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Deadlock Analysis of Improved Round Robin CPU Scheduling Algorithm Using Markov Chain Model

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Abstract - **CPU scheduling defines the rules for deciding which of the available process in ready queue will be selected next and dispatched by scheduler to CPU, so that the resource utilization and overall performance of the system could be improved. Various traditional CPU scheduling algorithms have been proposed by several researchers each having their pros and cons. The improved round robin CPU scheduling algorithm [24] is one of them that reduces the average waiting time and increases the throughput and maintains the same level of CPU utilization. In this paper we evaluated the performance of this algorithm by incorporating the deadlock condition. The transition from one process to another process is done under markovian concept and data set based Markov chain model is proposed to study these different transition states. The overall performance in terms of unequal and equal numerical data are calculated and then comparative analysis is performed to justify the conclusion further with the help of some numerical illustrations, simulation study has been performed.**

Keywords —Deadlock Analysis, CPU Scheduling, Improved Round Robin Algorithm, Markov Chain Model, Numerical Data Model, Transition Probability Matrix.

I. INTRODUCTION

Concurrent execution is one of the important feature provided by operating system that improves the utilization of system resources by distributing them among many concurrently executing processes. Since the deadlock problem is a logical one it can arise in different contexts. The deadlock problem becomes more complex when a system has different resource types and, in general, more than one resource of the same type. In this case resources of the same type are not labeled differently. When a request is made for a number of resources of a particular type, any resources of that type may be granted. Thus, the situation of only one resource of each type is a special case for this [7, 8].

In a multiprogramming environment, several processes may be executing and requesting a finite number of common resources simultaneously. When some resources are requested by a process and if that resources are not available at that time then the process enter into a resting state. Sometimes a resting process is never again able to change state because of the resources it has requested are held by another resting processes. This situation is called a deadlock. Deadlocks arise in process synchronization when processes wait for each other's signals, or in resource sharing when they wait for other processes to release resources that they need. Deadlocked processes remain blocked indefinitely, which adversely affects user service, throughput and resource efficiency.

This deadlock situation has arisen only because of the following general conditions [3-5] are operative:

a. The access of resources by different processes is exclusive in time. ("mutual exclusion" condition).
b. Few resources are already held by a process and is waiting for additional resources to complete its execution ("wait for" condition).
c. Resources cannot be forcibly removed from the process holding them until the resources are released voluntarily after completion ("no preemption" condition).
d. A circular chain of processes exists, such that each process holds one or more resources that are being requested by the next process in the chain ("circular wait" condition). The existence of these conditions effectively defines a state of deadlock [1, 2]. In general, three methods Deadlock Prevention, Deadlock Avoidance and Deadlock detection are used to deal with deadlock condition.

II. RELATED WORK

A deadlock is an undesirable situation where processes of a set of states that hold schedulers are locked indefinitely from access to schedulers held by other processes within the states. No processes of the states can release its own schedulers before completing its tasks. Therefore, the deadlock will last forever, unless a deadlock resolution procedure is performed. Researchers have studied different types of anomalies found in deadlock and introduced various models to precisely define deadlock condition and also provided comprehensive analysis of deadlock. Banker's algorithm for avoiding deadlock has been improved by using a waiting (resting) state processes over there and avoid deadlock in particular study [10, 15].

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Researcher's presented deadlock detection technique using wait for graph through propagating the probe messages along the edges of wait for graph and deadlock resolution algorithm, which is based on the mutual cooperation of the transactions [12]. Few other researchers [23] also developed an algorithm for deadlock detection at local and global level. One more researcher presented magiclock, a deadlock detection technique which is eliminates removable lock dependencies into thread specific partitions, consolidates equivalent lock dependencies and searches over the set of lock dependency, and produced numerical based results to show that magiclock is significant and also detecting potential deadlock in multithreaded programs [13, 14]. Author's proposed a hierarchy of deadlock models and deadlock detection problems and gives a comparative study between deadlock models and deferent types gates (OR, AND, etc.) models [11]. Also, resource allocation used at the infrastructure level and gives the comparative analysis of physical resources versus virtual resources, and then implemented algorithm using CloudSim simulator and generate the experimental results for numerical data basis [16].

Author's studied a model-based strategy on binary decision diagram for efficiently retrieving the boundary unsafe states. Demonstrate that symbolic computation enables with large structure and state spaces with limited time and memory requirements and also presented computational costs are substantially reduced through the pertinent exploitation of the special structure that exist in the considered problem. Demonstrate the efficacy of the developed approaches through a series of computational experiments also establish the ability of the proposed methodology to effectively compute tractable implementation [17, 18]. One more author's proposed deadlock-based study for timed Rebeca models and checking schedulability and also focus on events-based behavior for actor's action and predict some experimental result for that [19]. Also, studied different approaches to solve the state space explosion problem using heuristic and metaheuristic algorithms and generate some solutions and proposed two new algorithms to find deadlock in complex software systems and produced some experimental solution for that [20], and a dynamic priority task scheduling strategy based on value evaluation to handle the different threat in particular scenario. The strategy is based on some features related to multilevel ready queue and designed an algorithm for real time value computation and enhancing its adaptability and carry out its performance optimization using some experiment-based strategy [21], and another researcher developed novel version of the resource allocation systems to avoid the deadlock problem and produce new decomposed operational modes to provide new policies and running data set for particular resources that was enable the formal characteristics and the effective computation of this study mode [22].

A data set based markov chain model is presented to study the transition states and number of scheduling schemes are designed and treated as its particular cases and are compared under the setup of markov chain model and finding deadlock index measure, and again some simulation study is performed to evaluate the comparative merits of specific scheme has terms, conditions and restrictions over the general class [6]. The Improved Round Robin (IRR) policy reduces the average waiting time and increases the throughput and maintains the same level of CPU utilization like traditional Round Robin provides. In this paper a markov chain model is done in order to determine the performance of this suggested IRR algorithm. We have also proposed some other ways to assign the scheduler to the next ready process. These efforts have found very efficient and useful. Further some numerical studies have been done to justify the proposed suggestions [9].

The set of possible values of an individual random variable $X^{(n)}$ *(or* $X(t)$*)* of a stochastic process $\{X^{(n)}, n\geq 1\}$, $\{X(t), t\in T\}$ is known as state space, The stochastic process $\{X^{(n)}, n=0,1,2\ldots\}$ is called Markov chain, if, for $j, k, j1, \ldots, j(n-1)$ € N (or any subset of I). Medhi has been tested an elaborate study of a variety of stochastic processes and their applications in various fields and developed a Markov chain model for the study of uncertain rainfall phenomenon and also presented the use of stochastic process in the management of queues [25-27]. Naldi proposed and develop a Markov chain model for understanding the internet traffic sharing among various operators in a competitive market [28]. Researcher studied the use of Markov chain model for multilevel queue scheduler and also designed a scheduling scheme and compare through numerical based study [29, 30]. Proposed a linear data model-based study of improved RR CPU Scheduling algorithm with features of shortest job first scheduling with varying time quantum by using Markov chain model with different data set and performed some numerical based study [31, 32]. Author's worked on traditional round robin scheme to reduce the total waiting time of an any process which is spend in a ready queue and improve the performance of existing round robin algorithm to understand this waiting time difference using mathematical formulation and calculation [24].

III. PROPOSED SYSTEM

In Improved Round Robin (IRR) CPU scheduling policy, the basic functions of Round Robin with an improvement towards the priority assigned to the processes nearing completion are combined. Since the time requirement for completion of a process Pi after $(ri - 1)^{th}$ round is at the most one-time quantum. Hence, we consider a priority queue (to be referred as Q_2) in addition to the ready queue Q_1. An additional queue has been used by author [9] for dispatching priority in context of FCFS scheduling. All processes, after being served by the CPU in penultimate round, are sent to the rear end of Q_2 instead of Q_1. Thus, the processes which need only one quantum or less will be terminated in the first round itself from Q_1, while all others will be terminated on being dispatched from Q_2. Therefore, processes going to CPU through Q_1, if not terminated, may return back to the rear end of either Q_1 or Q_2. As shown in Fig. 3.1, this approach organizes the pending requests in two queues. Deadlock analysis of IRR CPU scheduling policy assumes cycle of four queues (Q_1, Q_2, Q_3 & Q_4) for the purpose of sequential allocation to scheduler; it starts with two processes from Q_1 one process from Q_2 and one process from Q_3 (deadlock process) and one process from Q_4 (waiting process).

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The scheduling policy can further be improved by adopting some different cycle. Precise idea is to appropriately choose a pair of numbers p and q (p>q) that determine the number of processes from Q_1 and Q_2 for allocation to CPU in the cycle. An optimal choice may however, depend on the number of processes and the size of their CPU bursts. In the present work, we shall confine our discussion to p and q. This scheduling policy provides better estimates than the conventional RR policy in respect of all performance measures, including the throughput, without any significant increase in the overheads [9, 24].

