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Common Fixed Point Theorem for Semi-Compatible mappings in Symmetric Spaces

[1]Thirunavukkarasu .P, [2]Eswari.P.K,[3]Manjula R

[1]Assistant Professor, Department of Mathematics, Periyar E.V.R College (Autonomous), Tiruchirappalli, India.

[2]Associate Professor, Department of Mathematics, Mookambigai College of Engineering, Pudukottai, India.

[3]Lecturer (Senior Grade)/Mathematics, NPA Centenary Polytechnic College, Kotagiri, The Nilgiris, India .

Abstract: In this paper we introduce semi-compatible mapping in symmetric spaces and utilize the same to prove some results on existence and uniqueness of fixed point in symmetric spaces with an example. Some related theorems are also proved.

Keywords: Symmetric spaces, semi-compatible maps, weakly compatible mapping, common fixed point.

I. INTRODUCTION

Banach contraction principle is a well known fundamental result in fixed point theory which has been used and extended in many different directions. There have been several generalization of metric space. Among which one of the generalization is semi-metric space(symmetric space) initated by In 1986, Jungck [8] gave more generalized commuting and weakly commuting maps called compatible maps. A and B are called compatible if $\lim_{n\to\infty} d(ABx_n, BAx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = t$, for some $t \in X$. Clearly, weakly commuting maps are compatible, but the implication is not reversible.

In 1995, authors[3] introduced the concept of semi-compatibility and obtained the first result that established a situation in which a collection of mappings has a fixed point. They defined a pair of self-maps (S, T) to be semi-compatible if

1. $Sx = Tx \Rightarrow STx = TSx$ and
2. $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$, for some $x \in X$, implies $\lim_{n\to\infty} STx_n = Tx$ holds.

B. Sing and S. Jain [12] observe that (2) implies (1). Hence they defined the semi-compatibility by condition (2) only.

Singh & Sharma [13] generalized the result of Vasuki [14], Grabiec [6] and Singh & Chauhan [11] where the sufficient conditions for the existence of a common fixed points of two self-maps in fuzzy metric spaces.

Further, we generalize the results of Singh & Sharma [13] where the sufficient conditions for the existence of a common fixed point of six self-maps in fuzzy metric space.

M.Frechet[5], K..Menger[9] and W.A.Wilson[15]. M.Cicchese[4] introduced the notion of contractive mapping in semi-metric space and proved the first fixed point theorem for this class of space.

In order to obtain common fixed point theorems in symmetric spaces, some axioms are needed. In [7], the authors assumed axiom (W3) and in [2] the author assumed axioms (W3), (W4) and (H.E). M.Aamri and D.El.Moutawakil [1] gave a notion of E.A property which generalize the concept of non-compatible mapping in metric space and proved some common fixed point theorems for non-compatible mappings under strict contractive conditions.

Sessa [10] generalized the notion of commuting maps and introduced the concept of weakly commuting maps. He defines A and B to be weakly commuting if $d(ABx, BAx) \le d(Ax, Bx)$ for all $x \in X$. Obviously, commuting maps are weakly commuting but the converse is not true.

II. PRELIMINARIES

Let X be a non-empty set and $d : X \times X \to [0, \infty)$. (X,d) be a semi-metric space (symmetric space) if and only if it satisfies the following:

[15} W1: $d(x, y) = 0$ if and only if x = y

[15] W2 : $d(x, y) = d(y, x)$ if and only if x = y for any $x, y \in X$.

The difference of a semi-metric and a metric comes from the triangle inequality. In order to obtain fixed point theorems on a semi-metric space, we need some additional axioms which are as follows:

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Let (X, d) be a semi- metric space. Then, for sequences $\{x_n\}$, $\{y_n\}$ in X and $x \in X$, we have

[15] W.3: Given $\{x_n\}$, x and y in X, $d(x_n, x) \to 0$ and $d(x_n, y) \to 0$ imply that x = y.

