form large papillary ducts.

BLOOD SUPPLY

The cortex of the kidney obtains the majority of renal blood flow. The renal artery divided into parts into anterior posterior divisions, which give rise to five segmental arteries which arrives at the hilum of the kidneys and branches into the interlobar artery. The interlobar artery travels between the pyramids and continues as arcuate artery, which curves along the corticomedullary junction. The interlobular artery is a stem off of the arcuate artery and goes through the cortex towards the capsule. As the interlobular artery rises towards the cortex, branches of afferent arterioles reach to each glomerulus. The afferent arterioles form the capillaries that form the glomerulus. The glomerular capillaries reunify and form the efferent arteriole which departures at the vascular pole. The efferent arterioles of cortical nephrons form peritubular capillaries. In the juxtamedullary nephrons, the peritubular capillaries give rise to the ascending and descending vasa recta. The peritubular capillaries and vasa recta then drain into the interlobular vein, which combine with others to make the arcuate vein. Then the interlobular vein departures the kidney as the renal vein. In adult kidney (in resting state) 1.2l to 1.3l blood per minute or 25%of cardiac output reaches. Renal blood flow can be measured with electromagnetic or other type of flow meter or it can be evaluated by applying the fick principle [12]. From renal plasma flow, the renal blood flow can be calculated by below formula. Hematocrit (HCT)-45%

The renal blood flow=RPF× 1/(1-HCT) ⟶700×1/ (1-0.45) =1273ml/Minute [45]

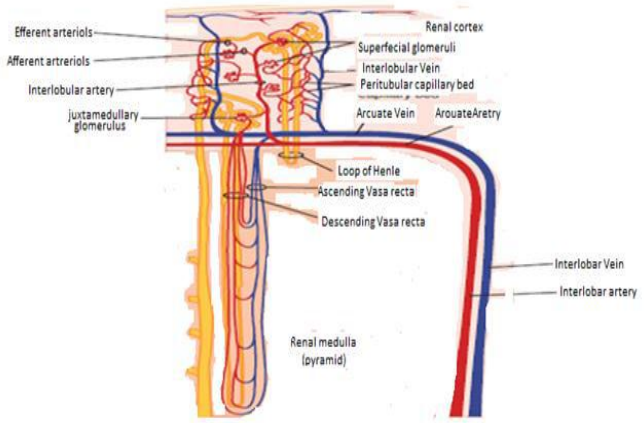


Figure-2

NEPHRON

Approximately one million nephrons are present in each human kidney, each have capacity to form urine. Kidney cannot regenerate fresh nephrons, so there is decrease is nephron with age and due to injury, disease.

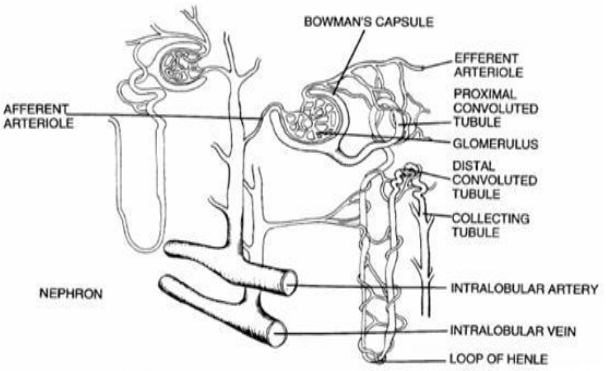


Fig 1.: Kidney anatomy and enhanced view of nephron. [13] [14]

Figure-3

III. PART-2

BLOOD

7% of body weight is contributed by blood [24] [25] which is a constantly circulating fluid and provides the body with nutrition, oxygen, and waste removal. Blood is a liquid, with numerous cells and proteins flowing in it, making blood denser then water with average density of appr. 1060kg/m^3[26]. The average healthy person has about 5 liters (more than a gallon) of blood. Plasma which contains Protein along with glucose and other dissolved nutrients makes up about 50% of blood, this protein help blood to clot, transport substances through the blood, and perform other functions.

