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On the Homogeneous Bi-Quadratic Equation with Five Unknowns

$$[2k(x^2+y^2)-(4k-1)xy](x^2-y^2)=8k(z^2-w^2)T^2$$

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Abstract - In this paper, the biquadratic equation with five unknowns given by $[2k(x^2+y^2)-(4k-1)xy](x^2-y^2)=8k(z^2-w^2)T^2$ is examined for its framework of non-zero well defined integer solutions. A few excitements connection between the solutions and especially a closed plane figure numbers are showed.

Keywords - Quadratic equation, Integral solutions, special Polygonal numbers, Pyramidal numbers.

I. INTRODUCTION

Biquadratic Diophantine equations, uniform dimensions and not of the same dimensions, have evoked the involvement of innumerable Mathematicians since ancient times as can seen from [1-8]. In the conditions one may refer [9-15] for various of problems on the Diophantine equations with at most four variables. This keeping in touch deals with the problems to control non-zero integer solutions of until now another biquadratic equation in 5 unknowns represented by $[2k(x^2+y^2)-(4k-1)xy](x^2-y^2)=8k(z^2-w^2)T^2$. A few excitements connection between the solutions and especially a closed plane figure numbers are showed.

II. NOTATIONS USED

- $T_{m,n}$ - Polygonal number of rank n with size m.
- P_m^n - Pyramidal number of rank n with size m.
- Pr_n - Pronic number of rank n.
- $CP_{m,n}$ - Centered Pyramidal number of rank n with size m.
- Ky_n - Keynea number of rank n.
- PP_n - Pentagonal Pyramidal number of rank n.

III. METHOD OF ANALYSIS

The Diophantine equation instead of the biquadratic equation with five unknowns under concern is

$$[2k(x^2+y^2)-(4k-1)xy](x^2-y^2)=8k(z^2-w^2)T^2 \tag{1}$$

The replacement of the linear transformations

x=u+v, y=u-v, z=uv+1,w=uv-1 (2)

in (1) leads to $u^2+(8k-1)v^2=8kT^2$ (3)

Different model of solutions of (1) are offered below

Pattern-1

Write (3) as $u^2-v^2=8k(T^2-v^2)$ (4)

Choice: I

Which implies $\frac{u+v}{k(T+v)}=\frac{4(T-v)}{u-v}=\frac{p}{q}, q\neq 0$ (5)

Using the system of cross ratio, we get

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$$u = u(k,p,q) = -kp^2 - 8q^2 + 16kpq$$
$$v = v(k,p,q) = -kp^2 + 8q^2$$
$$T = T(k,p,q) = kp^2 + 8q^2 - 2pq$$

Hence in view of (2) the related solutions of (1) are

$$x = x(k,p,q) = -2kp^2 + 16kpq$$
$$y = y(k,p,q) = -16q^2 + 16kpq$$
$$z = z(k,p,q) = k^2p^4 - 16k^2p^3q + 128kpq^3 - 64q^4 + 1$$
$$w = w(k,p,q) = k^2p^4 - 16k^2p^3q + 128kpq^3 - 64q^4 - 1$$
$$T = T(k,p,q) = kp^2 + 8q^2 - 2pq$$

A few mathematical examples are presented in the table below:

p	q	x	y	z	w	T
1	3	$46k$	$-144+48k$	$-47k^2+3456k-5183$	$-47k^2+3456k-5185$	$k+66$
2	1	$24k$	$-16+32k$	$-112k^2+256k-63$	$-112k^2+256k-65$	$4k+4$
1	2	$30k$	$-64+32k$	$-31k^2+1024k-1023$	$-31k^2+1024k-1025$	$k+28$
2	2	$56k$	$-64+64k$	$-240k^2+2048k-1023$	$-240k^2+2048k-1025$	$4k+24$
1	1	$14k$	$-16+16k$	$-15k^2+128k-63$	$-15k^2+128k-65$	$k+6$

From the table it is experimental that $x^2 + w^2 = y^2 + z^2$

And by definition, the numbers $2209k^4 - 324864k^3 + 12433442k^2 - 35838720k + 26884225$,
$12544k^4 - 57344k^3 + 80672k^2 - 33280k + 4225$,
$961k^4 - 63488k^3 + 1113026k^2 - 2099200k + 1050625$,
$57600k^4 - 983040k^3 + 4689440k^2 - 4198400k + 1050625$,
and $225k^4 - 3840k^3 + 18530k^2 - 16640k + 4225$
represent the second order Ramanujan numbers.
Thus, one may well obtain considerably many second orders Ramanujan numbers.

