

Three grade Manpower models having two sources of Depletion with Loss of Manpower as a Geometric process and an Extended Threshold

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Abstract A three grade marketing organization in which attrition lead to random loss of manpower takes place due to policy decisions and transfer decisions is considered. This attrition of personnel will adversely affect the smooth functioning of the organization. Frequent recruitment is not advisable as it involves more cost. Since the loss of manpower and the inter-decision times are probabilistic, the organization requires a suitable recruitment policy to plan for recruitment. In this paper, the problem of time to recruitment based on shock model approach is studied by considering two different models of inter-policy decision times with loss of manpower as a geometric process and the extended threshold gives a better allowable cumulative loss of manpower in the organization. Analytical expressions for the performance measures namely mean and variance for the time to recruitment are obtained. The results are numerically illustrated by assuming specific distributions and relevant conclusions are made.

Keywords — Extended threshold, Inter-policy decision times, Inter-transfer decision times, Loss of manpower as Geometric process, Shock model approach, Three grade manpower system, Univariate CUM policy of recruitment.

AMS Subject Classification (2010): Primary: 90B70, Secondary: 60H30, 60K05.

I. INTRODUCTION

Exit of personnel is a common phenomenon in any marketing organization. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. Frequent recruitments may also be expensive due to the cost of recruitments and training. As the loss of manpower is unpredictable, suitable recruitment policy has to be designed to overcome this loss. Several researchers have studied the problem of time to recruitment for a two grade manpower system using shock model approach. In this context, the authors in [1], [2] and [3] have given the stochastic models for manpower planning and social processes for the usefulness to construct the manpower models. In [7], the authors have obtained the stochastic model for two grade manpower system with wastage as a geometric process which is here referred to find the analytical results for three grade manpower system. The authors in [8] have obtained the mean and variance of the time to recruitment for an organization consisting of two grades (two grade man power system) by assuming that the distribution of loss of man manpower in different decisions and that of the inter-decision times as exponential according as the threshold for the loss of man power in the organization is maximum (minimum) of the exponential

thresholds in the two grades. Recently, in [4] and [5], the authors have obtained the time to recruitment for stochastic model under two sources of depletion of manpower using univariate policy of recruitment by considering various assumptions for breakdown thresholds, loss of manpower and the inter-policy decisions. This paper analyses the research work by assuming that the loss of manpower due to policy decisions forms geometric process, the stochastic models are constructed and the inter-policy decisions times are in the following cases: (Model-I) as a sequence of exchangeable and constantly correlated exponential random variables and (Model-II) as a geometric process. An extended threshold is introduced to give a better allowable cumulative loss of manpower in the manpower system. It is assumed that the inter-policy decisions times for the three grades form the same ordinary renewal process, the intertransfer decisions times for the three grades form the same ordinary renewal process which is different from that of inter-policy decisions. The conventional breakdown threshold used in all the earlier studies is identified as the level of alertness in the present paper. If the organisation is not alert when the cumulative loss of manpower exceeds this level of alertness, an allowable loss of manpower of magnitude D is permitted. However, Recruitment is made whenever the cumulative loss of man power exceeds the extended threshold. A univariate recruitment policy,



usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory is used. Analytical results related to time to recruitment are derived and the influence of the nodal parameters on the performance measures are studied and relevant conclusions are made with the help of numerical illustrations.

II. MODEL DESCRIPTION

Consider an organization with three grades taking policy and transfer decisions at random epochs in $(0,\infty)$. Let $X_{ni}, n = A, B, C \& i = 1, 2, 3, ...$ be a sequence of exponential random variables representing the loss of manpower in grade A,B,C due to ith policy decision which forms geometric process with a positive constant $a_1, a_2, a_3 > 0$, respectively. Let $\overline{X_m}$ be the cumulative loss of manpower for the three grades in the first m policy decisions. Let Y_{Aj} , Y_{Bj} and Y_{Cj} be independent and identically distributed exponential random variables representing the loss of manpower in the organization due to jth transfer decision with mean $\frac{1}{\alpha_{2A}}$, $\frac{1}{\alpha_{2B}}$ and $\frac{1}{\alpha_{2C}}$, α_{2A} , α_{2B} , $\alpha_{2C} > 0$. Let $\overline{Y_n}$ be the cumulative loss of manpower for the three grades in the first n transfer decisions. For i=1,2,..., let U_i be (Model-I) a sequence of exchangeable and constantly correlated exponential random variables with mean λ and correlation $R \in [-1,1]$ with relation $b = \lambda(1-R)$ and (Model-II) a geometric process with a positive constant a > 0 and independent and identically distributed hyper-exponential distribution representing the time between (i-1)th and ith policy decisions with mean $(p_1/\lambda_1) + ((1-p_1)/\lambda_2)$, $0 < p_1 < 1$, where p_1 is the proportion of policy decisions having high attrition rate $\lambda_1 > 0$ and $(1 - p_1)$ is the proportion of policy decisions having low attrition rate $\lambda_2 > 0$. For j=1,2,..., let V_j be independent and identically distributed hyper-exponential random variable representing the time between $(j-1)^{th}$ and j^{th} transfer decisions with mean $(p_2/\lambda_3) + ((1-p_2)/\lambda_4), \quad 0 < p_2 < 1$, where p_2 is the proportion of transfer decisions having high attrition rate $\lambda_3 > 0$ and $(1 - p_2)$ is the proportion of transfer decisions having low attrition rate $\lambda_4 > 0$. Let $N_p(t)$ and $N_{Tr}(t)$ be the number of policy and transfer decisions taken in the organization during the period of recruitment (0,t]. Let $X_{N_{p}(t)}$ and $Y_{N_{T_{r}}(t)}$ be the total loss of manpower in $N_{p}(t)$ decisions and $N_{Tr}(t)$ decisions. Let the cumulative distribution function of the random variable K be $W_{\mathbf{k}}(.)$ (density function $w_{\mathbf{k}}(.)$), and the Laplace transform of $w_{K}(.)$ be $\overline{w}_{K}(.)$. Assume that Z_{A}, Z_{B} and Z_{C} represents the threshold levels for the cumulative loss of manpower in grade Α, В and С with mean $1/\theta_A$, $1/\theta_B$, $1/\theta_c$, respectively, where $\theta_A, \theta_B, \theta_C > 0$. Let Z be the threshold level for the cumulative loss of manpower (alertness level) and D represents the magnitude of allowable loss of manpower.