About half of blood volume is composed of blood cells:
- Red blood cells to carry oxygen to the tissues
- White blood cells to fight infections
- Platelets, smaller cells to help blood to clot

Blood travels through blood vessels (arteries and veins). Blood is prevented from clotting in the blood vessels by their smoothness, and the proper balance of clotting factors. Blood with its components are most important part of patient management treatment protocols [19][36].The viscosity of

 DOI : 10.18231/2454-9150.2018.0451

blood depends on acting shear force and it is determined by Hematocrit value. Blood as per biological point of view can be considered as tissue to be made up of various cells RBC,WBC and PLETLETS and PLASMA but from rheological point of view blood is a two phased liquid.

PRESSURE IN RENAL VESSELS

The behavior of pressure in glomerular capillary has been measured in the rat and has been found to be considerably lower than the foretold on thye basis of indirect measurement. When the mean systolic arterial pressure is 100 mmhg, then glomerular capillary pressure is around 45 mmhg.The pressure drop across the glomerulas is only 1 to 3 mmhg, but more drop occurs in the efferent arteriole so that the pressure in the peritubular capillary is about 8 mmhg.The pressure in renal vein is about 4mmhg.The pressure gradient are similar for squirrel ,monkey and presumably in human with glomerular capillary pressure that is about 40% of systolic arterial pressure.[12]

DIEBETIES

Diabetes is the chronic disease that occurs in two cases, when the pancreas does not produce enough insulin or when the body cannot effectively use the insulin it produces. [29] [30]

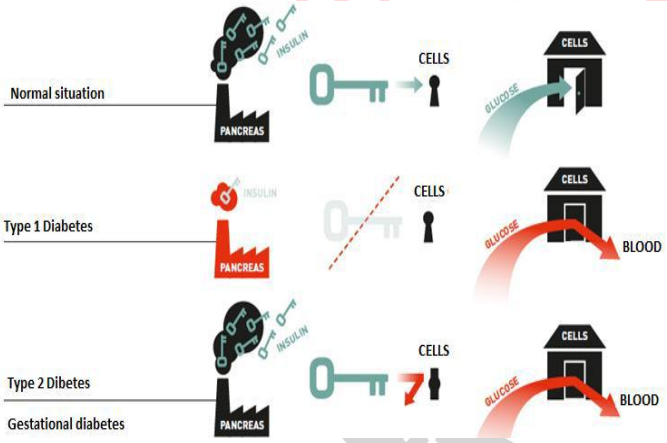


Figure-4

STRUCTURE AND FUNCTION OF RENAL CAPILLARIES

The glomerular and peritubular capillaries are two capillary beds in Renal circulation, which are arranged in series and separated by the efferent arterioles that regulates the hydrostatic pressures .High hydrostatic pressure in the glomerular capillaries(about 60mm Hg) causes rapid fluid filtration, on the other hand much lower hydrostatic pressure in the peritubular capillaries(about 13mm Hg)allows rapid fluid reabsorption[32].The peritubular capillaries drains into the vessels of the venous system, which is parallel to the arteriolar vessels and forms the interlobular vein arcuate vein, inter lober vein and renal vein, which leaves kidney beside the renal artery and ureter. The glomerulus is a tuft of small blood vessels called capillaries placed within Bowman's capsule in the kidney [33].

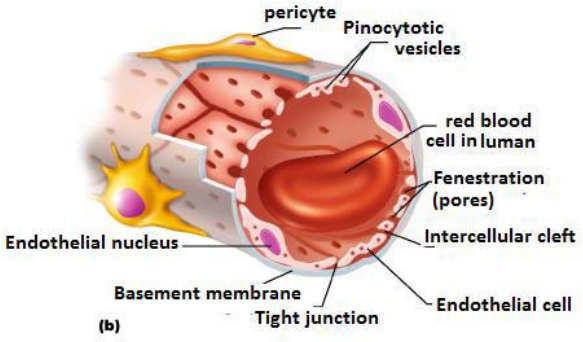


Figure-5

IV.PART 3

DESCRIPTION OF PROBLEM

The flow of blood is possible in capillaries, as we know that these vessels are marginally far from the heart and thin also. It's natural because the blood flows very slowly in arterioles where there is high viscosity and this is well explained by Fahreaus-Lindqvist effect. As per this effect the blood flows in two separated layers when it through capillaries. The plasma layer containing no blood cells and the second layer is that of blood cells, which float in plasma on the axis of the capillary. In this process the effective blood viscosity depends upon radius of the capillary. so the effective viscosity decreases, with radius and thus the blood flow becomes possible.