Generalized Markov chain models in CPU scheduling

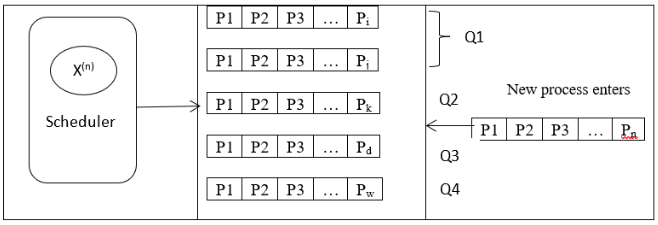

Fig. 3.1: Generalized Markov chain models in CPU scheduling

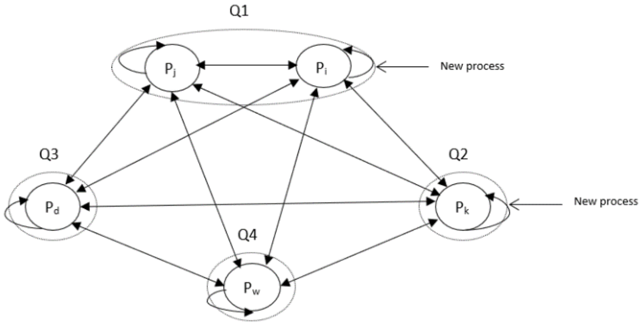

Fig. 3.2: Unrestricted transition diagram

Let $X^{(n)}$, $n \geq 1$, be a Markov chain where $X^{(n)}$ denotes the state of the scheduling at the quantum of time. The state space for the random variable $X^{(n)}$ is $\{Q_1, Q_2, Q_3, Q_4\}$ where $Q_1 = P_i$, P_j are combine process in first state, $Q_2 = P_k$ is second state, $Q_3 = P_d$ is third (deadlock) state and $Q_4 = P_w$ is fourth (waiting) state and scheduler X move stochastically over different processing states and waiting within different quantum of time. And fig. 3.2 shows the transition diagram performing transition from one state to another state according to CPU scheduling algorithm. Unit step transaction probability matrix for $X^{(n)}$ under general model is:

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ P_j & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ P_k & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ P_d & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ P_w & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{array}$$

Predefined selection for initial probabilities of states are:

$P [X^{(n)} = P_i] = P_{r1}$; $P [X^{(n)} = P_j] = P_{r2}$; $P [X^{(n)} = P_k] = P_{r3}$; $P [X^{(n)} = P_d] = 0$; $P [X^{(n)} = P_w] = 0$ ……… *eq 1*

Let S_{ij} $(i, j = 1, 2, 3,\ldots)$ be the unit step transition probabilities of scheduler over three states then transition probability depend on subject to condition:

$S_{15} = (1 – \sum_{i=1}^{4} S1i)$; $S_{25} = (1 – \sum_{i=1}^{4} S2i)$; $S_{35} = (1 – \sum_{i=1}^{4} S3i)$; $S_{45} = (1 – \sum_{i=1}^{4} S4i)$; $S_{55} = (1 – \sum_{i=1}^{4} S5i)$; & $0 \leq S_{ij} \leq 1$,

The state probabilities, after the first quantum can be obtained by a simple relationship:

$P [X^{(1)} = P_i] = P [X^{(0)} = P_i] P [X^{(1)} = P_i / X^{(0)} = P_i] + P [X^{(0)} = P_j] P [X^{(1)} = P_i / X^{(0)} = P_j] + P [X^{(0)} = P_k] P [X^{(1)} = P_i / X^{(0)} = P_k] + P [X^{(0)} = P_d] P [X^{(1)} = P_i / X^{(0)} = P_d] + P [X^{(0)} = P_w] P [X^{(1)} = P_i / X^{(0)} = P_w]$

$P [X^{(1)} = P_i] = \sum_{i=1}^{4} Pri\ Si1$; $P [X^{(1)} = P_j] = \sum_{i=1}^{4} Pri\ Si2$; $P [X^{(1)} = P_k] = \sum_{i=1}^{4} Pri\ Si3$; $P [X^{(1)} = P_d] = \sum_{i=1}^{4} Pri\ Si4$; $P [X^{(1)} = P_w] = \sum_{i=1}^{4} Pri\ Si5$ ……….*eq. 2*

Similarly, state probabilities after second quantum can be obtained by simple relationship:

$P [X^{(2)} = P_i] = P [X^{(1)} = P_i] P [X^{(2)} = P_i / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_i / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_i / X^{(1)} = P_k] + P [X^{(1)} = P_d] P [X^{(2)} = P_i / X^{(1)} = P_d] + P [X^{(1)} = P_w] P [X^{(2)} = P_i / X^{(1)} = P_w]$

$P [X^{(2)} = P_i] = \sum_{i=1}^{5} (\sum_{j=1}^{4} Prj\ Sji) S_{i1}$; $P [X^{(2)} = P_j] = \sum_{i=1}^{5} (\sum_{j=1}^{4} Prj\ Sji) S_{i2}$;
$P [X^{(2)} = P_k] = \sum_{i=1}^{5} (\sum_{j=1}^{4} Prj\ Sji) S_{i3}$; $P [X^{(2)} = P_d] = \sum_{i=1}^{5} (\sum_{j=1}^{4} Prj\ Sji) S_{i4}$; $P [X^{(2)} = P_w] = \sum_{i=1}^{5} (\sum_{j=1}^{4} Prj\ Sji) S_{i5}$ …………*eq. 3*

The generalized expressions for n quantum are:

$P [X^{(n)} = P_i] = \sum_{m=1}^{5} \ldots\ldots\ldots \sum_{l=1}^{5} \sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{4} Prj\ Sji\ \ Sik\ \ Skl\ \ldots\ldots Sm1$;
$P [X^{(n)} = P_j] = \sum_{m=1}^{5} \ldots\ldots\ldots \sum_{l=1}^{5} \sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{4} Prj\ Sji\ \ Sik\ \ Skl\ \ldots\ldots Sm2$;
$P [X^{(n)} = P_k] = \sum_{m=1}^{5} \ldots\ldots\ldots \sum_{l=1}^{5} \sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{4} Prj\ Sji\ \ Sik\ \ Skl\ \ldots\ldots Sm3$;
$P [X^{(n)} = P_d] = \sum_{m=1}^{5} \ldots\ldots\ldots \sum_{l=1}^{5} \sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{4} Prj\ Sji\ \ Sik\ \ Skl\ \ldots\ldots Sm4$
$P [X^{(n)} = P_w] = \sum_{m=1}^{5} \ldots\ldots\ldots \sum_{l=1}^{5} \sum_{k=1}^{5} \sum_{i=1}^{5} \sum_{j=1}^{4} Prj\ Sji\ \ Sik\ \ Skl\ \ldots\ldots Sm5$ …………*eq. 4*

IV. DEADLOCK ANALYSIS OF IRR CPU SCHEDULING SCHEMES

The following are the schemes that are obtained by imposing some restrictions and condition on the transition model of IRR algorithm under possibility of deadlock.

A. Scheme - I

At any stage, after dispatching two processes from Q_1, if Q_2 is found to be empty, another pair of processes will be dispatched from Q_1. this scheme is described in fig. 4.1.

- A new process can only enter to first queue Q_1 and executing the two processes P_i and P_j , if state Q_2 (i.e. process P_k) is found to be empty, then another pair of processes (P_i and P_j) will be dispatched from state Q_1. Scheduler comes to Q_4 only if state Q_1 and Q_2 are empty.
- Define $Q_4 = P_w$ is a waiting state.

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- When two or more cooperating process from queue Q_1 and Q_2 are assigned to CPU and executed concurrently then neither of the process may able to execute towards completion as there are some resources that are commonly used by them and required to complete their execution but due to unavailability of these resources all the process from Q_1 and Q_2 are reached to a deadlock situation. Under these circumstances the processes from Q_1 and Q_2 are permanently blocked.
- Define $Q_3 = P_d$ is a deadlock state.

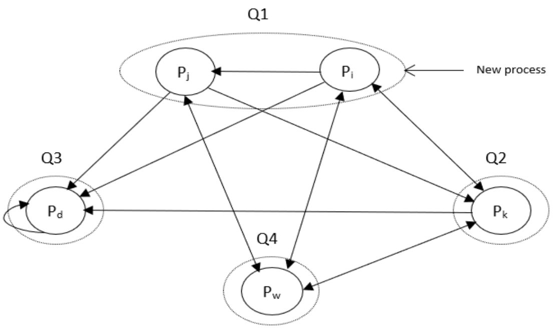

Fig. 4.1: Restricted transition diagram

Thus, the initial probabilities under scheme-I are:

$P\ [\ X^{(0)} = P_i\ ] = 1\ ;\ P\ [\ X^{(0)} = P_j\ ] = 0\ ;\ P\ [\ X^{(0)} = P_k\ ] = 0\ ;$
$P\ [\ X^{(0)} = P_d\ ] = 0\ ;\ P\ [\ X^{(0)} = P_w\ ] = 0$

Unit step transaction probability matrix for $X^{(n)}$ under scheme-I is:

$$P = \begin{array}{c} \\ \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ P_j & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ P_k & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ P_d & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ P_w & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{array}} = \begin{array}{c} \\ \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & S_{12} & S_{13} & S_{14} & S_{15} \\ P_j & 0 & 0 & S_{23} & S_{24} & S_{25} \\ P_k & S_{31} & 0 & 0 & S_{34} & S_{35} \\ P_d & 0 & 0 & 0 & S_{44} & 0 \\ P_w & S_{51} & S_{52} & S_{53} & 0 & 0 \end{array}}$$

By using eq. 2 the state probabilities after the first-time quantum are:

$P\ [\ X^{(1)} = P_i\ ] = 0\ ;\ P\ [\ X^{(1)} = P_j\ ] = S_{12}\ ;\ P\ [\ X^{(1)} = P_k\ ] = S_{13}\ ;\ P\ [\ X^{(1)} = P_d\ ] = S_{14}\ ;\ P\ [\ X^{(1)} = P_w\ ] = S_{15}$

By using eq. 3 the state probabilities after the second time quantum are:

$$P\ [\ X^{(2)} = P_i\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_w\ ]$$
$$P\ [\ X^{(2)} = P_i\ ] = S_{13}\ S_{31} + S_{15}\ S_{51}$$
$$P\ [\ X^{(2)} = P_j\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_w\ ]$$
$$P\ [\ X^{(2)} = P_j\ ] = S_{25}\ S_{52}$$
$$P\ [\ X^{(2)} = P_k\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_w\ ]$$
$$P\ [\ X^{(2)} = P_k\ ] = S_{13}\ S_{31} + S_{35}\ S_{53}$$
$$P\ [\ X^{(2)} = P_d\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_w\ ]$$
$$P\ [\ X^{(2)} = P_d\ ] = S_{44}\ S_{44}$$
$$P\ [\ X^{(2)} = P_w\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_w\ ]$$
$$P\ [\ X^{(2)} = P_w\ ] = S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53}$$

Similarly, third time quantum are:

$$P\ [\ X^{(3)} = P_i\ ] = (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{31} + (\ S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{51}$$
$$P\ [\ X^{(3)} = P_j\ ] = (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{12} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{52}$$
$$P\ [\ X^{(3)} = P_k\ ] = (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{13} + (\ S_{25}\ S_{52})\ S_{23} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{53}$$
$$P\ [\ X^{(3)} = P_d\ ] = (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{14} + (\ S_{25}\ S_{52}\ )\ S_{24} + (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{34} + (S_{44}\ S_{44})\ S_{44}$$
$$P\ [\ X^{(3)} = P_w\ ] = (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{15} + (\ S_{25}\ S_{52})\ S_{25} + (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{35}$$