A small number of fascinating properties experimental are as follows:

1. $z(q,1,q) - 16T(q,1,q^2) + 4x(q,1,q^3) + y(q,q,q) - S_q - 11t_{4,q} \equiv 0 \pmod{18}$
2. $x(p,p,-p) - y(p,p,-p) - 2T(p,p,-p) + 8PP_p = 0$
3. $z(k,1,k) - 3x(k,1,k^2) + 4y(k,1,k^2) - t_{4,k} - 2gn_k = 3 \pmod 2$
4. $x(k,p,-1) - y(k,p,-1) + 2T(k,p,-1) = 32 \pmod 4$
5. $x(1,p,1) + y(1,p,1) + 2t_{4,p} = -16 \pmod{32}$
6. $x(1,p,-1) + y(1,p,-1) + 32Obl_p = -16 \pmod{32}$
7. $x(k,2q^2+1,q) + 2T(k,2q^2+1,q) - 12(4k\,OH_q - OH_q) - t_{4,4q} = 0$
8. $x(k,p,p+1) + y(k,p,p+1) + 2T(k,p,p+1) = 4(8k\Pr_p - \Pr_p)$
9. $y(16p^2,p,p^2) - x(16p^2,p,p^2)$ is a biquadratic integer.
10. Each of the following represents a nasty number:

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$$a)\ 6\{x(k,1,k^2)+4y(k,1,k^2)+z(k,1,k)\}$$
$$b)\ x(k,q,q)-y(k,q,q)+2T(k,q,q)$$
$$c)\ 6\{y(1,p,-p)+3T(1,p,-p)\}$$

Choice: II

Following a related process as in Choice I, the solutions of (1) are

$$x = x(k,p,q) = -2p^2 + 16kpq$$
$$y = y(k,p,q) = -16kq^2 + 16kpq$$
$$z = z(k,p,q) = p^4 - 16kp^3q + 128k^2pq^3 - 64k^2q^4 + 1$$
$$w = w(k,p,q) = p^4 - 16kp^3q + 128k^2pq^3 - 64k^2q^4 - 1$$
$$T = T(k,p,q) = p^2 + 8kq^2 - 2pq$$

Properties

1. $x(1,-p,p+1)+y(1,-p,p+1)+2T(1,-p,p+1)+z(k,p,q)-w(k,p,q)+Ct_{56,p}=1$
2. $-4\{x(q^2,q,q)+y(q^2,q,q)\}-w(1,1,q)-8\{t_{4,q}-6CP_{16,n}\}\equiv 0 \pmod{64}$
3. $-8\{x[k^2(2k^2-1),1,2k^2-1]-y[k^2(2k^2-1),1,2k^2-1]-T[k^2(2k^2-1)^2,1,2k^2-1]\}+w[k,1,2k^2-1]-4=4gn_{k^2+2}-32SO_k$
4. $y(q+1,2q,q)+T(q+1,2q,q)=48P_q^5$
5. $x(k,p,-1)-y(k,p,-1)-2T(k,p,-1)+4\Pr_p=0$
6. $x(2,4p^2,p^2)-y(2,4p^2,p^2)=0$

For the other choices of (4) are given below

Choice: III

Following a like method as in choice I, the solutions of (1) are

$$x = x(k,p,q) = -4kp^2 + 16kpq$$
$$y = y(k,p,q) = -8q^2 + 16kpq$$
$$z = z(k,p,q) = 4k^2p^4 - 32k^2p^3q + 64kpq^3 - 16q^4 + 1$$
$$w = w(k,p,q) = 4k^2p^4 - 32k^2p^3q + 64kpq^3 - 16q^4 - 1$$
$$T = T(k,p,q) = 2kp^2 + 4q^2 - 2pq$$

