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Performance of Sample Standard Deviation (s) chart under Three Delta Control Limits and Six Delta Initiatives

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ABSTRACT: A control chart is a graphical device for representation of the data for knowing the extent of variations from the expected standard. The technique of control chart was suggested by W.A. Shewhart of Bell Telephone Company based on three sigma limits. M. Harry, the engineer of Motorola has introduced the concept of six sigma in 1980. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities. Moderate distribution proposed by Naik V.D and Desai J.M, is a sound alternative of normal distribution, which has mean and mean deviation as pivotal parameters and which has properties similar to normal distribution. Naik V.D and Tailor K.S. have suggested the concept of 3-delta control limits and developed various control charts based on this distribution. K.S.Tailor has introduced the concept of six-delta initiatives. In this paper an attempt is made to construct a control chart based on six delta initiatives for sample standard deviation. Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

Keywords: Control charts, Mean Deviation, Moderate distribution, Process control, Six Delta, Six Sigma,

I. INTRODUCTION

The technique of quality control was developed by W. A. Shewhart (1931). It was based on three sigma(3σ) control limits. The concept of six-sigma was introduced by Motorola by the engineer Mikel Harry in 1980. He developed methods for problem solving that combined formal techniques, particularly relating to measurement, to achieve measurable savings in millions of dollars. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities.

R.Radhakrishnan and P.Balamurugan (2010, 2011, and 2016) have developed six sigma based control charts for mean, exponentially weighted moving average (EWMA), X bar using standard deviation, sample standard deviation, range and moving averages.

Naik V.D and Desai J.M (2015) have proposed an alternative of normal distribution called moderate distribution, which has mean(μ) and mean deviation(δ) as pivotal parameters and which has properties similar to normal distribution. V.D.Naik and K.S.Tailor (2015, 2016) have suggested 3δ(3 mean deviation) control limits based on moderate distribution. On the basis of 3δ control limits, they have developed $\bar{X}$-chart, R-chart, s-chart and d-chart. K.S.Tailor (2016) has also developed moving average and moving range chart and exponentially moving average chart under moderateness assumption.

Similar to six sigma concept, K.S.Tailor (2017) has introduced the concept of six delta initiatives. The six sigma concept is based on normality assumption and the control limits are determined by using standard deviation (σ-sigma) of the statistic, whereas the six delta concept is based on moderateness assumption and the control limits are determined by using mean deviation (δ-delta) of the statistic. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities whereas in six delta initiatives, it is expected to produce 1.7 or less number of defects per million of opportunities. If the companies practicing Six Delta initiatives use the control limits, then no point fall outside the control limits because of the improvement in the quality of the process. K.S.Tailor (2017, 2018) has proposed X-bar chart associated with mean deviation, sample mean deviation (d) chart and exponentially weighted moving average (EWMA) chart based on six delta initiatives.

Here an attempt is made to construct a control chart based on six delta initiatives for sample standard deviation specially designed for the companies who want to apply Six Delta initiatives in their organization. Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

II. CONCEPTS AND TERMINOLOGIES

The following terms are used for constructing the control charts under the moderateness assumption.

A. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

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B. Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

C. Tolerance level (TL)

It is the difference between USL and LSL,

TL = USL - LSL

D. Process capability (Cp)

We use mean deviation instead of standard deviation to calculate process capability which is the ratio of tolerance level to six times mean deviation of the process.

$$Cp = (TL/\ 6\sqrt{\frac{\pi}{2}}\delta) = (TL/10.6369\delta)$$

$$= (USL\text{-}LSL)/10.6369\delta$$

E. Mean deviation (δ): For many purposes mean deviation is the most useful measure of dispersion of a set of numbers. It is the mean of absolute deviation.

F. Quality Control Constant[1] (K_{md})

The constant K_{md} is introduced in this paper to determine the control limits based on six delta initiatives for sample mean deviation.

G. Quality Control Constant[2] ($B_3{}'$and $B_4{}'$)

The constants $B_3{}'$and $B_4{}'$ introduced in this paper to determine the control limits based on three delta for standard deviation.