Similarly, fourth time quantum are:

$$P\ [\ X^{(4)} = P_i\ ] = \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{13} + (\ S_{25}\ S_{52})\ S_{23} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{53}\ \}\ S_{31} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{15} + (\ S_{25}\ S_{52})\ S_{25} + (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{35}\ \}\ S_{51}$$
$$P\ [\ X^{(4)} = P_j\ ] = \{\ (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{31} + (\ S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{51}\ \}\ S_{12} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{15} + (\ S_{25}\ S_{52})\ S_{25} + (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{35}\ \}\ S_{52}$$
$$P\ [\ X^{(4)} = P_k\ ] = \{\ (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{31} + (\ S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{51}\ \}\ S_{13} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{12} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{52}\ \}\ S_{23} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{15} + (\ S_{25}\ S_{52})\ S_{25} + (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{35}\ \}\ S_{53}$$
$$P\ [\ X^{(4)} = P_d\ ] = \{\ (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{31} + (\ S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{51}\ \}\ S_{14} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{12} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{52}\ \}\ S_{24} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{13} + (\ S_{25}\ S_{52})\ S_{23} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{53}\ \}\ S_{34} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{14} + (\ S_{25}\ S_{52}\ )\ S_{24} + (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{34} + (S_{44}\ S_{44})\ S_{44}\ \}\ S_{44}$$
$$P\ [\ X^{(4)} = P_w\ ] = \{\ (S_{13}\ S_{31} + S_{35}\ S_{53})\ S_{31} + (\ S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{51}\ \}\ S_{15} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{12} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{52}\ \}\ S_{25} + \{\ (S_{13}\ S_{31} + S_{15}\ S_{51})\ S_{13} + (\ S_{25}\ S_{52})\ S_{23} + (S_{15}\ S_{51} + S_{25}\ S_{52} + S_{35}\ S_{53})\ S_{53}\ \}\ S_{35}$$

Similarly, we can find fifth, sixth and so on time quantum.

B. *Scheme - II*

If Q_1 is left with a single process, Q_2 will have its turn immediately after the dispatch of the single process from Q_1. Fig 4.2 described the scheme under restriction of some transitions. These restrictions are:

- A new process enters to queue Q_1 only.
- Scheduler cannot jump to Q_4 from Q_1 without passing Q_2.
- If Q_1 is left with a single process, Q_2 will have its turn immediately after the dispatch of the single process from Q_1.

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- Now a process from Q_1 is assigned and executed and it is assumed that this process is not able to complete execution within first time quantum due to the occurrence of any I/O request or any halt condition, so it gets suspended. and then process from Q_2 is assigned and started execution but its execution is also get suspended due to the same reason for which the process from Q_1 was suspended. So, both the processes from Q_1 and Q_2 are not able to execute towards completion and reaches to a deadlock condition.
- Resting of scheduler on Q_4 ends up only if a new process enters in Q_1, otherwise resting continues.

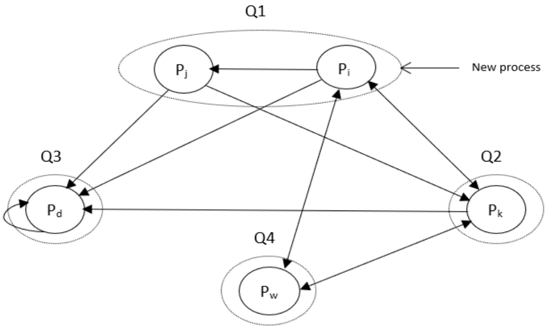

Fig. 4.2: Restricted transition diagram

Thus, the initial probabilities under scheme-II are:

$P [X^{(0)} = P_i] = 1 ; P [X^{(0)} = P_j] = 0 ; P [X^{(0)} = P_k] = 0 ;$
$P [X^{(0)} = P_d] = 0 ; P [X^{(0)} = P_w] = 0$

Unit step transaction probability matrix for $X^{(n)}$ under scheme-II is:

$$P = \overset{\longleftarrow X^{(n)} \longrightarrow}{X^{(n-1)}\updownarrow \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ P_j & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ P_k & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ P_d & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ P_w & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{array}} = \overset{\longleftarrow X^{(n)} \longrightarrow}{X^{(n-1)}\updownarrow \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & S_{12} & S_{13} & S_{14} & S_{15} \\ P_j & 0 & 0 & S_{23} & S_{24} & 0 \\ P_k & S_{31} & 0 & 0 & S_{34} & S_{35} \\ P_d & 0 & 0 & 0 & S_{44} & 0 \\ P_w & S_{51} & 0 & S_{53} & 0 & 0 \end{array}}$$

By using eq. 2 the state probabilities after the first-time quantum are:

$P [X^{(1)} = P_i] = 0 ; P [X^{(1)} = P_j] = S_{12} ; P [X^{(1)} = P_k] = S_{13} ; P [X^{(1)} = P_d] = S_{14} ; P [X^{(1)} = P_w] = S_{15}$

By using eq. 3 the state probabilities after the second time quantum are:

$P [X^{(2)} = P_i] = P [X^{(1)} = P_i] P [X^{(2)} = P_i / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_i / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_i / X^{(1)} = P_k] + P [X^{(1)} = P_d] P [X^{(2)} = P_i / X^{(1)} = P_d] + P [X^{(1)} = P_w] P [X^{(2)} = P_i / X^{(1)} = P_w]$
$P [X^{(2)} = P_i] = S_{13} S_{31} + S_{15} S_{51}$
$P [X^{(2)} = P_j] = P [X^{(1)} = P_i] P [X^{(2)} = P_j / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_j / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_j / X^{(1)} = P_k] + P [X^{(1)} = P_d] P [X^{(2)} = P_j / X^{(1)} = P_d] + P [X^{(1)} = P_w] P [X^{(2)} = P_j / X^{(1)} = P_w]$
$P [X^{(2)} = P_j] = 0$
$P [X^{(2)} = P_k] = P [X^{(1)} = P_i] P [X^{(2)} = P_k / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_k / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_k / X^{(1)} = P_k] + P [X^{(1)} = P_d] P [X^{(2)} = P_k / X^{(1)} = P_d] + P [X^{(1)} = P_w] P [X^{(2)} = P_k / X^{(1)} = P_w]$
$P [X^{(2)} = P_k] = S_{13} S_{31} + S_{35} S_{53}$
$P [X^{(2)} = P_d] = P [X^{(1)} = P_i] P [X^{(2)} = P_d / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_d / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_d / X^{(1)} = P_k] + P [X^{(1)} = P_d] P [X^{(2)} = P_d / X^{(1)} = P_d] + P [X^{(1)} = P_w] P [X^{(2)} = P_d / X^{(1)} = P_w]$
$P [X^{(2)} = P_d] = S_{44} S_{44}$
$P [X^{(2)} = P_w] = P [X^{(1)} = P_i] P [X^{(2)} = P_w / X^{(1)} = P_i] + P [X^{(1)} = P_j] P [X^{(2)} = P_w / X^{(1)} = P_j] + P [X^{(1)} = P_k] P [X^{(2)} = P_w / X^{(1)} = P_k] + P [X^{(1)} = P_d] P [X^{(2)} = P_w / X^{(1)} = P_d] + P [X^{(1)} = P_w] P [X^{(2)} = P_w / X^{(1)} = P_w]$
$P [X^{(2)} = P_w] = S_{15} S_{51} + S_{35} S_{53}$

Similarly, third time quantum are:

$P [X^{(3)} = P_i] = (S_{13} S_{31} + S_{35} S_{53}) S_{31} + (S_{15} S_{51} + S_{35} S_{53}) S_{51}$
$P [X^{(3)} = P_j] = (S_{13} S_{31} + S_{15} S_{51}) S_{12}$
$P [X^{(3)} = P_k] = (S_{13} S_{31} + S_{15} S_{51}) S_{13} + (S_{15} S_{51} + S_{35} S_{53}) S_{53}$
$P [X^{(3)} = P_d] = (S_{13} S_{31} + S_{15} S_{51}) S_{14} + (S_{13} S_{31} + S_{35} S_{53}) S_{34} + (S_{44} S_{44}) S_{44}$
$P [X^{(3)} = P_w] = (S_{13} S_{31} + S_{15} S_{51}) S_{15} + (S_{13} S_{31} + S_{35} S_{53}) S_{35}$

Similarly, fourth time quantum are:

$P [X^{(4)} = P_i] = \{ (S_{13} S_{31} + S_{15} S_{51}) S_{13} + (S_{15} S_{51} + S_{35} S_{53}) S_{53} \} S_{31} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{15} + (S_{13} S_{31} + S_{35} S_{53}) S_{35} \} S_{51}$
$P [X^{(4)} = P_j] = \{ (S_{13} S_{31} + S_{35} S_{53}) S_{31} + (S_{15} S_{51} + S_{35} S_{53}) S_{51} \} S_{12}$
$P [X^{(4)} = P_k] = \{ (S_{13} S_{31} + S_{35} S_{53}) S_{31} + (S_{15} S_{51} + S_{35} S_{53}) S_{51} \} S_{13} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{12} \} S_{23} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{15} + (S_{13} S_{31} + S_{35} S_{53}) S_{35} \} S_{53}$
$P [X^{(4)} = P_d] = \{ (S_{13} S_{31} + S_{35} S_{53}) S_{31} + (S_{15} S_{51} + S_{35} S_{53}) S_{51}\} S_{14} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{12} \} S_{24} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{13} + (S_{15} S_{51} + S_{35} S_{53}) S_{53} \} S_{34} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{14} + (S_{13} S_{31} + S_{35} S_{53}) S_{34} + (S_{44} S_{44}) S_{44}\} S_{44}$
$P [X^{(4)} = P_w] = \{ (S_{13} S_{31} + S_{35} S_{53}) S_{31} + (S_{15} S_{51} + S_{35} S_{53}) S_{51}\} S_{15} + \{ (S_{13} S_{31} + S_{15} S_{51}) S_{15} + (S_{13} S_{31} + S_{35} S_{53}) S_{35} \} S_{35}$

Similarly, we can find fifth, sixth and so on time quantum.