REAL MODEL

Blood is non-Newtonian complex fluid containing red blood cells (RBCs), white blood cells (WBCs) and platelets suspended in it. Blood has two phases one phase is plasma (55%) and 2nd phase is RBCs (45%) [35], and approximately 98% of RBCs in 45% of blood cells and there are a few parts (approximately 2%) of the other cells. Which are ignorable, so the foremost reason that the blood is not an ideal fluid but it is a mixture of the two phases. These blood cells, semi permeable packages of liquid have density greater than that of plasma, can change their shape and size while flowing through different blood vessels [36]. Plasma is a liquid having semi permeable packages of RBCs the nature of blood is almost Newtonian at high shear rate, but at low shear rate the blood exhibits yield stress and shows non- Newtonian behavior [37]. We have considered three-dimensional orthogonal curvilinear co-ordinate system, prescribed as E3 called as 3-dim Euclidean space. Here we have some quantities related to moving blood in cylindrical vessels:Blood velocity $V^k=V^k(x^i,t),k=1,2,3$ blood pressure $P=p(x^i,t)$ and density $\rho=\rho(x^i,t)$ where x^i be the coordinates of any point in space and $i=1,2,3$. Blood can be treated as homogeneous mixture, if we consider that both phases-plasma and blood cells are equally distributed in whole blood.

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EQUATION ON CONTINUITY

In the whole circuit of the human blood circulatory system, the heart behaves merely like a pumping station with no source and sink, so the law of conservation of mass can well be applied to hemodynamic [38], as in absence of source and sink in any region of flowing fluid, the fluid mass is conserved in that region. The Renal Circulatory System is whole blood flow circuit of the kidney, so it is a sub system of human circulatory system. Blood enter in kidney by arteries and out by veins and in a kidney with no source or sink. Since Mass of enter the blood is equal to mass of outer the blood, therefore law of conservation of mass can also be applied for renal circulatory system.

The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells.

Let X is the volume portion enclosed by the blood cells in unit volume. And X can be replaced by H/100, where H is the hematocrit the volume percentage of blood cells. Then the volume portion covered by plasma will be 1-X. if the mass ratio of blood cells to plasma is r, then clearly

$$r = \frac{x\rho_c}{(1-x)\rho_p}$$

Where ρ_c and ρ_p and are densities of blood cells and blood plasma respectively. Though Usually this mass ratio is not constant; but we consider it to be constant in present context [39].The both phase of blood,(blood cells and plasma) move with a common velocity. Campbell and Pitcher have presented a model for this situation. According to this model we consider the two phases of blood separately [40]. Hence according to principle of conservation of mass, the equations of continuity for the two phases are as follows [41].

$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c V^i),_i = 0$

$\frac{\partial(1-X)\rho_p}{\partial t} + ((1\text{-}X)\rho_p V^i),_i = 0$

Where v is the common velocity of the two phases blood cells and plasma $(X\rho_c V^i),_i$ is co-variant derivative of $(X\rho_c V^i)$ with respect to X^iIn the same way $((1\text{-}X)\rho_p V^i)$ with respect to X^i.If we define uniform density ρ_m as follows:

$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p}$

…….(3.1)

Then the equations can be combined together as follows,

$\frac{\partial(\rho_m)}{\partial t} + (\rho_m c),_i = 0$

As blood is incompressible fluid, hence ρ_m will be a constant quantity. Thus, the equation of continuity for blood flow takes the following form:

$V,_i^i = 0$

$$\frac{\partial v^i}{\partial x^i} + \frac{v^i \partial\sqrt{g}}{\sqrt{g}\partial x^i} = \frac{1}{\sqrt{g}}\left(\sqrt{g}V^i\right),_i = 0$$

EQUATION OF MOTION

According to law of conservation of momentum, the total momentum of any fluid system is conserved if there is no external force hence it can well apply to renal circulatory system. In other words, the rate of change of momentum of a fluid particle with respect to time equals to external force exerted on it. This is also called Newton's 2nd law of motion.

So, the rate of change of momentum is equal to sum of about two mentioned forces, which may be symbolically presented as follows.