III. THREE DELTA CONTROL LIMITS FOR SAMPLE MEAN DEVIATION CHART

Kalpesh S Tailor has proposed sample standard deviation (s) chart under the moderateness assumption. Suppose that the main variable of the process x follows **moderate distribution**. The mean of x is E(x) = μ and mean deviation of x is $\delta_x = \delta'$. For s chart the values of sample standard deviation(s) are obtained from each subgroup taken at regular interval of time, from a production process. As the value of process S.D. σ' is unknown, its estimator $\bar{s}$ is used in its place.

i.e. E (s) = $C_2\sigma' = \bar{s}$ or $\sigma' = \frac{E(s)}{C_2} = \frac{\bar{s}}{C_2}$.

The following important results of σ (when the underlying distribution is moderate) are used for calculating the control limits of this chart.

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})^2 \text{ is the sampling variance} \quad (1)$$

$$\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i \quad (2 \quad E(s)$$

$$= \bar{s} \quad (3) \quad E\left(\frac{s}{\sigma'}\right)$$

$$= C_2 \quad (4)$$

Where σ' is the process standard deviation and C_2 is a constant which depends on the size of the sample.

$$\therefore E(s) = C_2\sigma' = \sqrt{\frac{\pi}{2}}C_2\delta' \quad (5)$$

$$\text{Or } \sigma' = \frac{E(s)}{C_2} = \frac{\bar{s}}{C_2} \text{ and } \delta' = \frac{\bar{s}}{\sqrt{\frac{\pi}{2}}C_2} \quad (6)$$

Where δ' is the process mean deviation.

$$S.D\left(\frac{s}{\sigma'}\right) = [2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{1}{\sqrt{2n}} \quad (7)$$

$$\therefore S.D(s) = [2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{\sigma'}{\sqrt{2n}} \quad (8)$$

$$M.D(s) = \delta_s = [2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\sqrt{\frac{\pi}{2}}\frac{\delta'}{\sqrt{2n}} \quad (9)$$

Hence on the basis of 3δ criteria the control limits of s chart can be represented as follows.

Central line (C.L) = E(s)

$= \bar{s}$ (10)

Lower control limit(L.C.L) = E (s) - $3\delta_s$

$$= \bar{s} - 3\sqrt{\frac{2}{\pi}}\sigma_s$$

$$= \bar{s}\text{-}3\sqrt{\frac{2}{\pi}}[2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{\sigma'}{\sqrt{2n}}$$

$$= \bar{s} - 3\sqrt{\frac{2}{\pi}}\left[[2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{\bar{s}}{C_2\sqrt{2n}}\right]$$

= As $\sigma' = \frac{E(s)}{C_2} = \frac{\bar{s}}{C_2}$

$$\bar{s}\left\{1 - 3\sqrt{\frac{2}{\pi}}[2(n-1)2nC_2{}^2]^{\frac{1}{2}}.\frac{1}{C_2\sqrt{2n}}\right\}$$

$$= B'_3\bar{s} \quad (11)$$

Where $B'_3 = 1 - 3\sqrt{\frac{2}{\pi}}[2(n-1)2nC_2{}^2]^{\frac{1}{2}}.\frac{1}{C_2\sqrt{2n}}$

Upper control limit(U.C.L)

= E (s) + $3\delta_s$

$$= \bar{s} + 3\sqrt{\frac{2}{\pi}}\sigma_s$$

$$= \bar{s} + 3\sqrt{\frac{2}{\pi}}[2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{\sigma'}{\sqrt{2n}}$$

$$= \bar{s} + 3\sqrt{\frac{2}{\pi}}\left[[2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{\bar{s}}{C_2\sqrt{2n}}\right]$$

As $\sigma' = \frac{E(s)}{C_2} = \frac{\bar{s}}{C_2}$

$$= \bar{s}\left\{1 + 3\sqrt{\frac{2}{\pi}}[2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{1}{C_2\sqrt{2n}}\right\}$$

$$= B'_4\bar{s} \quad (12)$$

Where $B'_4 = 1 + 3\sqrt{\frac{2}{\pi}}[2(n-1) - 2nC_2{}^2]^{\frac{1}{2}}.\frac{1}{C_2\sqrt{2n}}$

Values of $B_3{}'$, $B_4{}'$ for different values of n are given in the table 1.