[15] W.4: Given $\{x_n\}$, $\{y_n\}$ and x in X, $\lim_{n\to\infty} d(x_n, x) = 0$ and $d(y_n, x_n) = 0$ imply that $\lim_{n\to\infty} d(y_n, x) = 0$.

Definition 2.1 [1]: Let A and B be two self-mappings of a semi metric space (X, d). Then A and B satisfy the property E.A if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} d(Ax_n, t) = \lim_{n\to\infty} d(Bx_n, t) = 0$, for some $t \in X$.

Definition 2.2: Let (X,d) be the metric space and let A and B be two self-maps. A and B are commuting maps if $ABx = BAx$ for all x in X.

Definition 2.3[1]: Let A and B be twp self-mappings of a symmetric space (x,d). Then A and B are said to be compatible if $\lim_{n\to\infty} d(ABx_n, BAx_n) = 0$, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} d(Ax_n, t) = \lim_{n\to\infty} d(Bx_n, t) = 0$, for some $t \in X$.

Definition:2.4: Semi-compatible maps[3]:A pair of self-maps (S, T) is said to be semi-compatible if

1. $Sx = Tx \Rightarrow STx = TSx$ and
2. $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$, for some $x \in X$, implies $\lim_{n\to\infty} STx_n = Tx$.

In order to establish our result, we need a function $\phi : R^+ \to R^+$ satisfying $0 < \phi(t) < t,\ t > 0$

III. MAIN RESULTS

Theorem 3.1:

Let (X,d) be a symmetric (semi-metric) space that satisfies (W3) and (H.E), and let A, B, S, T, P and Q be the self-mappings of X such that

1. $AX \subset QTX$ and $BX \subset PSX$
2. The pairs (A, PS) and (B, QT) satisfy (E.A) property
3. For any $x,y \in X, (x \neq y)$,

$$d(Ax, By) \le \phi\left(\max\begin{Bmatrix} d(PSx, QTy), d(PSx, Ax), \\ d(QTy, By), d(PSx, By), \\ d(QTy, Ax) \end{Bmatrix} \right)$$

4. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then there exist u, v ∈ X such that $Au = PSu = Bv = QTv$.

PROOF:

From (2), there exists a sequence $\{x_n\}$ in X, and a point $t \in X$ such that $\lim_{n\to\infty} d(Bx_n, t) = \lim_{n\to\infty} d(QTx_n, t) = 0$.

From (1), there exists a sequence $\{y_n\}$ in X such that $Bx_n = PSy_n$ and hence $\lim_{n\to\infty} d(PSy_n, t) = 0$.

By (H.E), $\lim_{n\to\infty} d(Bx_n, QTx_n) = \lim_{n\to\infty} d(PSy_n, QTx_n) = 0$.

From (4), there exists a point $u \in X$ such that PSu=t. Also, we have $\lim_{n\to\infty} d(Bx_n, PSu) = \lim_{n\to\infty} d(QTx_n, Psu) = \lim_{n\to\infty} d(PSy_n, PSu) = 0$.

From (3), we have

$$d(Au, Bx_n) \le \phi\left(\max\begin{Bmatrix} d(PSu, QTx_n), d(PSu, Au), \\ d(QTx_n, Bx_n), d(PSu, Bx_n), \\ d(QTx_n, Au) \end{Bmatrix} \right)$$

By taking limit as $n\to\infty$, we have $d(Au, Bx_n) = 0$.

By (W3), we get Au = PSu. Since $AX \subset QTX$, there exists a point $v \in X$ such that Au = QTv. We show that $QTv = Bv$. If not, from (3), we have

$$d(Au, Bv) \le \phi\left(\max\begin{Bmatrix} d(PSu, QTv), d(PSu, Au), \\ d(QTv, Bv), d(PSu, Bv), \\ d(QTv, Au) \end{Bmatrix} \right)$$

$$\le \phi\left(\max\{d(Au, Au), d(Au, Au), d(Au, Bv), d(Au, Bv), d(Au, Au)\}\right)$$

$$\le \phi\left(d(Au, Bv)\right)$$

$= d(Au, Bv)$ Which is a contradiction.