$\frac{dp}{dt}$ =-P+F where, $\frac{dp}{dt}$ =rate of change of momentum

P=Internal pressures=viscous force

The hydro dynamical pressure p between the two phases of blood can be assumed to be uniform as both phases i.e. blood cells and plasma are always in equilibrium state in blood [42]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^i)v,_j^i = -Xp,_j g^{ij} + X\eta_c\left(g^{jk}v,_k^i\right),_j$$

…….

(3.2)

Similarly, taking the viscosity coefficient of plasma to be η_p the equation of motion of plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \left((1-X)\rho_p v^j\right)v,_j^i = -(1-X)p,_j g^{ij} + (1-X)\eta_p\left(g^{jk}v,_k^i\right),_j$$ …...

(3.3)

Now adding equation (3.2) and (3.3) and using relation (3.1), the

equation of blood flow with the both phases will be as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j)v,_j^i = -p,_j g^{ij} + \eta_m\left(g^{jk}v,_k^i\right),_j$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

DIFFERENT CONSECUTIVE EQUATION FOR BLOOD

Blood is non-homogeneous mixture of plasma and blood cells. But for practical purposes it may be considered to be homogeneous two-phase mixture of plasma and blood cells. The constitutive equations proposed for whole blood mixture are as follows:

1.Newtonian equation

$\tau = \eta e$,

where η is the viscosity coefficient

which is defined well in the broad blood vessels where there is low hematocrit [43].

2.The non-Newtonian power law equation

$\tau = \eta e^n$

This is found to be conformable for strain rate between 5 and 200

$0.68 \leq n \leq 0.80$ [44]

 DOI : 10.18231/2454-9150.2018.0451

The non -Newtonian Herschel-Bulkley equation [45]

$\tau=\eta e^n+\tau_0(\tau \geq \tau_0)$

e =0($\tau < \tau_0$)

It is defined when blood shows yield stress τ_0.

The yield stress arise because blood cells form aggregates in the form of rouleaux at low strain rate.

If $\tau < \tau_0$,no blood flow takes place. The yield stress is given by the following formula

$$\tau_0^{\frac{1}{3}} = \frac{A(H - H_m)}{100}$$

Where,

$A= (.008 \pm 0.002 dyne/cm^2)^{\frac{1}{3}}$

H is normal haematocrit and H_m is the haematocrit below which there is no yield stress.

HEMOTOCRATIC

It is a ratio of the volume of red cells to the volume of whole blood. Normal range for hematocrit varies between the sexes and is approximately 45% to 52% for men and 37% to 48% for women. This is usually measured by spinning down a sample of blood in a test tube, which causes the red blood cells to pack at the bottom of the tube. The hematocrit (expressed as percentage points) is normally about three times the hemoglobin concentration (reported as grams per decilitre). [24] [32] [33] [34] [35]

BOUNDARY CONDITION

1.The velocity of blood flow on the axis of capillaries at r=0 will be maximum and finite,

V_0 = maximum velocity.

2.The velocity of blood flow on the wall blood vessel at r=R, where, R is the radius of capillary, will be zero. This condition is well known as no-slip condition.

V. Part-4

MATHEMATICAL MODELLING: TWO LAYERED BLOOD FLOWS – ONE IS NEWTONIAN WHILE OTHER IS NON- NEWTONIAN POWER LAW FLOW: -

In thin blood capillaries blood flows in blood vessels in two layers. One layer made up of only plasma is attached with wall of the vessels is considered to be Newtonian. The second core layer where ratio of blood cells is too high in comparison to plasma, may be supposed to be non-Newtonian power law. Equation of continuity for power law flow is as under:

$$\frac{1}{\sqrt{g}\left(\sqrt{g}v^i\right)_{,i}} = 0 \qquad \ldots\ldots\ldots\ldots\ldots$$

(4.1)

Equation of the motion is extended as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^i v^i_{,j} = T^{ij}_{,j}$$

……(4.2)

Where T^{ij} is taken from constitutive Equation of per law flow

$$T^{ij} = -pg^{ij} + \eta_m (e^{ij})^n = -pg^{ij} + T'^{ij}.$$

$$\rho_m = X\rho_c + (1-X)\rho_p$$

and $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity of mixture of the blood.

X=H/100 is volume ratio of the blood cell; H is the Hematocrit. Other symbols have their usual meanings.