C. *Scheme - III*

If Q_1 is left with no process, Q_2 will function as a single ready queue. The following transition are restricted in this scheme:

- A new process can only enter to Q_2.
- Transition from Q_1 to Q_4 is restricted.
- Now a process from Q_2 is assigned and executed and it is assumed that this process is not able to complete execution within first time quantum due to the occurrence of any I/O request or any halt condition, so it gets suspended. and then process from Q_1 is assigned and started execution but its execution is also get suspended due to the same reason for which the process from Q_2 was suspended. So, both the processes from Q_2 and Q_1

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are not able to execute towards completion and reaches to a deadlock condition.

- Transition must occur in sequence from Q_2 to Q_1, Q_1 to Q_2, Q_1 to Q_4 and then Q_2 to Q_4 to be shown in fig. 4.3.

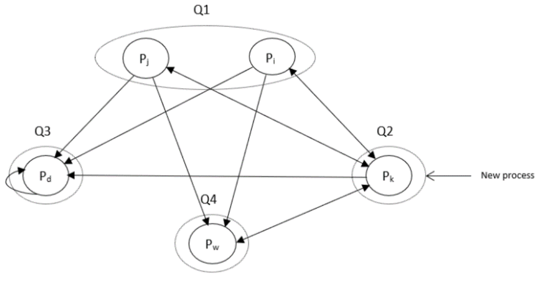

Fig. 4.3: Restricted transition diagram

Thus, the initial probabilities under scheme-III are:

$P\ [\ X^{(0)} = P_i\ ] = 0\ ;\ P\ [\ X^{(0)} = P_j\ ] = 0\ ;\ P\ [\ X^{(0)} = P_k\ ] = 1\ ;$
$P\ [\ X^{(0)} = P_d\ ] = 0\ ;\ P\ [\ X^{(0)} = P_w\ ] = 0$

Unit step transaction probability matrix for $X^{(n)}$ under scheme-III is:

$$P = \begin{array}{c} \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ P_j & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ P_k & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ P_d & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ P_w & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{array}} = \begin{array}{c} \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & S_{13} & S_{14} & S_{15} \\ P_j & 0 & 0 & S_{23} & S_{24} & S_{25} \\ P_k & S_{31} & S_{32} & 0 & S_{34} & S_{35} \\ P_d & 0 & 0 & 0 & S_{44} & 0 \\ P_w & 0 & 0 & S_{53} & 0 & 0 \end{array}}$$

By using eq. 2 the state probabilities after the first-time quantum are:

$P\ [\ X^{(1)} = P_i\ ] = 0\ ;\ P\ [\ X^{(1)} = P_j\ ] = 0\ ;\ P\ [\ X^{(1)} = P_k\ ] = S_{13}\ ;$
$P\ [\ X^{(1)} = P_d\ ] = S_{14}\ ;\ P\ [\ X^{(1)} = P_w\ ] = S_{15}$

By using eq. 3 the state probabilities after the second time quantum are:

$P\ [\ X^{(2)} = P_i\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_i / X^{(1)} = P_w\ ]$
$P\ [\ X^{(2)} = P_i\ ] = S_{13}\ S_{31}$
$P\ [\ X^{(2)} = P_j\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_j / X^{(1)} = P_w\ ]$
$P\ [\ X^{(2)} = P_j\ ] = S_{23}\ S_{32}$
$P\ [\ X^{(2)} = P_k\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_k / X^{(1)} = P_w\ ]$
$P\ [\ X^{(2)} = P_k\ ] = S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53}$
$P\ [\ X^{(2)} = P_d\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_d / X^{(1)} = P_w\ ]$
$P\ [\ X^{(2)} = P_d\ ] = S_{44}\ S_{44}$
$P\ [\ X^{(2)} = P_w\ ] = P\ [\ X^{(1)} = P_i\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_i\ ] + P\ [\ X^{(1)} = P_j\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_j\ ] + P\ [\ X^{(1)} = P_k\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_k\ ] + P\ [\ X^{(1)} = P_d\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_d\ ] + P\ [\ X^{(1)} = P_w\ ]\ P\ [\ X^{(2)} = P_w / X^{(1)} = P_w\ ]$
$P\ [\ X^{(2)} = P_w\ ] = S_{35}\ S_{53}$

Similarly, third time quantum are:

$P\ [\ X^{(3)} = P_i\ ] = (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{31}$
$P\ [\ X^{(3)} = P_j\ ] = (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{32}$
$P\ [\ X^{(3)} = P_k\ ] = (S_{13}\ S_{31})\ S_{13} + (\ S_{23}\ S_{32})\ S_{23} + (S_{35}\ S_{53})\ S_{53}$
$P\ [\ X^{(3)} = P_d\ ] = (S_{13}\ S_{31})\ S_{14} + (\ S_{23}\ S_{32})\ S_{24} + (\ S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{34} + (S_{44}\ S_{44})\ S_{44}$
$P\ [\ X^{(3)} = P_w\ ] = (S_{13}\ S_{31})\ S_{15} + (\ S_{23}\ S_{32})\ S_{25} + (\ S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{35}$

Similarly, fourth time quantum are:

$P\ [\ X^{(4)} = P_i\ ] = \{\ (S_{13}\ S_{31})\ S_{13} + (\ S_{23}\ S_{32})\ S_{23} + (S_{35}\ S_{53})\ S_{53}\}\ S_{31}$
$P\ [\ X^{(4)} = P_j\ ] = \{\ (S_{13}\ S_{31})\ S_{13} + (\ S_{23}\ S_{32})\ S_{23} + (S_{35}\ S_{53})\ S_{53}\}\ S_{32}$
$P\ [\ X^{(4)} = P_k\ ] = \{\ (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{31}\ \}\ S_{13} + \{\ (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{32}\ \}\ S_{23} + \{\ (S_{13}\ S_{31})\ S_{15} + (\ S_{23}\ S_{32})\ S_{25} + (\ S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{35}\}\ S_{53}$
$P\ [\ X^{(4)} = P_d\ ] = \{\ (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{31}\ \}\ S_{14} + \{\ (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{32}\ \}\ S_{24} + \{\ (S_{13}\ S_{31})\ S_{13} + (\ S_{23}\ S_{32})\ S_{23} + (S_{35}\ S_{53})\ S_{53}\ \}\ S_{34} + \{\ (S_{13}\ S_{31})\ S_{14} + (\ S_{23}\ S_{32})\ S_{24} + (\ S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{34} + (S_{44}\ S_{44})\ S_{44}\ \}\ S_{44}$
$P\ [\ X^{(4)} = P_w\ ] = \{\ (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{31}\ \}\ S_{15} + \{\ (S_{13}\ S_{31} + S_{23}\ S_{32} + S_{35}\ S_{53})\ S_{32}\ \}\ S_{25} + \{\ (S_{13}\ S_{31})\ S_{13} + (\ S_{23}\ S_{32})\ S_{23} + (S_{35}\ S_{53})\ S_{53}\ \}\ S_{35}$

Similarly, we can find fifth, sixth and so on time quantum.

V. SIMULATION STUDY WITH NUMERICAL ANALYSIS USING DATA SETS

In order to analyze three schemes mentioned in section 4.1, 4.2 and 4.3 under Markov chain model with unequal and equal transition following different data sets are used:

A. *Data Set – I*

Scheme I: Let initial probabilities are: $Pr_1 = 1;\ Pr_2 = 0\ ;\ Pr_3 = 0\ ;\ Pr_4 = 0$ *and* $Pr_5 = 0$
Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c} \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.28 & 0.26 & 0.07 & 0.39 \\ P_j & 0 & 0 & 0.35 & 0.1 & 0.55 \\ P_k & 0.38 & 0 & 0 & 0.05 & 0.57 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.33 & 0.32 & 0.35 & 0 & 0 \end{array}}$$

Equal

$$P = \begin{array}{c} \uparrow \\ X^{(n-1)} \\ \downarrow \end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.05 & 0.05 & 0.05 & 0.85 \\ P_j & 0 & 0 & 0.05 & 0.05 & 0.9 \\ P_k & 0.05 & 0 & 0 & 0.05 & 0.9 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.05 & 0.05 & 0.9 & 0 & 0 \end{array}}$$

Table 5.1.1: The transition probabilities *P[$X^{(n)}$ = Qi]* for unequal and equal cases:

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Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.28	0.26	0.07	0.39	0	0.05	0.05	0.05	0.85
n = 2	0.2275	0.1248	0.2345	0.111	0.3022	0.045	0.0425	0.7675	0.055	0.09
n = 3	0.1888	0.1604	0.2086	0.1357	0.291	0.0429	0.0068	0.0854	0.0978	0.7673
n = 4	0.1753	0.1459	0.2071	0.1754	0.281	0.0426	0.0405	0.6931	0.1046	0.1194
n = 5	0.1714	0.139	0.195	0.2126	0.2667	0.0406	0.0081	0.1116	0.1434	0.6964
n = 6	0.1621	0.1333	0.1866	0.2482	0.2544	0.0404	0.0369	0.6293	0.1514	0.1422
n = 7	0.1549	0.1268	0.1778	0.2822	0.2429	0.0386	0.0091	0.1318	0.1867	0.6339

Scheme II: Let initial probabilities are: *Pr₁ = 1; Pr₂ = 0 ; Pr₃ = 0 ; Pr₄ = 0 and Pr₅ = 0*

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.3 & 0.25 & 0.08 & 0.37 \\ P_j & 0 & 0 & 0.85 & 0.15 & 0 \\ P_k & 0.34 & 0 & 0 & 0.06 & 0.6 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.62 & 0 & 0.38 & 0 & 0 \end{array}}$$

Equal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.05 & 0.05 & 0.05 & 0.85 \\ P_j & 0 & 0 & 0.05 & 0.95 & 0 \\ P_k & 0.05 & 0 & 0 & 0.05 & 0.9 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.05 & 0 & 0.95 & 0 & 0 \end{array}}$$