Hence, $Au = Bv$ and hence $Au = PSu = Bv = QTv$.

Theorem 3.2

Let (X, d) be a symmetric space that satisfies (W3) and (H.E), and let A, B, S, T, P and Q be the self- mappings of X such that

1. $AX \subset QTX$ and $BX \subset PSX$
2. The pairs (A, PS) and (B, QT) satisfy (E.A) property
3. The pairs (A, PS) and (B, QT) are semi-compatible.
4. One of A, B, PS and QT is continuous.
5. For any $x,y \in X, (x \neq y)$,

$$d(Ax, By) \le \phi\left(\max\begin{Bmatrix} d(PSx, QTy), d(PSx, Ax), \\ d(QTy, By), d(PSx, By), \\ d(QTy, Ax) \end{Bmatrix} \right)$$

6. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then there exist $u \in X$ such that u is a common fixed point of A, PS, B and QT.

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PROOF:

By theorem 3.1, we have $Au = PSu = Bv = QTv$ .First, we have to show that there exists a common fixed point for the mapping A, PS, B & QT.

By (4), Suppose PS is continuous, then $PSAx_n \to PSu$ and $(PS)^2 y_n \to PSu$ and by (3), semi compatibility of (A,PS), we have $APSy_n \to PSu$ .Since we have $Au = PSu$,then $APSu = PSAu$. Now we have $AAu = APSu = PSAu = (PS)^2 u$. Now we claim that Au is a common fixed point of A and PS, if $AAu \neq Au$.

Using condition (5), we have

$$d(Au, AAu) = d(AAu, Bv) \leq \phi\left(\max\left\{d(PSAu, QTv), d(PSAu, AAu), d(QTv, Bv), d(PSAu, Bv), d(QTv, AAu)\right\}\right)$$

$$\leq \phi\left(\max\left\{d(AAu, Au), d(AAu, AAu), d(Au, Au), d(AAu, Au), d(Au, AAu)\right\}\right)$$

$$< \phi\left(d(AAu, Au)\right)$$

$$= d(AAu, Au)$$ Which is a contradiction.

Therefore, we have $AAu = Au$ which implies $AAu = PSAu = Au$, hence Au is a common fixed point of A and PS. Since we have $BQTv = QTBv$. Thus, we have $(QT)^2 v = QTBv = BQTv = BBV$. Similarly, we can show that Bv is a common fixed point of B and QT. Since $Au = Bv$, we conclude that Au is a common fixed point of A, PS, B and QT.

The proof is similar when QTX is assumed to be a complete subspace of X. The cases in which AX or BX is a complete subspace of X are similar to the cases in which QTX or SX respectively is complete since $AX \subset QTX$ and $BX \subset PSX$.

If $Au = Bu = QTu = PSu = u$ and $Av = Bv = QTv = PSv = v$ and $u \neq v$, then (5) gives

$$d(u, v) = d(Au, Bv) \leq \phi\left(\max\begin{Bmatrix} d(PSu, QTv), d(PSu, Au), \\ d(QTv, Bv), d(PSu, Bv), \\ d(QTv, Au) \end{Bmatrix}\right)$$

$$\leq \phi\left(\max\left\{d(u, v), d(u, u), d(v, v), d(u, v), d(v, u)\right\}\right)$$

$$< \phi\left(d(u, v)\right)$$

$$= d(u, v)$$ Which is a contradiction.

Therefore u=v and the common fixed point is unique for A, B, QT and PS.

