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A Comparison of Software Reliability Growth Models Based on Kumaraswamy Modified Inverse Weibull Distribution and Lehmann-Type Laplace Distribution-Type II

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Abstract - Kumaraswamy Modified Inverse Weibull distribution and Lehmann -Type Laplace distribution-Type II based software reliability growth models are framed in this paper, for early detection of software failure based on time between failure observations. A set of software failure data is assumed to follow Kumaraswamy Modified Inverse Weibull distribution and Lehmann-Type Laplace distribution-Type II. Unconstrained optimization technique is used to estimate the parameters of Kumaraswamy Modified Inverse Weibull distribution and the parameters of Lehmann-Type Laplace distribution-Type II are estimated by Profile Likelihood Method. It is proved that Lehmann-Type Laplace distribution-Type II fits better for the software failure data than the Kumaraswamy Modified Inverse Weibull distribution by using AIC and BIC techniques. Kumaraswamy Modified Inverse Weibull distribution and Lehmann-Type Laplace distribution-Type II based control mechanisms are framed to detect the failure points for a set of software data taken.

Keywords - Lehmann-Type Laplace distribution Type II (LLD-II), Profile Likelihood, Kumaraswamy Modified Inverse Weibull distribution (KMIW), Unconstrained optimization technique, Akaike Information Criterion(AIC), Bayesian Information Criterion (BIC), Non-Homogeneous Poisson Process (NHPP).

I. INTRODUCTION

Assessing software is important to evaluate and predict the reliability and performance of software system. By identifying the failures in the software, they can be eradicated and hence in turn increases the reliability and life time of the assessed software. To assess software, Software Reliability Growth Models (SRGM) are framed [4].

In practical software engineering, the Non-Homogeneous Poisson Process (NHPP) [4] based SRGM are proved to be successful. To evaluate the mean value function $m(x)$ i.e., the expectation of the number of failures experienced upto a certain point is the main issue in NHPP model. It is assumed that the number of failures follows Poisson distribution.

Statistical Process Control (SPC) [11] is used to monitor software reliability process. An efficient and appropriate SPC tool in testing software reliability is the control chart. Mean value control chart taking failure number along X-axis, successive differences of $m(x)$ along Y-axis and three parallel lines to X-axis for Lower Control Limit(LCL), Upper Control Limit(UCL) and Control Limit(CL) is used. Alarming signal and better quality are indicated by points below LCL and above UCL respectively. It is indicated that the software process is in stable condition by the points falling within the control limits,.

In recent years, several authors framed SRGM based on NHPP models. Out of them the commonly used are Weibull, Exponential Geometric, Goel-Okumoto [7], Lehmann Type Laplace Distribution-Type I (LLD-I) [2], Pareto type III [10], , Lehmann-Type Laplace distribution-Type II [3] models.

In this paper, it is proved that Lehmann-Type Laplace distribution-Type II (LLD-II) has a better fit when compared to Kumaraswamy Modified Inverse Weibull distribution for a Software failure data [7] using Akaike Information Criterion (AIC) [1] and Bayesian Information Criterion (BIC) [9]. Control mechanisms are developed using KMIW and LLD-Type II for the two datesets of software failure data.

The paper is organized as follows. Section 2 describes the models viz., KMIW and LLD-II along with their parameter estimation. In section 3, a set of software failure data are considered to show that LLD-II is a better fit than KMIW. In section 4, NHPP model for KMIW and LLD-II are given and to find the failure detection points based on

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control limits of the software, KMIW and LLD-II control mechanisms are framed. Section 5 concludes the paper.