Choice: IV

Following a parallel method as in choice I, the solutions of (1) are

$$x = x(k,p,q) = -8p^2 + 16kpq$$
$$y = y(k,p,q) = -4kq^2 + 16kpq$$
$$z = z(k,p,q) = 16p^4 - 64kp^3q + 32k^2pq^3 - 4k^2q^4 + 1$$
$$w = w(k,p,q) = 16p^4 - 64kp^3q + 32k^2pq^3 - 4k^2q^4 - 1$$
$$T = T(k,p,q) = 2kp^2 + 4q^2 - 2pq$$

Pattern-2

Assume $T = p^2 + (8k-1)q^2$ (6)

where p and q are different integers.

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$$write\ 8k\ as \quad 8k = (1+i\sqrt{8k-1})\ (1-i\sqrt{8k-1}) \qquad (7)$$

Using (6) and (7) in (3) and employing the technique of factorization, describe

$$(u+i\sqrt{8k-1}\ v) = (1+i\sqrt{8k-1}\ )\ (p+i\sqrt{8k-1}\ q)^2$$

Equating the real and imaginary parts, we have

$$u = u(k,p,q) = p^2 - (8k-1)q^2 + 2(8k-1)pq$$

$$v = v(k,p,q) = p^2 - (8k-1)q^2 + 2pq$$

Hence in observation of (2), the related solutions of (1)are

$$x = x(k,p,q) = 2p^2 - 2(8k-1)q^2 + 2(2-8k)pq$$

$$y = y(k,p,q) = -16kpq$$

$$z = z(k,p,q) = p^4 + 2(2-8k)p^3q - 6(8k-1)p^2q^2 + 2(8k-2)(8k-1)pq^3 + (8k-1)^2q^4 + 1$$

$$w = w(k,p,q) = p^4 + 2(2-8k)p^3q - 6(8k-1)p^2q^2 + 2(8k-2)(8k-1)pq^3 + (8k-1)^2q^4 - 1$$

$$T = T(k,p,q) = p^2 + (8k-1)q^2$$

Properties

1) $8w(k,1,q) + y[(8k-1)(8k-2),1,q^3] - 8T(k,(8k-1)q^2,q) = 32(1-4k)\Pr_q - 99k(gn_{q^2} - 1)$

2) $x(1,p,-1) + 10T(1,p,-1) = 12\Pr_p + J_7 + 13$

3) $x(1,p,-p) + T(1,p,-p) = 8t_{4,p}$

4) $8x(k,p^2,q) + y(2-8k,p^2,q) + 16T(k,p^2,q) = 32t_{4,p^2}$

Pattern-3

From (3)

$$u^2 = 8kT^2 - (8K-1)v^2 \qquad (8)$$

Introducing the linear transformations

$$T = P \pm (8k-1)Q$$

$$v = P \pm 8kQ \qquad (9)$$

In (8), it simplifies to

$$P^2 = 8k(8k-1)Q^2 + u^2$$

Whose solution is given by

$$u = u(k,r,s) = 8k(8k-1)r^2 - s^2$$

$$P = P(k,r,s) = 8k(8k-1)r^2 - s^2$$

$$Q = Q(k,r,s = 2rs)$$

Using the values of P and Q in (9) and taking (2), the related solutions of (1) are given by

$$x = x(k,r,s) = 16k(8k-1)r^2 + 16krs$$

$$y = y(k,r,s) = -2s^2 - 16krs$$

$$z = z(k,r,s) = 64k^2(8k-1)^2r^4 + 128k^2r^3s - 16krs^3 + 1$$

$$w = w(k,r,s) = 64k^2(8k-1)^2r^4 + 128k^2r^3s - 16krs^3 - 1$$

$$T = T(k,r,s) = 8k(8k-1)r^2 + 2(8k-1)rs + s^2$$

IV. CONCLUSION

In this paper, we have offered dissimilar choices of numeral solutions to uniform biquadratic equation with five unknowns,

$$[2k(x^2+y^2) - (4k-1)xy](x^2-y^2) = 8k(z^2-w^2)T^2$$

.To finish as biquadratic equations are well-to-do in multiplicity; individual possibly will think about other forms of biquadratic equations and look for equivalent properties

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