Blood vessels are cylindrical, so the above equations have to be transformed into cylindrical co-ordinates.

Now we have to transform the equations (4.1) and (4.2) in cylindrical form. As we know.

For cylindrical Co-ordinates, $X^1=r, X^2=\theta, X^3=z$

Matrix of metric tensor in cylindrical co-ordinates are as follows:

$$\left[g_{ij}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While the matrix of conjugate, metric tensor is as follows

$$\left[g^{ij}\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where the Christoffel's symbols of 2nd kind are as follows

$$\begin{Bmatrix} 1 \\ 2\ 2 \end{Bmatrix} = -r, \quad \begin{Bmatrix} 2 \\ 2\ 1 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1\ 2 \end{Bmatrix} = \frac{1}{r}, \text{rest are zero,}$$

Relation between contra variant physical components of the velocity of the blood flow will be as follows:

$$\sqrt{g_{11}}v^1 = v_r \Rightarrow v_r = v^1$$

$$\sqrt{g_{22}}v^2 = v_0 \Rightarrow v_0 = r\,v^2$$

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And $\sqrt{g}_{33}v^3 = v_z \Rightarrow v_z = v^3$

Again the physical components of $p_{,j}g^{ij}$ is $-\sqrt{g_{ii}}\, p_{,j}g^{ij}$

The matrix of the physical components of shearing stress-tensor

$$T^{'ij}=\eta_m(e^{ij})^n=\eta_m(g^{ik}v_{,}{}^{i}{}_{k}+g^{jk}v^{j}{}_{,k})^n$$

Will be as follows

$$\begin{bmatrix} 0 & 0 & \eta_m(dv/dr)^n \\ 0 & 0 & 0 \\ \eta_m(dv/dr)^n & 0 & 0 \end{bmatrix}$$

The covarint derivative $T^{'ij}$

$$T_{,j}^{'ij}=\frac{1}{\sqrt{g}}\frac{\partial}{\partial X^j}(\sqrt{g}\,T^{'ij})+\begin{Bmatrix} i \\ j\ \ k \end{Bmatrix}T^{'ij}$$

Keeping in view the above fact, tonsorial equation can be transformed into cylindrical form which are as follows:

The equation of continuity

$$\frac{\partial v}{\partial z}=0 \qquad \text{.....(4.3)}$$

Equation of motion,r-component

$$-\frac{\partial p}{\partial r}=0 \qquad \text{.....(4.4)}$$

θ-component

$0=0$ .....(4.5)

Z-component

$$0=-\frac{\partial p}{\partial z}+\frac{\eta_m}{r}\frac{\partial}{\partial r}\left(r\left\{\frac{\partial v_z}{\partial r}\right\}^n\right) \qquad \text{.....(4.6)}$$

Keeping in mind that blood flow is axially symmetric in arteries concerned i.e

$V_\theta = 0, V_z$ and p do not depend upon θ. Also, the blood flow steadily, i.e.

$$\frac{\partial p}{\partial t}=\frac{\partial v_r}{\partial t}=\frac{v_\theta}{\partial t}=\frac{\partial v_z}{\partial t}=0 \qquad \text{.....}$$

(4.7)

SOLUTION

On integrating equation, we get $v_z = v(r)$ because v does not dependupon θ. ....

(4.8)

on integrating equation of motion we get P=p (z) since p does not depend upon θ ... (4.9)

Now, with the help of equation (4.8) and (4.9) the equations of motion (4.6) convert in the following form

$$0=-\frac{dp}{dz}+\frac{\eta_m}{r}\frac{d}{dr}\left\{r\left(\frac{dv}{dr}\right)^n\right\} \qquad \text{.....}$$

(4.10)

The pressure –gradient – (dp/dz) =p of blood flow in the arteries remote the heart can be supposed to be constant and hence the equation (4.10) takes the following form

$$\frac{d}{dr}\left\{r\left(\frac{dv}{dr}\right)^n\right\}=-\frac{pr}{\eta_m} \qquad \text{....}$$

(4.11)