Table 5.1.2: The transition probabilities *P[$X^{(n)}$ = Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.3	0.25	0.08	0.37	0	0.05	0.05	0.05	0.85
n = 2	0.3144	0	0.3956	0.14	0.15	0.045	0	0.81	0.1	0.045
n = 3	0.2275	0.0943	0.1356	0.1889	0.3537	0.0428	0.0023	0.045	0.1428	0.7673
n = 4	0.2654	0.0683	0.2714	0.2294	0.1655	0.0406	0.0021	0.7312	0.1494	0.769
n = 5	0.1949	0.0796	0.1873	0.2772	0.261	0.0404	0.002	0.0752	0.19	0.6926
n = 6	0.2255	0.0585	0.2156	0.316	0.1845	0.0384	0.002	0.6601	0.1977	0.102
n = 7	0.1877	0.0677	0.1762	0.3558	0.2128	0.0381	0.002	0.0989	0.2345	0.6267

Scheme III: Let initial probabilities are: *Pr₁ = 0 ; Pr₂ = 0 ; Pr₃ = 1 ; Pr₄ = 0 and Pr₅ = 0*

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.3 & 0.1 & 0.6 \\ P_j & 0 & 0 & 0.26 & 0.06 & 0.68 \\ P_k & 0.4 & 0.35 & 0 & 0.08 & 0.17 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}}$$

Equal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.05 & 0.05 & 0.9 \\ P_j & 0 & 0 & 0.05 & 0.05 & 0.9 \\ P_k & 0.05 & 0.05 & 0 & 0.05 & 0.85 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}}$$

Table 5.1.3: The transition probabilities *P[$X^{(n)}$ = Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0	0.3	0.1	0.6	0	0	0.05	0.05	0.9
n = 2	0.12	0.105	0.6	0.124	0.051	0.0025	0.0025	0.9	0.0525	0.0425
n = 3	0.24	0.21	0.1143	0.1903	0.2454	0.045	0.045	0.04275	0.0978	0.7695
n = 4	0.0457	0.04	0.372	0.236	0.3062	0.0021	0.0021	0.774	0.1044	0.1173
n = 5	0.1488	0.1302	0.3303	0.2727	0.1179	0.0387	0.0387	0.1175	0.1433	0.6617
n = 6	0.1321	0.1156	0.1964	0.3218	0.234	0.0059	0.0059	0.6656	0.153	0.1695
n = 7	0.0786	0.0687	0.3037	0.3577	0.1913	0.0333	0.0333	0.1701	0.1869	0.5764

B. Data Set – II

Scheme I: Let initial probabilities are: *Pr₁ = 1; Pr₂ = 0 ; Pr₃ = 0 ; Pr₄ = 0 and Pr₅ = 0*

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.25 & 0.22 & 0.12 & 0.41 \\ P_j & 0 & 0 & 0.4 & 0.14 & 0.46 \\ P_k & 0.38 & 0 & 0 & 0.11 & 0.51 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.35 & 0.42 & 0.23 & 0 & 0 \end{array}}$$

Equal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.1 & 0.1 & 0.1 & 0.7 \\ P_j & 0 & 0 & 0.1 & 0.1 & 0.8 \\ P_k & 0.1 & 0 & 0 & 0.1 & 0.8 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.1 & 0.15 & 0.8 & 0 & 0 \end{array}}$$

Table 5.2.1: The transition probabilities *P[$X^{(n)}$ = Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.25	0.22	0.12	0.41	0	0.1	0.1	0.1	0.7
n = 2	0.2271	0.1722	0.1943	0.1792	0.2272	0.08	0.07	0.57	0.12	0.16
n = 3	0.1534	0.1522	0.1711	0.2519	0.2714	0.073	0.024	0.143	0.192	0.568
n = 4	0.16	0.1523	0.1571	0.3104	0.2202	0.0711	0.0641	0.4641	0.216	0.1847
n = 5	0.1368	0.1325	0.1468	0.3682	0.2158	0.0649	0.0256	0.1613	0.2759	0.4723
n = 6	0.1313	0.1248	0.1327	0.4193	0.1919	0.0634	0.0537	0.3869	0.3011	0.195
n = 7	0.1176	0.1134	0.1229	0.4671	0.1789	0.0582	0.0259	0.1677	0.3515	0.3969

Scheme II: Let initial probabilities are: *Pr₁ = 1; Pr₂ = 0 ; Pr₃ = 0 ; Pr₄ = 0 and Pr₅ = 0*

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.3 & 0.22 & 0.1 & 0.38 \\ P_j & 0 & 0 & 0.62 & 0.38 & 0 \\ P_k & 0.34 & 0 & 0 & 0.12 & 0.54 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.52 & 0 & 0.48 & 0 & 0 \end{array}}$$

Equal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.1 & 0.1 & 0.1 & 0.7 \\ P_j & 0 & 0 & 0.1 & 0.9 & 0 \\ P_k & 0.1 & 0 & 0 & 0.1 & 0.8 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.1 & 0 & 0.9 & 0 & 0 \end{array}}$$

Table 5.2.2: The transition probabilities *P[$X^{(n)}$ = Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.3	0.22	0.1	0.38	0	0.1	0.1	0.1	0.7
n = 2	0.2724	0	0.3684	0.2404	0.1188	0.08	0	0.64	0.2	0.08
n = 3	0.187	0.0817	0.117	0.3118	0.3024	0.072	0.008	0.08	0.272	0.568
n = 4	0.1971	0.0561	0.2369	0.3756	0.1342	0.0648	0.0072	0.5192	0.2944	0.1144
n = 5	0.1504	0.0591	0.1426	0.4451	0.2028	0.0634	0.0065	0.1102	0.3593	0.4607
n = 6	0.154	0.0451	0.1671	0.4997	0.1342	0.0571	0.0063	0.4216	0.3825	0.1325
n = 7	0.1266	0.0462	0.1263	0.5523	0.1488	0.0554	0.0057	0.1256	0.436	0.3773

Scheme III: Let initial probabilities are: *Pr₁ = 0 ; Pr₂ = 0 ; Pr₃ = 1 ; Pr₄ = 0 and Pr₅ = 0*

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.24 & 0.13 & 0.63 \\ P_j & 0 & 0 & 0.18 & 0.11 & 0.71 \\ P_k & 0.28 & 0.36 & 0 & 0.12 & 0.24 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}}$$

Equal

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.1 & 0.1 & 0.8 \\ P_j & 0 & 0 & 0.1 & 0.1 & 0.8 \\ P_k & 0.1 & 0.1 & 0 & 0.1 & 0.7 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}}$$

Table 5.2.3: The transition probabilities *P[$X^{(n)}$ = Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0	0.24	0.13	0.63	0	0	0.1	0.1	0.8
n = 2	0.0672	0.0864	0.63	0.1588	0.0576	0.01	0.01	0.8	0.11	0.07
n = 3	0.1764	0.2268	0.0893	0.2526	0.2549	0.08	0.08	0.072	0.192	0.576
n = 4	0.025	0.0321	0.3381	0.3112	0.2936	0.0072	0.0072	0.592	0.2152	0.1784
n = 5	0.0947	0.1217	0.3054	0.3586	0.1197	0.0592	0.0592	0.1798	0.2758	0.4259
n = 6	0.0855	0.1099	0.1643	0.4209	0.2194	0.018	0.018	0.4377	0.3056	0.2206
n = 7	0.046	0.0591	0.2597	0.4638	0.1713	0.0438	0.0438	0.2242	0.353	0.3352

C. Data Set – III

Scheme I: Let initial probabilities are: $Pr_1 = 1; Pr_2 = 0 ; Pr_3 = 0 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.2 & 0.24 & 0.16 & 0.4 \\ P_j & 0 & 0 & 0.38 & 0.18 & 0.44 \\ P_k & 0.42 & 0 & 0 & 0.14 & 0.44 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.28 & 0.42 & 0.3 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.15 & 0.15 & 0.15 & 0.55 \\ P_j & 0 & 0 & 0.15 & 0.15 & 0.7 \\ P_k & 0.15 & 0 & 0 & 0.15 & 0.7 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.15 & 0.15 & 0.7 & 0 & 0 \end{array}$$

Table 5.3.1: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.2	0.24	0.16	0.4	0	0.15	0.15	0.15	0.55
n = 2	0.2128	0.168	0.196	0.2296	0.1936	0.105	0.0825	0.4075	0.195	0.21
n = 3	0.1365	0.1239	0.173	0.3213	0.2453	0.0926	0.0473	0.1751	0.2843	0.4008
n = 4	0.1413	0.1303	0.1534	0.3897	0.1852	0.0864	0.07401	0.3015	0.3316	0.2066
n = 5	0.1163	0.106	0.139	0.4572	0.1813	0.0762	0.044	0.1687	0.4009	0.3104
n = 6	0.1092	0.0994	0.1226	0.5143	0.1543	0.0719	0.058	0.2353	0.4442	0.1908
n = 7	0.0947	0.0866	0.1103	0.5668	0.1414	0.0639	0.0394	0.153	0.499	0.2449

Scheme II: Let initial probabilities are: $Pr_1 = 1; Pr_2 = 0 ; Pr_3 = 0 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.22 & 0.28 & 0.12 & 0.38 \\ P_j & 0 & 0 & 0.58 & 0.42 & 0 \\ P_k & 0.19 & 0 & 0 & 0.16 & 0.65 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.62 & 0 & 0.38 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.15 & 0.15 & 0.15 & 0.55 \\ P_j & 0 & 0 & 0.15 & 0.85 & 0 \\ P_k & 0.15 & 0 & 0 & 0.15 & 0.7 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.15 & 0 & 0.85 & 0 & 0 \end{array}$$

Table 5.3.2: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.22	0.28	0.12	0.38	0	0.15	0.15	0.15	0.55
n = 2	0.2888	0	0.272	0.2572	0.182	0.105	0	0.49	0.3	0.105
n = 3	0.1645	0.0635	0.15	0.3354	0.2865	0.0893	0.0158	0.105	0.3893	0.4008
n = 4	0.2062	0.0362	0.1918	0.4058	0.16	0.0759	0.0134	0.3564	0.4319	0.1226
n = 5	0.1356	0.0454	0.1395	0.4764	0.203	0.0719	0.0114	0.1176	0.5081	0.2912
n = 6	0.1524	0.0298	0.1414	0.5341	0.1422	0.0613	0.0108	0.26	0.5462	0.1219
n = 7	0.115	0.0335	0.114	0.5875	0.1498	0.0573	0.0092	0.1144	0.6036	0.2155