Theorem 3.3:

Let (X, d) be a symmetric space that satisfies (W3) and (H.E), and let A, B, S, T, P and Q be the self-mappings of X such that

1. $AX \subset QTX$ and $BX \subset PSX$
2. The pairs (A, PS) and (B, QT) satisfy (E.A) property
3. The pairs (A, PS) and (B, QT) are semi-compatible.
4. One of A, B, PS and QT is continuous.
5. The pairs (P, A), (S, A), (Q, B) and (T, B) are weakly compatible maps.
6. For any $x, y \in (X \neq y)$,

$$d(Ax, By) \leq \phi\left(\max\begin{Bmatrix} d(PSx, QTy), d(PSx, Ax), \\ d(QTy, By), d(PSx, By), \\ d(QTy, Ax) \end{Bmatrix}\right)$$

7. The pairs (P, S) and (Q, T) are commutative maps.
8. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then A, B, S, T, P and Q have a unique common fixed point.

PROOF:

By theorem 3.1 & 3.2, we have u is a common fixed point for A, B, QT, PS.

From (5) & (7) we have

$Pu = P(PSu) = P(SPu) = (PS)Pu$; $Pu = PAu = APu$ and

$Su = S(PSu) = (SP)Su = (PS)Su$; $Su = SAu = ASu$;

implies that Pu and Su are common fixed points of (A, Ps). Therefore $u = Pu = Su = Au = PSu$. Similarly, Qu and Tu are common fixed points of(B, QT). Therefore $u = Qu = Tu = Bu = QTu$. Hence u is common fixed point of A, B, S, T, P&Q.

Theorem 3.4

Let (X,, d) be a symmetric space that satisfies (W3) and (H.E), and let A, B, S, T, P and Q be the self-mappings of X such that

1. $AX \subset QTX$ and $BX \subset PSX$
2. The pairs (A, PS) and (B, QT) satisfy (E.A) property.
3. For any $x, y \in X, (x \neq y)$,

$$d(Ax, By) \leq \phi\left(\max\begin{Bmatrix} d(PSx, Ty), \\ \left(\frac{k}{2}\right)(d(PSx, Ax) + d(QTy, By)), \\ \left(\frac{k}{2}\right)(d(PSx, By) + d(QTy, Ax)) \end{Bmatrix}\right),$$

$0 < k < 2$

4. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then there exist $u, v \in X$ such that $Au = PSu = Bv = QTv$.

PROOF:

From (2), there exists a sequence $\{x_n\}$ in X, and a point t $\in X$ such that $\lim_{n\to\infty} d(Bx_n, t) = \lim_{n\to\infty} d(QTx_n, t) = 0$.

From (1), there exists a sequence $\{y_n\}$ in X such that $Bx_n = PSy_n$ and hence $\lim_{n\to\infty} d(PSy_n, t) = 0$.

By (H.E), $\lim_{n\to\infty} d(Bx_n, QTx_n) = \lim_{n\to\infty} d(PSy_n, QTx_n) = 0$.

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From (4), there exists a point $u \in X$ such that $PSu = t$. Also, we have $\lim_{n\to\infty} d(Bx_n, PSu) = \lim_{n\to\infty} d(QTx_n, Psu) = \lim_{n\to\infty} d(PSy_n, PSu) = 0.$

From (3), we have

$$d(Au, Bx_n) \le \phi\left(\max\begin{Bmatrix} d(PSu, QTx_n), d(PSu, Au), \\ d(QTx_n, Bx_n), d(PSu, Bx_n), \\ d(QTx_n, Au) \end{Bmatrix} \right)$$

By taking limit as $n\to\infty$, we have. $d(Au, Bx_n) = 0$ .By (W3), we get $Au = PSu$.

Since $AX \subset QTX$, there exists a point $v \in X$ such that $Au = QTv$. We show that $QTv = Bv$.