Values of $B_3{}'$and $B_4{}'$ for different values of n		
n	$B_3{}'$	$B_4{}'$
2	0	2.8080
3	0	2.2569
4	0	2.0099
5	0.1308	1.8691
6	0.2263	1.7737
7	0.2961	1.7039
8	0.3499	1.6501

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9	0.3911	1.6088
10	0.4281	1.5719
11	0.4591	1.5409
12	0.4838	1.5162
13	0.5069	1.4931
14	0.5265	1.4735
15	0.5438	1.4562
16	0.5603	1.4397
17	0.5736	1.4264
18	0.5847	1.4152
19	0.5983	1.4017
20	0.6086	1.3914
21	0.6196	1.3804
22	0.6294	1.3706
23	0.6377	1.3623
24	0.6459	1.3541
25	0.6520	1.3480

Table 1

IV. SIX DELTA BASED CONTROL LIMITS FOR SAMPLE STANDARD DEVIATION CHART

Fix the tolerance level (TL) and process capability (C_p) to determine the process mean deviation (δ). Apply the value of δ in the control limits $\bar{s} \pm K_{md}\delta$ to get the control limits for the six delta based control chart for the sample standard deviation. The value of K_{md} is obtained by using $P(Z \leq K_{md})$= 1- α_1, where $\alpha_1 = 1.7 \times 10^{-6}$ and Z is a standard moderate variate. For a specified TL and C_p of the process, the value of δ is calculated, which is presented in table 2 for various combination of TL and C_p. Thus, the control limits for six delta based control chart for sample standard deviation are determined as,

$$CL_{6\delta} = \bar{s} \tag{13}$$
$$LCL_{6\delta} = \bar{s} - K_{md}\delta \tag{14}$$
$$UCL_{6\delta} = \bar{s} + K_{md}\delta \tag{15}$$

V. AN EMPIRICAL STUDY FOR SAMPLE STANDARD DEVIATION (s) CONTROL CHART AND COMPARISION OF THREE DELTA LIMITS AGAINST SIX DELTA INITIATIVES

To illustrate sample standard deviation (s) chart with three delta and six delta limits, a data set is taken from E .L. Grant and R. S. Leavenworth (1988) which is given in table 1. Three sigma and six sigma control limits are computed from this data set, and control charts are plotted under these two limits.

Data set

Lot	X1	X2	X3	X4	X5	s
1	77	80	78	72	78	2.68
2	76	79	73	74	73	2.28
3	76	77	72	76	74	1.79
4	74	78	75	77	77	1.47
5	80	73	75	76	74	2.42
6	78	81	79	76	76	1.90
7	75	77	75	76	77	0.89
8	79	75	78	77	76	1.41
9	76	75	74	75	75	0.63
10	71	73	71	70	73	1.20
11	72	73	75	74	75	1.17
12	75	73	76	73	73	1.26
13	75	76	78	79	77	1.41
14	77	77	78	77	76	0.63
15	77	76	77	77	77	0.40
16	77	77	77	79	79	0.98

Table 2

(a) Three delta control limits for sample standard deviation (s) chart:

The three delta control limits calculated from data set given in table 2 by using equations (10), (11) and (12) as follows.

CL = 1.41, LCL = 0.18 and UCL = 2.64

(b) Control limits based on six delta initiatives for sample standard deviation (s) chart:

For a given data set USL = 2.68, LSL = 0.40, TL = 2.68-0.40 = 2.28 and C_p = 1.5**.** The value of δ = 0.1429, which is found from the table 3, K_{md} = 5.815 which is calculated from $P(Z \leq K_{md})$ = 1- α_1, where $\alpha_1 = 1.7 \times 10^{-6}$. Hence, the control limits based on six delta initiatives for sample standard deviation chart for a specified TL and K_{md} are determined as,

$CL_{6\delta}$= 1.41, $LCL_{6\delta}$ = 0.5790 and $UCL_{6\delta}$ = 2.2410

Values of δ for a specified C_p and TL

TL / C_p	2.25	2.26	2.27	2.28	2.29
1.0	0.2115	0.2125	0.2134	0.2143	0.2153
1.1	0.1923	0.1932	0.1940	0.1949	0.1957
1.2	0.1763	0.1771	0.1778	0.1786	0.1794
1.3	0.1627	0.1634	0.1642	0.1649	0.1656
1.4	0.1511	0.1518	0.1524	0.1531	0.1538
1.5	0.1410	0.1416	0.1423	0.1429	0.1435
1.6	0.1322	0.1328	0.1334	0.1340	0.1346
1.7	0.1244	0.1250	0.1255	0.1261	0.1266
1.8	0.1175	0.1182	0.1186	0.1191	0.1196
1.9	0.1113	0.1118	0.1123	0.1128	0.1133
2.0	0.1058	0.1062	0.1067	0.1072	0.1076
2.1	0.1007	0.1012	0.1016	0.1021	0.1025
2.2	0.0961	0.0966	0.0970	0.0974	0.0979
2.3	0.0920	0.0924	0.0928	0.0932	0.0936
2.4	0.0881	0.0885	0.0889	0.0893	0.0897
2.5	0.0846	0.0850	0.0854	0.0857	0.0861