Scheme III: Let initial probabilities are: $Pr_1 = 0 ; Pr_2 = 0 ; Pr_3 = 1 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.31 & 0.16 & 0.53 \\ P_j & 0 & 0 & 0.28 & 0.14 & 0.58 \\ P_k & 0.25 & 0.29 & 0 & 0.12 & 0.34 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.15 & 0.15 & 0.7 \\ P_j & 0 & 0 & 0.15 & 0.15 & 0.7 \\ P_k & 0.15 & 0.15 & 0 & 0.15 & 0.55 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Table 5.3.3: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0	0.31	0.16	0.53	0	0	0.15	0.15	0.7
n = 2	0.0775	0.0899	0.53	0.1972	0.1054	0.0225	0.0225	0.7	0.1725	0.0825
n = 3	0.1325	0.1537	0.1546	0.2858	0.2734	0.105	0.105	0.0893	0.2843	0.4165
n = 4	0.0387	0.0448	0.3575	0.3471	0.2119	0.0134	0.0134	0.448	0.3292	0.1961
n = 5	0.0894	0.1037	0.2364	0.4025	0.168	0.0672	0.0672	0.2001	0.4004	0.2652
n = 6	0.0591	0.0686	0.2248	0.4597	0.1879	0.03	0.03	0.2854	0.4506	0.2041
n = 7	0.0562	0.0652	0.2254	0.5057	0.1475	0.0428	0.0428	0.2131	0.5024	0.199

D. Data Set – IV

Scheme I: Let initial probabilities are: $Pr_1 = 1; Pr_2 = 0 ; Pr_3 = 0 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.2 & 0.3 & 0.18 & 0.32 \\ P_j & 0 & 0 & 0.32 & 0.25 & 0.43 \\ P_k & 0.35 & 0 & 0 & 0.35 & 0.3 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.4 & 0.25 & 0.35 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.2 & 0.2 & 0.2 & 0.4 \\ P_j & 0 & 0 & 0.2 & 0.2 & 0.6 \\ P_k & 0.2 & 0 & 0 & 0.2 & 0.6 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.2 & 0.2 & 0.6 & 0 & 0 \end{array}$$

Table 5.4.1: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.2	0.3	0.18	0.32	0	0.2	0.2	0.2	0.4
n = 2	0.233	0.08	0.176	0.335	0.176	0.12	0.08	0.28	0.28	0.24
n = 3	0.132	0.0906	0.1571	0.4585	0.1618	0.104	0.072	0.184	0.376	0.264
n = 4	0.1197	0.0669	0.1252	0.5599	0.1283	0.0896	0.0736	0.1936	0.448	0.1952
n = 5	0.0951	0.056	0.1022	0.642	0.1046	0.0778	0.057	0.1498	0.5194	0.1962
n = 6	0.0776	0.0452	0.0831	0.7089	0.0852	0.0692	0.0548	0.1447	0.5763	0.1552
n = 7	0.0632	0.0368	0.0676	0.7633	0.0692	0.06	0.0449	0.1179	0.63	0.1474

Scheme II: Let initial probabilities are: $Pr_1 = 1; Pr_2 = 0 ; Pr_3 = 0 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.28 & 0.22 & 0.18 & 0.32 \\ P_j & 0 & 0 & 0.35 & 0.65 & 0 \\ P_k & 0.35 & 0 & 0 & 0.35 & 0.3 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.35 & 0 & 0.65 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.2 & 0.2 & 0.2 & 0.4 \\ P_j & 0 & 0 & 0.2 & 0.8 & 0 \\ P_k & 0.2 & 0 & 0 & 0.2 & 0.6 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.2 & 0 & 0.8 & 0 & 0 \end{array}$$

Table 5.4.2: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

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Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.28	0.22	0.18	0.32	0	0.2	0.2	0.2	0.4
n = 2	0.189	0	0.306	0.439	0.066	0.12	0	0.36	0.4	0.12
n = 3	0.1302	0.053	0.0845	0.58	0.1523	0.096	0.024	0.12	0.496	0.264
n = 4	0.0829	0.0365	0.1462	0.6675	0.067	0.0768	0.0192	0.2352	0.5584	0.1104
n = 5	0.0746	0.0232	0.0746	0.7573	0.0704	0.0374	0.0154	0.1075	0.6362	0.1718
n = 6	0.0507	0.0209	0.0703	0.8119	0.0463	0.0559	0.0075	0.148	0.6775	0.0795
n = 7	0.0408	0.0142	0.0485	0.8592	0.0373	0.0455	0.0112	0.0763	0.7243	0.1111

Scheme III: Let initial probabilities are: $Pr_1 = 0$; $Pr_2 = 0$; $Pr_3 = 1$; $Pr_4 = 0$ *and* $Pr_5 = 0$
Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.35 & 0.35 & 0.3 \\ P_j & 0 & 0 & 0.25 & 0.42 & 0.33 \\ P_k & 0.2 & 0.32 & 0 & 0.18 & 0.3 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.2 & 0.2 & 0.6 \\ P_j & 0 & 0 & 0.2 & 0.2 & 0.6 \\ P_k & 0.2 & 0.2 & 0 & 0.2 & 0.4 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Table 5.4.3: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0	0.35	0.35	0.3	0	0	0.2	0.2	0.6
n = 2	0.07	0.112	0.3	0.413	0.105	0.04	0.04	0.6	0.24	0.08
n = 3	0.06	0.096	0.1575	0.5385	0.148	0.12	0.12	0.096	0.376	0.288
n = 4	0.0315	0.0504	0.193	0.6282	0.0969	0.0192	0.0192	0.336	0.3552	0.1824
n = 5	0.0386	0.0618	0.1205	0.6951	0.084	0.0672	0.0672	0.1901	0.4301	0.1574
n = 6	0.0241	0.0386	0.113	0.7563	0.0681	0.038	0.038	0.1843	0.495	0.1567
n = 7	0.0226	0.0362	0.0862	0.8013	0.0539	0.0369	0.0369	0.1719	0.5471	0.1193

E. Data Set – V

Scheme I: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$; $Pr_4 = 0$ *and* $Pr_5 = 0$
Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.28 & 0.32 & 0.22 & 0.18 \\ P_j & 0 & 0 & 0.41 & 0.29 & 0.3 \\ P_k & 0.42 & 0 & 0 & 0.3 & 0.28 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.38 & 0.45 & 0.17 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ P_j & 0 & 0 & 0.25 & 0.25 & 0.5 \\ P_k & 0.25 & 0 & 0 & 0.25 & 0.5 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.25 & 0.25 & 0.5 & 0 & 0 \end{array}$$

Table 5.5.1: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.28	0.32	0.22	0.18	0	0.25	0.25	0.25	0.25
n = 2	0.2028	0.081	0.1454	0.3972	0.1736	0.125	0.0625	0.1875	0.375	0.25
n = 3	0.2004	0.1349	0.1276	0.5089	0.1015	0.1094	0.0938	0.1719	0.4688	0.1563
n = 4	0.0922	0.1018	0.1367	0.6304	0.1123	0.082	0.0664	0.129	0.5626	0.1602
n = 5	0.1001	0.0763	0.0903	0.7212	0.0854	0.0723	0.0606	0.1172	0.632	0.1182
n = 6	0.0704	0.0665	0.0778	0.7924	0.0662	0.0589	0.0476	0.0923	0.6945	0.107
n = 7	0.0578	0.0495	0.061	0.8505	0.0544	0.0498	0.0415	0.0801	0.7442	0.0847

Scheme II: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$; $Pr_4 = 0$ *and* $Pr_5 = 0$
Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.3 & 0.28 & 0.15 & 0.25 \\ P_j & 0 & 0 & 0.4 & 0.6 & 0 \\ P_k & 0.45 & 0 & 0 & 0.25 & 0.3 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.42 & 0 & 0.58 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ P_j & 0 & 0 & 0.25 & 0.75 & 0 \\ P_k & 0.25 & 0 & 0 & 0.25 & 0.5 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.25 & 0 & 0.75 & 0 & 0 \end{array}$$

Table 5.5.2: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.3	0.28	0.15	0.25	0	0.25	0.25	0.25	0.25
n = 2	0.231	0	0.265	0.4	0.084	0.125	0	0.25	0.5	0.125
n = 3	0.1545	0.0693	0.1134	0.5009	0.1373	0.0938	0.0313	0.125	0.5938	0.1563
n = 4	0.1769	0.0464	0.1506	0.594	0.0726	0.0703	0.0235	0.1485	0.672	0.086
n = 5	0.0983	0.0531	0.1102	0.686	0.0894	0.0586	0.0176	0.0879	0.7443	0.0918
n = 6	0.0871	0.0295	0.1006	0.7602	0.0576	0.0449	0.0147	0.0879	0.7941	0.0586
n = 7	0.0695	0.0261	0.0696	0.8161	0.52	0.0366	0.0112	0.0589	0.8383	0.0552

Scheme III: Let initial probabilities are: $Pr_1 = 0$; $Pr_2 = 0$; $Pr_3 = 1$; $Pr_4 = 0$ *and* $Pr_5 = 0$
Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.45 & 0.28 & 0.27 \\ P_j & 0 & 0 & 0.42 & 0.32 & 0.26 \\ P_k & 0.35 & 0.38 & 0 & 0.12 & 0.15 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.25 & 0.25 & 0.5 \\ P_j & 0 & 0 & 0.25 & 0.25 & 0.5 \\ P_k & 0.25 & 0.25 & 0 & 0.25 & 0.25 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Table 5.5.3: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0	0.45	0.28	0.27	0	0	0.25	0.25	0.5
n = 2	0.1575	0.171	0.27	0.334	0.0675	0.0625	0.0625	0.5	0.3125	0.0625
n = 3	0.0945	0.1026	0.2102	0.4652	0.1275	0.125	0.125	0.0938	0.4688	0.1875
n = 4	0.0736	0.0799	0.2131	0.5497	0.0837	0.0235	0.0235	0.25	0.5548	0.1485
n = 5	0.0746	0.081	0.1504	0.6214	0.0726	0.0625	0.0625	0.1603	0.6291	0.086
n = 6	0.0526	0.0572	0.1402	0.6863	0.0638	0.0401	0.0401	0.1173	0.7004	0.1026
n = 7	0.0491	0.0533	0.1115	0.7362	0.0501	0.0293	0.0293	0.1227	0.7498	0.0694