If not, from (3), we have

$$d(Au, Bx_n) \le \phi\left(\max\begin{Bmatrix} d(PSu, QTx_n), \\ \left(k/2\right)(d(PSu, Au) + d(QTx_n, Bx_n)), \\ \left(k/2\right)(d(PSu, Bx_n) + d(QTx_n, Au)) \end{Bmatrix} \right)$$

$$\le \phi\left(\max\left\{0, \frac{k}{2}d(Au, PSu), \frac{k}{2}d(Au, PSu)\right\}\right)$$

$$= \phi\left(\frac{k}{2}d(Au, PSu)\right)$$

$$< \frac{k}{2}d(Au, PSu)$$

Since $0 < k/2 < 1$, we get $d(Au, PSu) = 0$ and hence $Au = PSu$. Since $AX \subset QTX$, there exists a point $v \in X$ such that $Au = QTv$ we show that $Au = Bv$. If not, from (3) we have

$$d(Au, Bv) \le \phi\left(\max\begin{Bmatrix} d(PSu, QTv), \\ \left(k/2\right)(d(PSu, Au) + d(QTv, Bv)), \\ \left(k/2\right)(d(PSu, Bv) + d(QTv, Au)) \end{Bmatrix} \right)$$

$$\le \phi\left(\max\begin{Bmatrix} d(Au, Au), \\ \left(k/2\right)(d(Au, Au) + d(Au, Bv)), \\ \left(k/2\right)(d(Au, Bv) + d(Au, Au)) \end{Bmatrix} \right)$$

$$\le \phi\left(\max\left\{0, \frac{k}{2}d(Au, Bv), \frac{k}{2}d(Au, Bv)\right\}\right)$$

$$= \phi\left(\frac{k}{2}d(Au, Bv)\right)$$

$$< \frac{k}{2}d(Au, Bv)$$

Since $0 < k/2 < 1$, we get $d(Au, Bv) = 0$. Hence $Au = Bv$ and therefore $Au = PSu = Bv = QTv$.

Theorem 3.5

Let (X, d) be a symmetric space that satisfies (W3) and (H.E), and let A, B, S, T, P and Q be the self-mappings of X such that

1. $AX \subset QTX$ and $BX \subset PSX$
2. The pairs (A, PS) and (B, QT) satisfy (E.A) property
3. The pairs (A, PS) and (B, QT) are semi-compatible.
4. One of A, B, PS and QT is continuous.
5. For any $x,y \in X, (x \ne y)$,

$$d(Ax, By) \le \phi\left(\max\begin{Bmatrix} d(PSx, Ty), \\ \left(k/2\right)(d(PSx, Ax) + d(QTy, By)), \\ \left(1/2\right)(d(PSx, By) + d(QTy, Ax)) \end{Bmatrix} \right)$$

6. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then there exist $u \in X$ such that u is a common fixed point of A, PS, B and QT.

PROOF:

By theorem 3.1, we have $Au = PSu = Bv = QTv$. First, we have to show that there exists a common fixed point for the mapping A, PS, B & QT.

By (4), Suppose PS is continuous, then $PSAx_n \to PSu$ and $(PS)^2 y_n \to PSu$ and by (3), semi compatibility of (A,PS), we have $APSy_n \to PSu$. Since we have $Au = PSu$, then $APSu = PSAu$. Now we have $AAu = APSu = PSAu = (PS)^2 u$. Now we claim that Au is a common fixed point of A and PS, if $AAu \ne Au$.

Using condition (5), we have

$$d(Au, AAu) = d(AAu, Bv)$$

$$\le \phi\left(\max\begin{Bmatrix} d(PSAu, QTv), \\ \left(k/2\right)(d(PSAu, AAu) + d(QTv, Bv)), \\ \left(1/2\right)(d(PSAu, Bv) + d(QTv, AAu)) \end{Bmatrix} \right)$$

$$\le \phi\left(\max\begin{Bmatrix} d(AAu, Au), \\ \left(k/2\right)(d(AAu, AAu) + d(Au, Au)), \\ \left(1/2\right)(d(AAu, Au) + d(Au, AAu)) \end{Bmatrix} \right)$$

$$= \phi\left(\max\left\{d(AAu, Au), 0, d(AAu, Au)\right\}\right)$$

$$= \phi\left(d(AAu, Au)\right)$$

$$< d(AAu, Au)$$

Which is a contradiction.