II. Software Reliability Growth Models

2.1 Kumaraswamy Modified Inverse Weibull (KMIW) Software Reliability Growth Model

Kumaraswamy Modified Inverse Weibull [5] distribution has five parameters. Its cumulative distribution function is

$$F(x)=1-\left[1-e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\right]^{b} \qquad (2.1.1)$$

Its probability density function is

$$f(x)=\gamma b\left[\frac{\phi}{x^{2}}+\frac{\alpha\theta}{x^{\alpha+1}}\right]e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\left[1-e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\right]^{b-1} \qquad (2.1.2)$$

where $x,\phi,\theta,\alpha,\gamma,b>0$

α - Shape parameter

θ - Scale parameter represents the characteristics life

ϕ - Scale parameter

γ, b - parameters whose role is to introduce symmetry and produce distribution with heavier tails.

Estimation of parameters

Maximum likelihood method is used to estimate the five parameters of KMIW distribution. KMIW distribution has the likelihood function

$$l=\prod_{i=1}^{n}\gamma b\left[\frac{\phi}{x_i^{2}}+\frac{\alpha\theta}{x_i^{\alpha+1}}\right]e^{-\gamma\left(\frac{\phi}{x_i}+\frac{\theta}{x_i^{\alpha}}\right)}\left[1-e^{-\gamma\left(\frac{\phi}{x_i}+\frac{\theta}{x_i^{\alpha}}\right)}\right]^{b-1} \qquad (2.1.3)$$

Then its corresponding log-likelihood function is

$$\log l=n\log\gamma+n\log b+\sum_{i=1}^{n}\log\left(\frac{\phi}{x_i^{2}}+\frac{\alpha\theta}{x_i^{\alpha+1}}\right)-\gamma\sum_{i=1}^{n}\left(\frac{\phi}{x_i}+\frac{\theta}{x_i^{\alpha}}\right)+(b-1)\sum_{i=1}^{n}\log\left[1-e^{-\gamma\left(\frac{\phi}{x_i}+\frac{\theta}{x_i^{\alpha}}\right)}\right] \qquad (2.1.4)$$

Using unconstrained optimization technique, the maximum of log l in (2.1.4) can be found. The parameter values that give this maximum value are the optimum values.

2.2 Lehmann-Type Laplace Type II Software Reliability Growth Model

LLD-II has the probability density function [8] as

$$f(x)=\begin{cases}\dfrac{\alpha}{2\phi}e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x\le\theta\\[2ex] \dfrac{\alpha}{\phi}\left(1-\dfrac{1}{2}e^{\left(\frac{\theta-x}{\phi}\right)}\right)e^{\alpha\left(\frac{\theta-x}{\phi}\right)} & x\ge\theta\end{cases} \qquad (2.2.1)$$

where the shape parameter $\alpha>0$, the scale parameter $\phi>0$, is the location parameter $\theta>0$.

Its corresponding cumulative distribution function is

$$F(x)=\begin{cases}\dfrac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x\le\theta\\[2ex] 1-e^{\alpha\left(\frac{\theta-x}{\phi}\right)}+\dfrac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x}{\phi}\right)} & x\ge\theta\end{cases} \qquad (2.2.2)$$

Parameter estimation

Profile Likelihood method is used to estimate the parameters of LLD-II. For the observed data, the log-likelihood function can be represented as

$$\log l=n_1\log\alpha-n_1\log 2\phi+\frac{\alpha+1}{\phi}\sum_{i=1}^{n_1}(x_i-\theta)+n_2\log\alpha-n_2\log\phi+\sum_{i=1}^{n_2}\log\left(1-\frac{1}{2}e^{\left(\frac{\theta-x_i}{\phi}\right)}\right)+\frac{\alpha}{\phi}\sum_{i=1}^{n_2}(\theta-x_i) \qquad (2.2.3)$$

where $I_1=\{i\mid x_i\le\theta\}$ and $I_2=\{i\mid x_i>\theta\}, |I_1|=n_1, |I_2|=n_2$ and $n=n_1+n_2$.