F. Data Set – VI

Scheme I: Let initial probabilities are: $Pr_1 = 1$; $Pr_2 = 0$; $Pr_3 = 0$; $Pr_4 = 0$ *and* $Pr_5 = 0$
Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.3 & 0.37 & 0.18 & 0.15 \\ P_j & 0 & 0 & 0.55 & 0.2 & 0.25 \\ P_k & 0.47 & 0 & 0 & 0.24 & 0.23 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.36 & 0.44 & 0.2 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \updownarrow X^{(n-1)} \quad \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.3 & 0.3 & 0.3 & 0.1 \\ P_j & 0 & 0 & 0.3 & 0.3 & 0.4 \\ P_k & 0.3 & 0 & 0 & 0.3 & 0.4 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.3 & 0.3 & 0.4 & 0 & 0 \end{array}$$

Table 5.6.1: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

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Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.3	0.37	0.18	0.15	0	0.3	0.3	0.3	0.1
n = 2	0.2279	0.066	0.195	0.3288	0.1601	0.12	0.03	0.13	0.48	0.24
n = 3	0.1493	0.1388	0.1524	0.4298	0.0955	0.111	0.108	0.141	0.564	0.076
n = 4	0.106	0.0868	0.1507	0.521	0.0921	0.0651	0.0561	0.0961	0.672	0.1107
n = 5	0.104	0.0723	0.1054	0.5936	0.0723	0.062	0.0527	0.0806	0.7372	0.0674
n = 6	0.0756	0.063	0.0927	0.6521	0.0579	0.0444	0.0388	0.0614	0.7958	0.0595
n = 7	0.0644	0.0482	0.0742	0.7006	0.0484	0.0554	0.0312	0.0488	0.8392	0.0445

Scheme II: Let initial probabilities are: $Pr_1 = 1; Pr_2 = 0 ; Pr_3 = 0 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.36 & 0.32 & 0.12 & 0.2 \\ P_j & 0 & 0 & 0.48 & 0.58 & 0 \\ P_k & 0.51 & 0 & 0 & 0.18 & 0.31 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.48 & 0 & 0.52 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0.3 & 0.3 & 0.3 & 0.1 \\ P_j & 0 & 0 & 0.3 & 0.7 & 0 \\ P_k & 0.3 & 0 & 0 & 0.3 & 0.4 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0.3 & 0 & 0.7 & 0 & 0 \end{array}$$

Table 5.6.2: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0.36	0.32	0.12	0.2	0	0.3	0.3	0.3	0.1
n = 2	0.2592	0	0.2768	0.3648	0.0992	0.12	0	0.16	0.6	0.12
n = 3	0.1888	0.0933	0.1345	0.4457	0.1376	0.084	0.036	0.12	0.684	0.076
n = 4	0.1347	0.0679	0.1768	0.5411	0.0795	0.0588	0.0252	0.0892	0.7704	0.0564
n = 5	0.1283	0.0485	0.117	0.6244	0.0817	0.0437	0.0176	0.0647	0.8324	0.0416
n = 6	0.0989	0.0462	0.1068	0.6861	0.0619	0.0319	0.0131	0.0475	0.8772	0.0303
n = 7	0.0842	0.0356	0.086	0.7412	0.0529	0.0233	0.0096	0.0347	0.9102	0.0222

Scheme III: Let initial probabilities are: $Pr_1 = 0 ; Pr_2 = 0 ; Pr_3 = 1 ; Pr_4 = 0$ *and* $Pr_5 = 0$

Consider data set of unequal and equal probabilities matrix are follows:

Unequal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.52 & 0.2 & 0.28 \\ P_j & 0 & 0 & 0.49 & 0.26 & 0.25 \\ P_k & 0.4 & 0.34 & 0 & 0.1 & 0.16 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Equal

$$\longleftarrow X^{(n)} \longrightarrow$$

$$P = \begin{array}{c}\uparrow\\ X^{(n-1)}\\ \downarrow\end{array} \begin{array}{c|ccccc} & P_i & P_j & P_k & P_d & P_w \\ \hline P_i & 0 & 0 & 0.3 & 0.3 & 0.4 \\ P_j & 0 & 0 & 0.3 & 0.3 & 0.4 \\ P_k & 0.3 & 0.3 & 0 & 0.3 & 0.1 \\ P_d & 0 & 0 & 0 & 1.0 & 0 \\ P_w & 0 & 0 & 1.0 & 0 & 0 \end{array}$$

Table 5.6.3: The transition probabilities *P[* $X^{(n)}$ *= Qi]* for unequal and equal cases:

Quantum No.	Unequal					Equal				
	P_i	P_j	P_k	P_d	P_w	P_i	P_j	P_k	P_d	P_w
n = 1	0	0	0.52	0.2	0.28	0	0	0.3	0.3	0.4
n = 2	0.208	0.1768	0.28	0.252	0.0832	0.09	0.09	0.4	0.39	0.03
n = 3	0.112	0.098	0.278	0.3676	0.1472	0.12	0.12	0.084	0.564	0.112
n = 4	0.1112	0.0945	0.2535	0.4433	0.1003	0.0252	0.0252	0.184	0.6612	0.1044
n = 5	0.1014	0.0862	0.2044	0.5155	0.0953	0.0552	0.0552	0.1195	0.7315	0.0386
n = 6	0.0818	0.0695	0.1903	0.5786	0.0826	0.0359	0.0359	0.0717	0.8005	0.0561
n = 7	0.0761	0.0647	0.1592	0.6321	0.0707	0.0215	0.0215	0.0776	0.8435	0.0359

VI. GRAPHICAL ANALYSIS

Graphical analysis is performed under above mentioned schemes in section 5.1, 5.2, 5.3, 5.4, 5.5 and 5.6 with different data sets considering unequal and equal probability matrix by gradually increasing the various quantum values. This analytical presentation on graphs about the variation *P [* $X^{(n)}$ *= Qi]* over six different lower and upper data sets are as follows:

A. Data Set – I

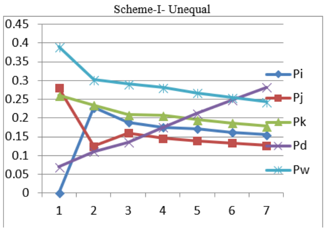

Fig. 6.1

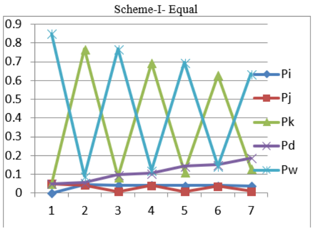

Fig. 6.4

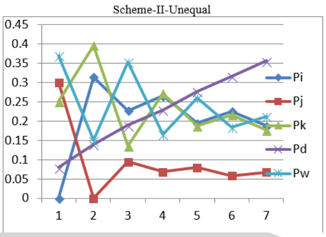

Fig. 6.2

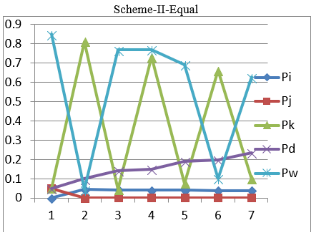

Fig. 6.5

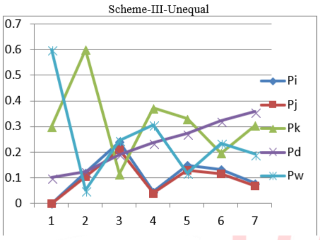

Fig. 6.3

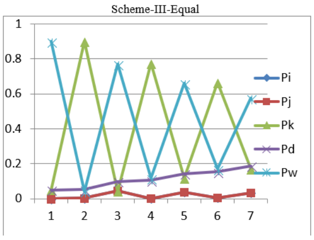

Fig. 6.6

Remark: In data set – I, we observed that, the transition states pattern in these graphs are identical and the probability of scheduler in the absorbing (deadlock) state is very low value as compare to other states, that means, the probability of reaching a deadlock state is getting low that increases the performance of the scheduler. The special remark for this process scheduling is that probability for the state P_k is very high. Therefore, there are more chance for jobs contained in state P_k to be processed than P_i and P_j.

B. Data Set – II

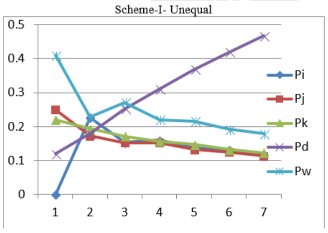

Fig. 6.7

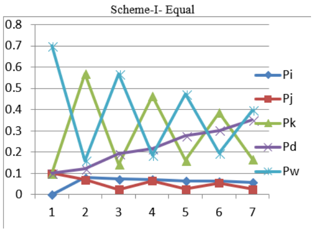

Fig. 6.10

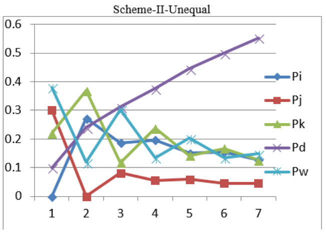

Fig. 6.8

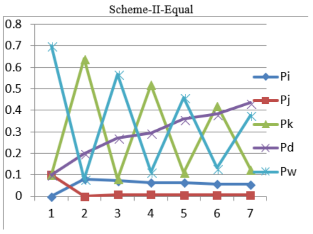

Fig. 6.11

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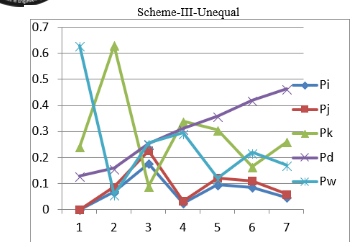

Fig. 6.9

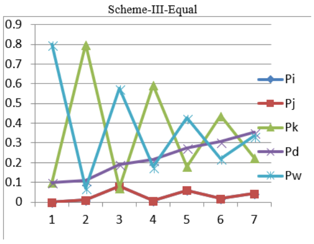

Fig. 6.12

Remark: In data set – II, we observed that, the probability of scheduler in the absorbing (deadlock) state is slightly high value as compare to other states over different quantum which is a sign of increasing the performance efficiency of the CPU scheduler in the data sets. The probability of state P_k is higher than the other data sets. Most of the transition probabilities are almost equal in fig. 6.10, 6.11 and 6.12. Therefore, this data set provides chance for job processing in deadlock state.