On integrating we get

$$r\left[\frac{dv}{dr}\right]^n=\frac{pr^2}{2\eta_m}+A \qquad \text{....}$$

(4.12)

velocity of the blood flow on the axis of cylindrical arteries is maximum and constant, So We can apply the boundary condition at r=0, $v=V_0$ (constant), on equation (4.12) which gives following form

$$r\left[\frac{dv}{dr}\right]^n=\frac{pr^2}{2\eta_m}\Rightarrow -\frac{dv}{dr}=\left[\frac{pr}{2\eta_m}\right]^{1/n}$$

...(4.13)

On integrating

$$v=-\left[\frac{P}{2\eta_m}\right]^{1/n}\frac{r^{1/n+1}}{1+1/n}+B \qquad \text{.....}$$

(4.14)

To determine the arbitrary constant B, we will apply the non-slip condition on the inner wall of the arteries at r=R, V=0, where R= radius of vessel, on equation (4.14) so as to get

$$B=\left[\frac{P}{2\eta_m}\right]^{1/n}\frac{nR^{1/n+1}}{(n+1)}$$

Hence the equation (14) takes the following form

$$v=\left[\frac{p}{2\eta_m}\right]^{1/n}\frac{n}{(n+1)}\left[R^{1/n+1}-r^{1/n+1}\right] \qquad \text{.....(4.15)}$$

Which determine the velocity of the blood flow in the artery remote from heart where P is gradient of blood pressure.

And η_m is the velocity of blood mixture

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Now the formula for velocity of blood flows can be obtained by replacing η_m with η_p in (4.10) of Newtonian model as follows:

$$v_p = \frac{P}{4\eta_p}\left(R^2 - r^2\right); \quad R-\delta \le r \le R \quad \text{..} \quad \text{.....(4.16)}$$

Where δ is the radius of core layer.The velocity of core layer is obtained as the formula (4.12) of power law model as follows

$$v_m = \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{(n+1)}\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) + \left[\frac{P}{4\eta_p}\left(R^2 - (R-\delta)^2\right) - \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{(n+1)}\left(R^{\frac{1}{n}+1} - (R-\delta)^{\frac{1}{n}+1}\right)\right]$$
$$0 \le r \le R-\delta \quad \text{.....(4.17)}$$

Where, the 2nd term is the relative velocity of plasma layer with respect to core layer.
Flow flux in Capillary is ….

$$Q = \int_0^{R-\delta} V_m 2\pi r dr + \int_{R-\delta}^{R} V_p 2\pi r dr$$

So from (4.16) and (4.17) we have

$$Q = \int_0^{R-\delta} \left[\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) + \left\{\frac{p}{4\eta_p}\left(R^2 - (R-\delta)^2\right) - \left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1}{n}+1} - (R-\delta)^{\frac{1}{n}+1}\right)\right\}\right] 2\pi r dr + \int_{R-\delta}^{R} \frac{p}{4\eta_p}\left(R^2 - r^2\right) 2\pi r dr \quad \text{.....(4.18)}$$

BIO PHYSICAL INTEPRETATION: -

Clinical data (male Diabetic, age 45yrs)

Sn	H.B.	Hematocrit	Pressure drop(mmhg)
1	9.1	26.8	140/90mmhg=18664.8/11998.8
2	9.2	27.8	140/90mmhg=18664.8/11998.8
3	9.4	28.9	130/90mmhg=17331.6/11998p
4	8.8	27	130/90mmhg=17331.6/11998p
5	9.8	29	140/90mmhg=18664.8/11998.8

Table 1

Average systolic Pressure =S= 136 mmhg
Average Diostolic pressure =D = 90 mmhg

Pressure at Capillary = $\left(\frac{\frac{D+S}{2}+D}{3}\right)$ =67.66

Pressure on Venules= $\frac{2}{3}$of Capillary = $\frac{2}{3}\times 67.66 = 45.11$mmhg

p_i =Blood Pressure on capillary $= 67.66\times 133.322 = 9020.56$ ps

p_f = Blood Pressure on Venules $= 45.11\times 133.322 = 6014.303$ ps

η_m = Viscosity of mixture $= 3.5\times 10^{-3}$ ps

η_p = Viscosity of plasma $= 1.2\times 10^{-3}$ ps

R= Radius of capillary $= 0.0965$m

δ = Thickness of RBC layer $= \frac{1}{3}R = 0.0322$

$R-\delta = 0.0643$

Average HB = 9.24 & Hematocrit $3\times 9.24 = 27.72$

Q=flow flux of blood = 0.01833 pa

Length of capillary = 19000 m

$$\eta_m = \eta_c X + (1-X)\eta_p \; ; \; X = \frac{H}{100},$$
$$3.5\times 10^{-3} = \eta_c \times \frac{27.72}{100} + \left(1 - \frac{27.72}{100}\right)\times 1.2\times 10^{-3}$$
$$\eta_c = 9.49\times 10^{-3}$$