Keeping θ fixed, the equations $\dfrac{\partial\log l}{\partial\alpha}=0$ and $\dfrac{\partial\log l}{\partial\phi}=0$ give

$$\alpha=\frac{n\phi}{(n_1-n_2)\theta+(n_2\overline{X}_2-n_1\overline{X}_1)} \qquad (2.2.4)$$

$$-(n_1+n_2)\phi+\left[(\alpha+1)n_1\theta-(\alpha+1)n_1\overline{X}_1+\alpha n_2\overline{X}_2-\alpha n_2\theta\right]+\frac{1}{2}\sum_{i=1}^{n_2}\frac{(\theta-x_i)e^{\left(\frac{\theta-x_i}{\phi}\right)}}{1-\frac{1}{2}e^{\left(\frac{\theta-x_i}{\phi}\right)}}=0 \qquad (2.2.5)$$

where $\overline{X}_1=\dfrac{\sum_{i=1}^{n_1}x_i}{n_1}$ and $\overline{X}_2=\dfrac{\sum_{i=1}^{n_2}x_i}{n_2}$.

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By evaluating,

$$\max_{\alpha,\theta,\phi} \log\, l(\alpha,\theta,\phi \mid x) = \max_{\theta}\left[\max_{\alpha,\phi}\, \log l(\alpha,\phi \mid x)\right]$$

(2.2.6)

using numerical techniques and MATLAB tools, the parameters α, θ, ϕ are estimated.

III. 3. Estimation and Goodness of Fit

3.1 Data Set

Let the cumulative time between failures be defined by the random variable X. Cumulative time between failures of a software product taken from AT & T is shown in Table 3.1.1.

Table 3.1.1 Cumulative Time between failures

Failure Number	Time between failure times in CPU units	Cumulative time between failures
1	5.5	5.5
2	1.83	7.33
3	2.75	10.08
4	70.89	80.97
5	3.94	84.91
6	14.98	99.89
7	3.47	103.36
8	9.96	113.32
9	11.39	124.71
10	19.88	144.59
11	7.81	152.4
12	14.59	166.99
13	11.42	178.41
14	18.94	197.35
15	65.3	262.65
16	0.04	262.69
17	125.67	388.36
18	82.69	471.05
19	0.45	471.5
20	31.61	503.11
21	129.31	632.42
22	47.6	680.02

For the data set in Table 3.1.1, the parameters, log likelihood values, AIC and BIC of corresponding distributions are given in the following Table 3.1.2.

Table 3.1.2. Goodness of fit using AIC and BIC

Distributions	Number of Parameters	Parameter values	Log likelihood value	AIC	BIC
KMIW	5	$\phi = 0.2913$ $\alpha = 10.0006$ $\theta = 10$ $\gamma = 87.6414$ $b = 0.5266$	-152.1983	318.1466	310.5787
LLD-II	3	$\theta = 5$ $\phi = 0.0578$ $\alpha = 2.5293\times10^{-58}$	-76.5826	160.4985	159.3473

IV. Software Failure Data Analysis

4.1 NHPP Model

The mean value function $m(x)$ and intensity function $\lambda(x)$ for finite value NHPP models are given as follows

$$m(x) = aF(x) \tag{4.1.1}$$

$$\lambda(x) = af(x)$$

Where, faults number in the software is shown as 'a'. NHPP models has the joint density function as

$$L = e^{-m(x_n)} \sum_{i=1}^{n} \lambda(x_i) \tag{4.1.2}$$

$$L = e^{-aF(x_n)} \prod_{i=1}^{n} af(x_i) \tag{4.1.3}$$

Partially differentiating $\log L$ with respect to 'a' and equating to zero

$$a = \frac{n}{F(x_n)} \tag{4.1.4}$$

For KMIW, the mean value function and intensity function using (2.1.1) and (2.1.2) are

$$m(x) = a\sum_{i=r}^{n}\binom{n}{i}\left(1-e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\right)^{b(n-i)}\left(1-\left(1-e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\right)^{b}\right)^{i}$$

(4.1.5)

$$\lambda(x) = \left\{\begin{array}{l} ar\binom{n}{r}\left(1-\left(1-e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\right)^{b}\right)^{r-1} \gamma b\left(\frac{\phi}{x^2}+\frac{\alpha\theta}{x^{\alpha+1}}\right) \\ e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\left(1-e^{-\gamma\left(\frac{\phi}{x}+\frac{\theta}{x^{\alpha}}\right)}\right)^{bn-br+b-1} \end{array}\right\}$$