C. *Data Set – III*

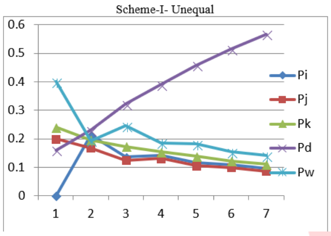

Fig. 6.13

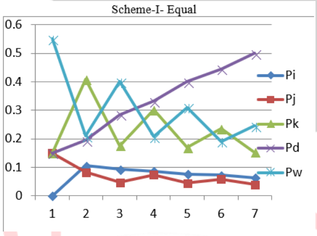

Fig. 6.16

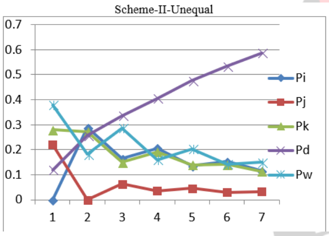

Fig. 6.14

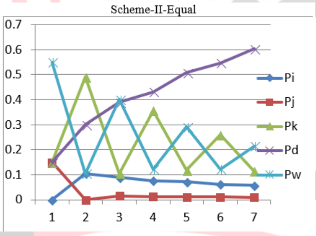

Fig. 6.17

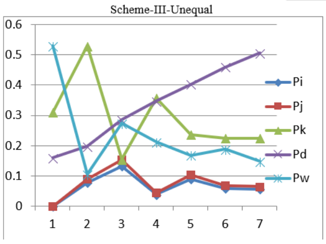

Fig. 6.15

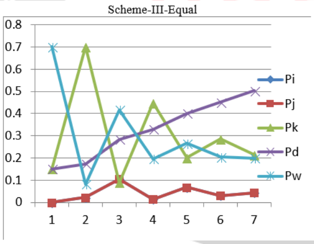

Fig. 6.18

Remark: In data set – III, we observed that, the graphical pattern (fig. 6.13, 6.14 and 6.15) state probability for unequal data set the absorbing (deadlock) state has higher chance of receiving the scheduler as compare to other working state (P_i P_j and P_k) and waiting state (P_w), the graphical pattern (fig. 6.16, 6.17 and 6.18) state probability for equal data set initially $n \geq 4$, we find equal chances of receiving the scheduler for all the queues. Therefore, it increased the performance of scheduler.

D. *Data Set – IV*

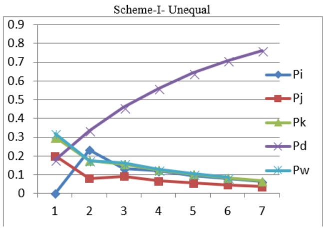

Fig. 6.19

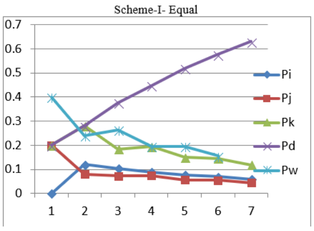

Fig. 6.22

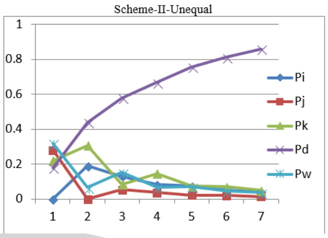

Fig. 6.20

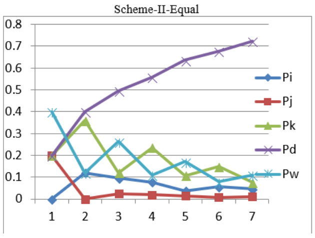

Fig. 6.23

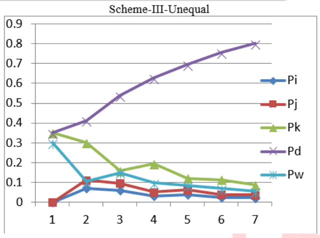

Fig. 6.21

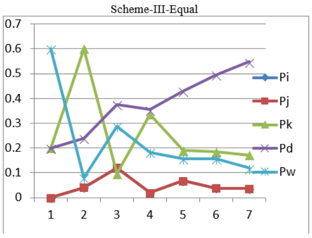

Fig. 6.24

Remark: In data set – IV, we observed that, the probability of system moving to absorbing state (deadlock state) is high. As no of quantum $n \geq 4$, reflect changing in pattern and the probability of working state (P_i , P_j and P_k) and including waiting state (P_w) are flying comparatively high. Therefore, we find that equal chance of receiving the job of scheduler.

E. *Data Set – V*

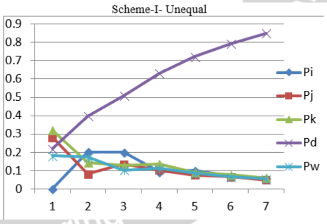

Fig. 6.25

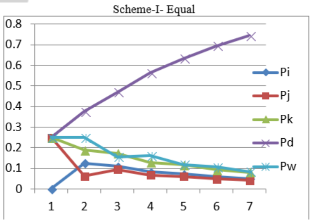

Fig. 6.28

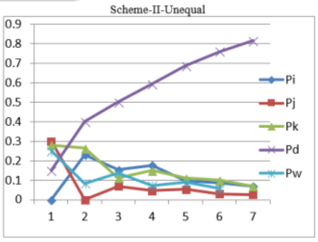

Fig. 6.26

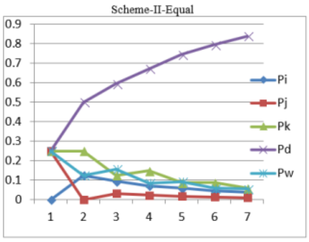

Fig. 6.29

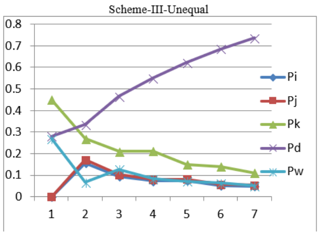

Fig. 6.27

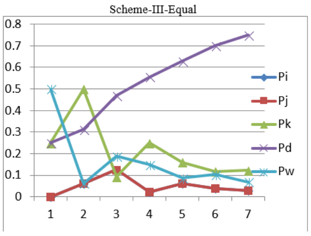

Fig. 6.30

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Remark: In data set – V, we observed that, the probability of system moving to absorbing (deadlock) state is very high. As number of quantum $n \geq 3$, reflect changing in pattern. The remarkable point is that the probability of deadlock state (P_d) is higher than other state (P_i , P_j , P_k and P_w). This shows a loss of efficiency so that scheduler spend more time on the deadlock state than working state.

F. *Data Set – VI*

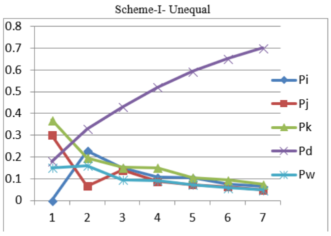

Fig. 6.31

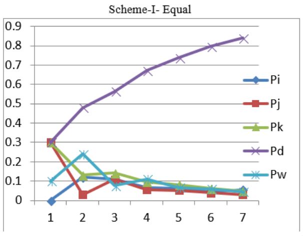

Fig. 6.34

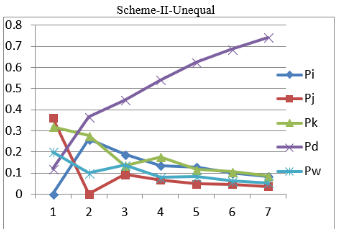

Fig. 6.32

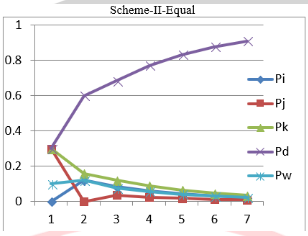

Fig. 6.35

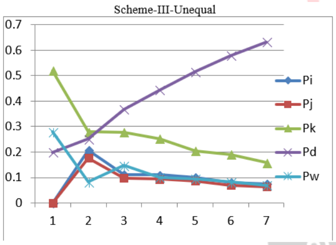

Fig. 6.33

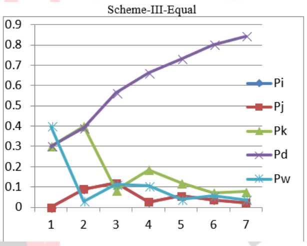

Fig. 6.36

Remark: In data set – VI, we observed that, the probability of system moving to absorbing (deadlock) state is very high. As number of quantum $n = 1, 2, 3, \ldots$, increase probability of P_i , P_j , P_k and P_w state constantly reduces that can see in fig. 6.31, 6.32, 6.33, 6.34, 6.35 and 6.36. At the same time chances for system shifting to deadlock state little high. This shows a loss of efficiency so that scheduler spend more time on the deadlock state than working state.

VII. CONCLUSION

In this paper, we evaluated the performance of improved round robin scheduling algorithm by introducing deadlock condition and did comparative analysis of three schemes using Markov chain model and analyzed unequal and equal probability matrix with number of data sets which have functions of restriction in terms of some state transition that effect of absorbing (deadlock) state.

In the initial probability of the transition state when we used lower values than we got a stable pattern of probability of variation over quantum almost in all the three data sets (data set-I, II and III). As we decreased the initial probabilities in terms of quantum then the probability of occurrence of deadlock also decreased proportionally. Therefore, there are more chance to get execute for jobs contained in state Q_1 and Q_2. Further, the transition state for higher value probability lead to quantum independency and the information overlapping in data sets (data set-IV, V and VI), which indicates a loss of system efficiency and serious degradation in performance of deadlock analysis of IRR scheduling algorithm. Therefore, data sets (data set-IV, V and VI) are not recommended for efficient utilization.

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