Now

$$p = -\frac{dp}{dz} \Rightarrow p\int_{z_i}^{z_f} dz = -\int_{p_i}^{p_f} dp \Rightarrow p\left(z_f - z_i\right) = \left(p_i - p_f\right)$$
$$\Rightarrow p = \frac{p_i - p_f}{z_f - z_i} = \frac{\Delta p}{\text{length of capillary}} = 0.158224$$

Now from (4.18)

$$Q = \int_0^{R-\delta} \left[\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left(R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) + \left\{\frac{p}{4\eta_p}\left(R^2 - (R-\delta)^2\right) - \left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1}{n}+1} - (R-\delta)^{\frac{1}{n}+1}\right)\right\}\right] 2\pi r dr + \int_{R-\delta}^{R} \frac{p}{4\eta_p}\left(R^2 - r^2\right) 2\pi r dr$$

 DOI : 10.18231/2454-9150.2018.0451

$$= \int_0^{0.0643} \left\{ \begin{array}{l} \left(\frac{0.158224}{2\times3.5\times10^{-3}}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} 2\pi r dr - 2\pi r^{\frac{n+1}{n}+1} dr\right) + \\ \left(\frac{0.158224}{4\times1.2\times10^{-3}}\right)\left(0.0965^2 - 0.0643^2\right)2\pi r dr \\ -\frac{0.158224}{2\times3.5\times10^{-3}} \times \frac{n}{n+1}\left(0.0965^{\frac{n}{n+1}} - 0.0643^{\frac{n}{n+1}}\right)2\pi r dr \end{array} \right\}$$

$$+ \int_{0.0643}^{0.0965} \frac{0.158224}{4\times1.2\times10^{-3}}\left(0.0965^2 2\pi r dr - 2\pi r^3 dr\right)$$

$$Q = \int_0^{0.0643} (22.6034)^{\frac{1}{n}} \times \frac{n}{n+1} \times 0.0965^{\frac{n+1}{n}} 2\pi \left[\frac{r^2}{2}\right]_0^{0.0643}$$

$$-2\pi\left[\frac{nr^{\frac{n}{3n+1}}}{3n+1}\right]_0^{0.0643} + .5359\left[\frac{r^2}{2}\right]_0^{0.0643} -$$

$$(23.03)^{\frac{1}{n}} \frac{n}{n+1}\left(0.0965^{\frac{n+1}{n}} - 0.0643^{\frac{n+1}{n}}\right)2\pi\left[\frac{r^2}{2}\right]_0^{0.0643}$$

$$+32.96\left[\begin{array}{l} 0.0585\left[\frac{r^2}{2}\right]_{0.0643}^{0.0965} \\ -2\pi\left(\frac{r^4}{4}\right)_{0.0643}^{0.0965} \end{array}\right]$$

$$= (22.6034\times0.0643)^{\frac{1}{n}} \frac{n}{n+1}\left[\frac{-n}{3n+1}(1.66\times10^{-3}) + (8.34\times10^{-3})\right] + 3.599$$

$$Q - 3.599\times10^{-3} = (1.4533)^{\frac{1}{n}} \frac{n}{n+1}\left[\frac{-n1.66\times10^{-3}}{3n+1} + 8.34\times10\right.$$

Taking $Q = .01833$

$$0.01473 = (1.4533)^{\frac{1}{n}} \times \frac{n}{n+1}\left[8.34\times10^{-4} - \frac{n\times1.66\times10^{-3}}{3n+1}\right]$$

Solving for n , n=0.064645

Substituting value of n in (4.18) we have

$$Q = \left(\frac{p}{2\eta_m}\right)^{15.469} \times (1.650451\times10^{-23}) + \left(\frac{p}{2\eta_p}\right) \times 1.092\times10^{-4}$$

$$0.01833 = (1.65\times10^{-23}) \times \left(\frac{p}{2\eta_m}\right)^{15.469} + 0.0227p \quad \ldots.(4.19)$$