(4.1.6)

Expected number of failures, using (2.1.1) and (4.1.4), is

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$$a = \frac{n}{\sum_{i=r}^{n}\binom{n}{i}\left(1-e^{-\gamma\left(\frac{\phi}{x_n}+\frac{\theta}{x_n^{\alpha}}\right)}\right)^{b(n-i)}\left(1-\left(1-e^{-\gamma\left(\frac{\phi}{x_n}+\frac{\theta}{x_n^{\alpha}}\right)}\right)^{b}\right)^{i}}$$

the value of 'a' for dataset is $a = 26.6937$.

For LLD-II, the mean value function and intensity function using (2.2.1) and (2.2.2) are

$$m(x)=\begin{cases} \dfrac{a\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x\le\theta \\ a\left(1-e^{\alpha\left(\frac{\theta-x}{\phi}\right)}+\dfrac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x}{\phi}\right)}\right) & x\ge\theta \end{cases} \qquad (4.1.7)$$

$$\lambda(x)=\begin{cases} \dfrac{a\alpha}{2\phi}e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x\le\theta \\ \dfrac{a\alpha}{\phi}e^{\alpha\left(\frac{\theta-x}{\phi}\right)}\left(1-\dfrac{1}{2}e^{\left(\frac{\theta-x}{\phi}\right)}\right) & x\ge\theta \end{cases} \qquad (4.1.8)$$

$$a = \frac{n}{\dfrac{\alpha e^{(\alpha+1)\left(\frac{x_{n_1}-\theta}{\phi}\right)}}{2(\alpha+1)}+\left(1-e^{\alpha\left(\frac{\theta-x_{n_2}}{\phi}\right)}+\dfrac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x_{n_2}}{\phi}\right)}\right)} \qquad (4.1.9)$$

From (4.1.9), the value of 'a' for the data set is 23.2099.

4.2 Control Mechanism

A control chart for the data set would be based on 0.9973 probability limits of the cumulative time between failures[6]. The solutions of the following equations are these probability limits and central line respectively, taking equitailed probabilities

$$F(x) = 0.99865$$
$$F(x) = 0.5$$
$$F(x) = 0.00135$$
(4.2.1)

The respective solutions of these equations in standard form be denoted as X_u, X_c, X_l. Then

$$X_u = F^{-1}(0.99865)$$
$$X_c = F^{-1}(0.5)$$
$$X_l = F^{-1}(0.00135)$$
(4.2.2)

The graph between failures' serial numbers and corresponding successive differences of $m(x)$ together with the 3 curves for X_u, X_c, X_l gives the control chart[11].

4.2.1 KMIW Control Mechanism

For the data set, the equitailed probabilities 0.99865, 0.5, 0.00135 are equated to the mean value function (4.2.2). Then the control limits are given as

$$UCL = 26.6577$$
$$LCL = 0.0360$$
$$CL = 13.3469$$

The successive differences of the mean value function is given in Table 4.2.1.

Table 4.2.1 Successive differences of the mean value function

Failure Number	Mean Value function $m(x)$	Successive differences of $m(x)$
1	0.1359	0.2992
2	0.4351	0.7037
3	1.1387	12.1478
4	13.2865	0.2834
5	13.5699	0.9389
6	14.5089	0.1912
7	14.7001	0.5042
8	15.2043	0.5079
9	15.7122	0.7479
10	16.4619	0.2565
11	16.7183	0.4331
12	17.1515	0.3035
13	17.4549	0.4470
14	17.9019	1.1654
15	19.0674	**5.8227×10^{-4}**
16	19.0679	1.3679
17	20.4359	0.5878
18	21.0236	**0.0028**
19	21.0264	0.1855
20	21.2119	0.6089
21	21.8208	0.1792
22	22	-

Figure 4.2.1 is mean value chart for KMIW SRGM. The chart shows that the at failure numbers 15 and 18, failures are detected.