Table 3

(c) s-charts for data set given in table 1 based on three delta and six delta limits:

Sample standard deviation s-chart is plotted under three delta limits and six sigma initiatives which are shown in figure 1 and figure 2. Sample numbers are taken on X-axis and values of s is taken on Y-axis and respective points are plotted on the chart.

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Figure 1: s-chart with 3-delta limits

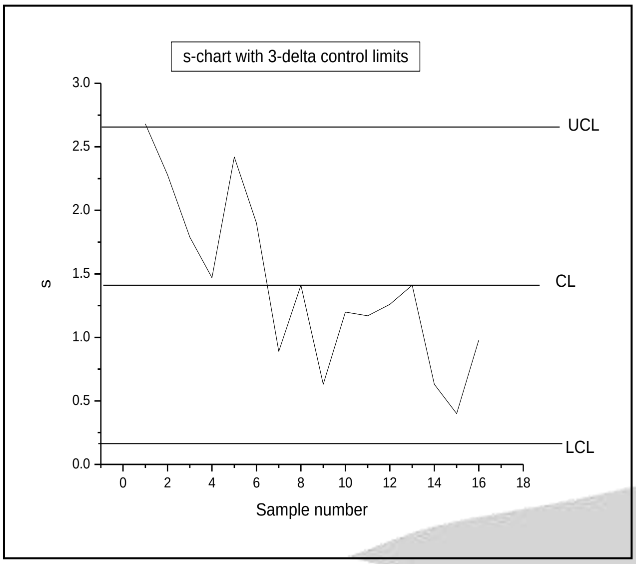
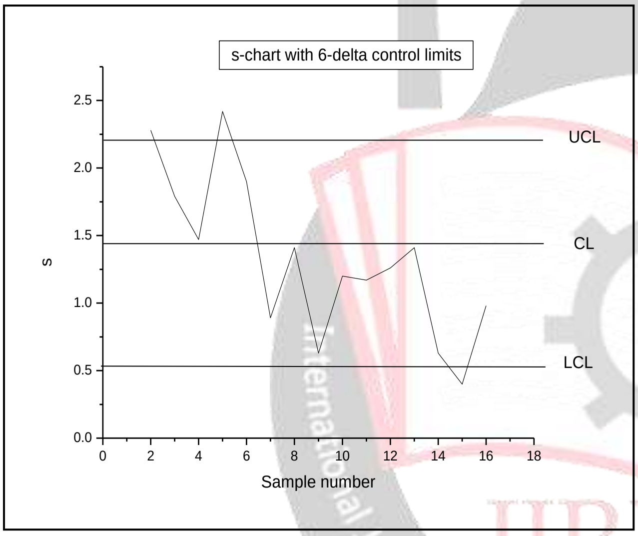


Figure 2: s-chart with 6-delta limits



VI. SUMMARY AND CONCLUSION

In this paper, sample standard deviation (s) chart is discussed under three delta and six delta control limits with an illustration. The main objective of this paper is to use mean deviation instead of standard deviation. As mean deviation is considered more efficient than standard deviation, the charts prepared by using mean deviation perform better than the charts prepared by using standard deviation. From figure 1, it can be seen that the production process is out of statistical control when we are applying 3-delta control limits. Also from figure 2, it can be seen that the process is out of the statistical control when we are using six-delta based control limits. It is very clear from the comparison that there are more points falling out the control limits in six delta initiatives than three delta control limits. So it can be concluded that the chart under six delta control limits are more effective towards detecting the shift in the value of sample standard deviation than the charts under three delta control limits. This is a next generation control chart technique and it can replace existing six sigma technique. So it is recommended that the control charts under six delta control limits should be used for the best results.

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