Now ignoring term 0.0227p (negligible value) and putting $p = \frac{\Delta p}{\text{length of capillary}}$, in (4.19) we have

$$0.01833 = (1.65\times10^{-23})\left[\frac{\Delta p}{2\eta_m 19000}\right]^{15.469}$$

(length of capillary 19000)

$$\Delta p = 871546.01\times\eta_m$$

$$\Delta p = 871546.01\times\left[\eta_c X + (1-X)\eta_p\right]$$

$$\Delta p = 871546.01\left[(9.49\times10^{-3})\frac{H}{100} + (1-\frac{H}{100})(1.2\times10^{-3})\right]$$

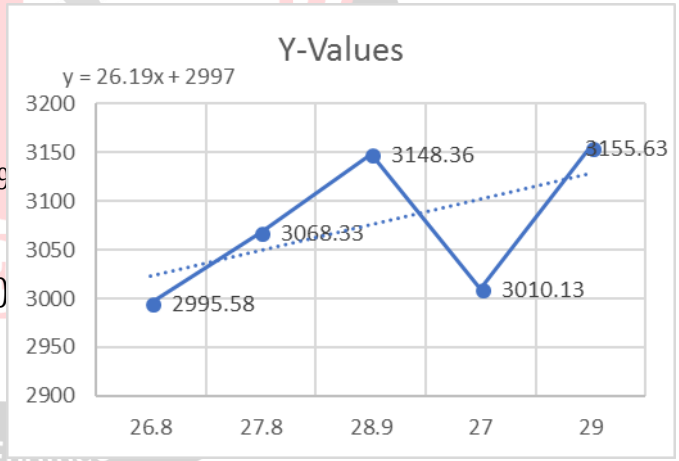
$$\Delta p = 72.7511H + 1045.855 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.20)$$

RESULTS AND DISCUSSION

Using relation (4.20) and substituting values of H

S.No	H	Δp
1	26.8	2995.58
2	27.8	3068.33
3	28.9	3148.36
4	27	3010.13
5	29	3155.63

Table-2



Graph-1

VI. CONCLUSION

Till now most of the work has been done on one phase model,But

in this paper we have verified two phase (one Plasma and other RBC) Non-Newtonian Model i.e Newtonian Power low flow with one layer Newtonian and other as non-Newtonian .

In Bio physical Interpretation ,We have taken clinical data of a 45 years old diabetic male and observed five readings of Blood Pressure and Hematocrit (Hematocrit=3×Heomoglobin).And we got the relation $\Delta p = 72.7511H + 1045.855$,by using the two phase Non-Newtonian Model and sketched the graph between

 DOI : 10.18231/2454-9150.2018.0451

Blood pressure drop in renal Capillary and Hematocrit , and trend of graph shows a linear relation between Blood Pressure drop and Hematocrit , given by **y=26.19x+2997**.This linear relation approves one phase model as well as the two phase relation $\eta_m = \eta_c X + \eta_p (1-X)$ where X= H/100 And slope of trend line is 26.19,by this slope of trend line we can suggest about condition of the blood pressure in concern tissue.Always we will obtain a linear curve but the slope give us clear picture of blood flow in tissue .A downward going trend line shows negative slope and indicate sudden change in blood pressure.

Remark:Our Research is beneficial for medical science,With the help of our data ,before operation a Doctor can tell that blood is required during the operation or not.Usually before operation Blood pressure of a patient is taken,this value gives average blood pressure of patient rather then Blood pressure drop in the tissue under operation,due to which complications may comes for the patient with normal blood pressure also.

In the model we discussed Kidney is a tissue and Diabeties is a disease related to Kidney.So we took clinical data of a Diabetic person and verified the model.With the help of this data we can conclude ,wheather while operating the particular tissue blood flow will be normal or not,which will help us to decide, If blood is required during operation or not , also we can judge that tissue is in condition to operate or not .In future we can work on other tissues also and disease concern with that tissue.For example we can consider lungs and TB as disease.

ACKNOWLEDGEMENT

I owe my sincere thanks to Dr.Divya Batra and Dr.Naveen Batra,Anantshree hospital Bhopal for the support and information they gave.

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