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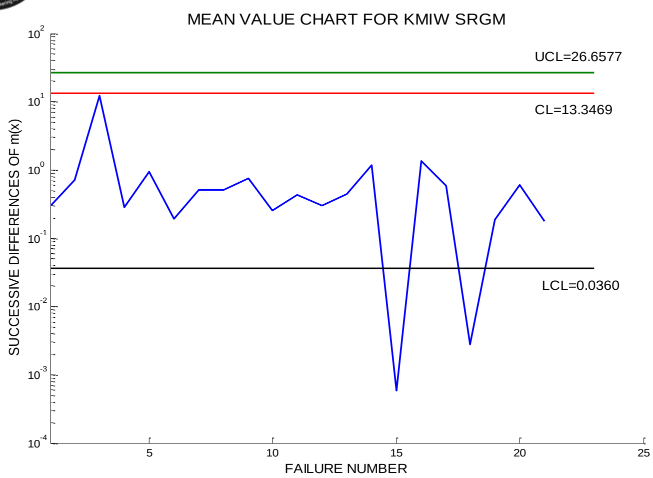

Figure 4.2.1

4.2.3 LLD-II Control Mechanism

For the data set,

$$UCL = 23.1785$$

$$LCL = 0.0313$$

$$CL = 11.6049$$

Table 4.2.3 gives successive differences of LLD-II.

Table 4.2.3 Successive differences of mean value function

Failure Number	Mean value function $m(x)$	Successive differences of $m(x)$
1	0.0507	0.1847
2	0.2354	0.2748
3	0.5103	6.0542
4	6.5644	0.2845
5	6.8489	1.0381
6	7.8870	0.2309
7	8.1180	0.6436
8	8.7616	0.7025
9	9.4641	1.1453
10	10.6093	0.4234
11	11.0327	0.7532
12	11.7859	0.5569
13	12.3427	0.8644
14	13.2071	2.4862
15	15.6933	**0.0013**
16	15.6946	3.1790
17	18.8736	1.3165
18	20.1901	**0.0059**
19	20.1961	0.3893
20	20.5854	1.1341
21	21.7195	0.2802
22	21.9997	-

Figure 4.2.3 is the Mean Value Chart from which it is found that LLD-II SRGM will detect the failures at 15 and 18 failure numbers.

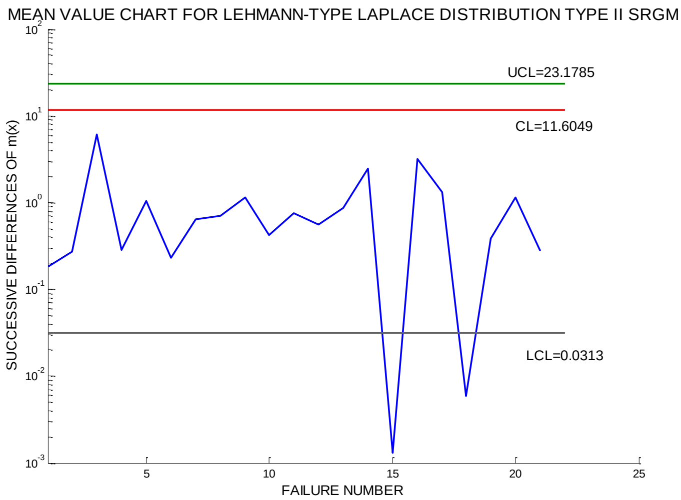

Figure 4.2.3

V. CONCLUSION

A set of software failure data is allowed to follow two distributions KMIW and LLD-II individually each. Estimation of parameters for the set of data is done using unconstrained optimization technique for KMIW and profile likelihood method for LLD-II. It is proved using AIC and BIC techniques that LLD-II is a better fit for the set of data when compared with KMIW. Then control mechanisms for KMIW and LLD-II are framed and the failure points are detected for the software failure data.

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