Therefore, we have $AAu = Au$ which implies $AAu = PSAu = Au$, hence Au is a common fixed point of A and PS. Since we have $BQTv = QTBv$. Thus, we have $(QT)^2 v = QTBv = BQTv = BBV$.

Similarly, we can show that Bv is a common fixed point of B and QT. Since $Au = Bv$, we conclude that Au is a common fixed point of A, PS, B and QT.

The proof is similar when QTX is assumed to be a complete subspace of X. The cases in which AX or BX is a complete subspace of X are similar to the cases in which

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QTX or SX respectively is complete since $AX \subset QTX$ and $BX \subset PSX$.

If $Au = Bu = QTu = PSu = u$ and $Av = Bv = QTv = PSv = v$ and $u \neq v$, then (5) gives

$$d(u,v) = d(Au,Bv)$$

$$\leq \phi \begin{pmatrix} \max\{d(PSu,QTv), \\ \left(k/2\right)(d(PSu,Au)+d(QTv,Bv)), \\ \left(1/2\right)(d(PSu,Bv)+d(QTv,Au))\} \end{pmatrix}$$

$$\leq \phi \begin{pmatrix} \max\{d(u,v), \\ \left(k/2\right)(d(u,u)+d(v,v)), \\ \left(1/2\right)(d(u,v)+d(v,u))\} \end{pmatrix}$$

$$< \phi(d(u,v))$$

$$= d(u,v)$$ Which is a contradiction.

Therefore u = v and the common fixed point is unique for A, B, QT and PS.

Theorem 3.6:

Let (X, d) be a symmetric space that satisfies (W3) and (H.E), and let A, B, S, T, P and Q be the self-mappings of X such that

1. $AX \subset QTX$ and $BX \subset PSX$
2. The pairs (A, PS) and (B, QT) satisfy (E.A) property
3. The pairs (A, PS) and (B, QT) are semi-compatible.
4. One of A, B, PS and QT is continuous.
5. The pairs (P, A), (S, A), (Q, B) and (T, B) are weakly compatible maps.
6. For any $x, y \in (X \neq y)$,

$$d(Ax,By) \leq \phi \begin{pmatrix} \max\{d(PSx,Ty), \\ \left(1/2\right)(d(PSx,Ax)+d(QTy,By)), \\ \left(1/2\right)(d(PSx,By)+d(QTy,Ax))\} \end{pmatrix}$$

7. The pairs (P, S) and (Q, T) are commutative maps.
8. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then A, B, S, T, P and Q have a unique common fixed point.

PROOF:

By using theorem3.4, 3.5 and 3.3, u is a unique common fixed point for the self-mappings A, B, P, Q, S and T.

Example 3.7:

Consider $X = [0,1]$ with semi-metric space (X, d) defined by $d(x,y) = (x-y)^2$. Define self-mappings A, B, P, Q, S, T as $Ax = Bx = \frac{x}{2}$; $Sx = Tx = x$ and $Px = Qx = \frac{x}{2}$ for all $x \in X$. Then we have the following:

1. (X, d) is a symmetric space satisfying the properties (W3) and (H.E)
2. $AX \subset QTX$ and $BX \subset PSX$
3. The pair (A, PS) and (B, QT) satisfy (E.A) property for the sequence $x_n = \frac{1}{n}, n = 1,2,3,\cdots$
4. The pairs (A, PS) and (B, QT) are semi-compatible.
5. A, PS, B, QT are continuous maps.
6. $$d(Ax,By) \leq \phi \begin{pmatrix} \max\{d(PSx,QTy), d(PSx,Ax), \\ d(QTy,By), d(PSx,By), \\ d(QTy,Ax)\} \end{pmatrix}$$
7. The ranges of the maps A, B, QT and PS are complete subspaces of X.

Then 0 is the unique common fixed point of the maps A, B, S, T, P and Q.

IV. CONCLUSION

In this paper, we presented the notion of semi-compatible mapping in symmetric space. Using this mapping, we proved a fixed point theorem for symmetric spaces in the sense of Singh and Sharma. We gave an example to support the concept defined in this paper.

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