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K-Normal Centrosymmetric Matrix

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Abstract: The basic concepts and theorems of K-Normal Centrosymmetric matrices are introduced with examples

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I. INTRODUCTION

Centrosymmetric matrix have practical applications in information theory ,linear system theory, linear estimate theory and numerical analysis(sec[1-3]). The concept of normal was introduced as a generalization of Hermitian matrices. Therefore the class of normal matrices includes the class of all Hermitian matrices. The class of normal matrices is important throughout matrix analysis. The K-Normal matrix have been discussed in sec[4-5].

In this paper we will discuss about the basic properties and theorems on the K-Normal Centro symmetric matrices, also we will discuss some results on centrosymmetric matrices.

Let C^{nxn} denote the set of all nxn complex matrix. A is K-Normal centrosymmetric matrix. A^* is calles conjugate transpose of A . Let K be a fixed product of disjoint transposition in S_n and K be the permutation matrix associated with K. Clearly K satisfies the following properties.

K^2= I, K^T=K.

II. DEFINITIONS AND THEOREMS.

Definition:1

A square matrix which is symmetric about the centre of its array of elements is called centrosymmetric thus $C=[a_{ij}]_{nxn}$ centrosymmetric if,

$$a_{ij}= a_{n-i+1,n-j+1}.$$

Definition:2

A centrosymmetric matrix $A\in C^{nxn}$ is said to be normal centrosymmetric If $AA^* = A^*A$.

Definition:3

A centrosymmetric matrix $A\in C^{nxn}$ is said to be K-normal centrosymmetric matrix If $AA^*K = KA^*A$.

Theorem:1

Let A, $B\in C^{nxn}$ are K-normal centrosymmetric matrix, then A±B is also K-normal centrosymmetric matrix.

Proof:

Let A, B are K-normal centrosymmetric matrix, Then
$AA^*K=KA^*A$; $BB^*K=KB^*B$

To prove: A±B is K-normal centrosymmetric matrix.
We will show that,

$$(A\pm B)(A\pm B)^*K=K(A\pm B)^*(A\pm B)$$

Now,

$$(A\pm B)(A\pm B)^*K=(A\pm B)(A^*\pm B^*)K$$

$$=(A\pm B)(A^*K\pm B^*K)$$

$$(A\pm B)^*K = K(A\pm B)^*(A\pm B).$$

Theorem:2

Let A, $B\in C^{nxn}$ are K-normal centrosymmetric matrices, and AB=BA,then AB is also K-normal centrosymmentric matrix.

Proof:

Let A,B are K-normal centrosymmetric matrix, Then
$AA^*K=KA^*A$; $BB^*K=KB^*B$
Given AB=BA.

To prove: AB is k-normal centrosymmentric matrix.
We will show that

$$(AB)(AB)^*K = K(AB)^*(AB)$$

Now, $(AB)(AB)^*K = ABA^*B^*K$

$$= BAA^*B^*K$$

$$= KA^*B^*BA$$

$$= K(AB)^*(AB).$$

Theorem:3

Let A, $B\in C^{nxn}$ are K-normal centrosymmentric matrix. and AB=BA. then AB^* is also K-normal centrosymmetric matrix.

Proof:

Let A, B are K-normal centrosymmetric matrices.
Then $AA^*K=KA^*A$ and $BB^*K=KB^*B$.
Given AB=BA.

To prove: AB^* is k-normal centrosymmetric matrix.
We will show that,

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$(AB^*)(AB^*)^*K = K(AB^*)^* (AB^*)$

Now,

$(AB)(AB^*)K = K(AB)^* (AB)$

$ABB^*A^*K = K(BA)^* (BA)$

$(AB^*) (AB^*)^*K = K(AB^*)^* (AB^*)$

Theorem:4

Let $A \in C^{nxn}$ be K-normal centrosymmetric matrix, then

i) iA is K-normal centrosymmetric matrix.

ii)-iA is K-normal centrosymmetric matrix.

Proof:

Let A be K-normal centrosymmetric matrix.

Then $AA^*K=KA^*A$.

To prove: i) iA is k-normal centrosymmetric matrix.

We will show that, $(iA) (iA)^*K=K(iA)^* (iA)$

Now,

$AA^*K = KA^*A$

$-i^2AA^*K = -i^2 KA^*A$

$(iA)(-i) A^*K = K(-i) A^* (iA)$

$(iA) (iA)^*K = K(iA)^* (iA)$

$\therefore$ iA is also K-normal centro symmetric matrix.

ii) -iA is K-normal centro symmetric matrix.

We will show that,

$(-iA) (-iA)^*K = K (-iA)^* (iA)$

Now,

$AA^*K = KA^*A$ $-i^2AA^*K = -i^2KA^*A$

$(-i)\ i AA^*K = K(i)(-i) A^*A$

$(-iA) (-iA)^*K = K (-iA)^* (-iA)$

$\therefore$ -iA is K-normal centrosymmetric matrix.

Theorem:5

Let $A \in C^{nxn}$ and A^+ be the moore penrose inverse of A, then A is K-normal centrosymmetric matrix, Iff A^+ is K-normal cenrosymmetric matrix.

Proof:

Let A be a K-normal centrosymmetric matrix.

Then $AA^*K=KA^*A$

To prove: A^+ is k-normal centrosymmetric matrix.

We will show that,

$(A^+) (A^+)^*K = K(A^+)^* (A^+)$

Now, $AA^*K = KA^*A$

$A^+(A^+)^*K = K(A^+)^*A^+$

$\therefore A^+$ is K-normal centrosymmetric matrix.

Let us assume that, A^+ is K-normal centrosymmetric matrix.

we will show that, $AA^*K=KA^*A$

Now, $(A^+) (A^+)^*K = K(A^+)^*(A^+)$

$AA^*K = KA^*A$

$\therefore$ A is K-normal centrosymmetric matrix.

III. Result

Let $A \in C^{nxn}$ are K-normal centrosymmetric matrix,then

i) $\overline{A}$ is K-normal centrosymmetric matrix.

ii) A^T is K-normal centrosymmetric matrix.

iii) A^* is K-normal centrosymmtric matrix.

Example:

1.Let $A=\begin{bmatrix} 1 & 2 & i \\ 2 & -i & 2 \\ i & 2 & 1 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 1 & i \\ 1 & -i & 1 \\ i & 1 & 0 \end{bmatrix}$

$k = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ then

$(A\pm B)(A\pm B)^*K= \begin{bmatrix} 5 & 3 & 6 \\ 3 & 1 & 3 \\ 6 & 3 & 5 \end{bmatrix}$

$=K(A\pm B)^*(A\pm B)$.

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