Z+D is the extended threshold for the cumulative loss of manpower with mean $\frac{1}{\theta_D}$ respectively, where $\theta_D > 0$. Let T be the time to recruitment for the entire organisation. Here, X_i and Y_j are linear and cumulative and Z, X_j , Y_j are statistically independent.

III. MAIN RESULTS

The event of time to recruitment is defined as follows: Recruitment occurs beyond t (t > 0) if and only if the total loss in manpower upto N_P(t) policy decisions and N_{Tr}(t) transfer decisions does not exceeds the breakdown threshold of the organization and it is given by $\{T > t\} \iff \{\widetilde{X}_{N_P(t)} + \widetilde{Y}_{N_{Tr}(t)} < Z\}$ (1)

Hence the probability of occurences of these two events are equal.

$$P(T > t) = P[\widetilde{X}_{N_{P}(t)} + \widetilde{Y}_{N_{Tr}(t)} < Z]$$

$$\tag{2}$$

Invoking the law of total probability and the result of renewal theory, the survival function of time to recruitment is determined. The rth moment for the time to recruitment is determined by taking the rth derivative of the Laplace transform of density function for the random variable with respect to s, and at s=0. Using this result the fundamental performance measures like mean and variance of time to recruitment is determined. Let $Z = (Z_A + Z_B + Z_C) + D$. Conditioning upon D, we get the distribution of the threshold and taking derivative for the Laplace transform of T at s=0, gives the mean time to recruitment.

$$E(T) = (C_1 - C_2 + C_3)T_D - C_1T_1 + C_2T_2 - C_3T_3$$
(3)

Model-I:

In Model-I, inter-policy decision times are assumed to form a sequence of exchangeable and constantly correlated exponential random variables with mean λ and correlation $R \in [-1,1]$ with relation $b=\lambda(1-R)$. Taking derivative for the Laplace transform of the random variable T with respect to s and at s = 0, the mean time to recruitment for the present case is derived. In Eq.(3), we substitute

$$T_{D} = \begin{bmatrix} \frac{\psi_{1}\psi_{2} - (1-F)f_{1}}{\psi_{1}\psi_{2}} \end{bmatrix}_{m=0}^{\infty} \left(\frac{-mb}{(1-R)} - \frac{(m+1)b}{(1-R)} \right) E^{m} \\ - \left[\frac{(1-F)(\psi_{1}f_{2} - f_{1})}{(\psi_{2} - \psi_{1})\psi_{1}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{1})^{1-m}}{(1-R) + b\psi_{1}(1-R+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(\psi_{1}f_{2} - f_{1})}{(\psi_{2} - \psi_{1})\psi_{1}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{1})^{-m}}{(1-R) + b\psi_{1}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{1-m}}{(1-R) + b\psi_{2}(1-R+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1-R+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-R)(1+b\psi_{2})^{-m}}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right]_{m=0}^{\infty} \left(\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right]_{m=0}^{\infty} \left(\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right) E^{m} \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right]_{m=0}^{\infty} \left(\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right) \\ + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(1-R) + b\psi_{2}(1+mR)} \right]_{m=0}$$

For i=1,2,3 and j=1,3,5



$$\begin{split} & \left| \frac{\gamma_{j}\gamma_{j+1} - (1 - B_{i})d_{j}}{\gamma_{j}\gamma_{j+1}} \right|_{m=0}^{\infty} \left(\frac{-mb}{(1 - R)} - \frac{(m + 1)b}{(1 - R)} \right) A_{i}^{m} \\ & - \left[\frac{(1 - B_{i})(\gamma_{j}d_{j+1} - d_{j})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1 - R)(1 + b\gamma_{j})^{1 - m}}{(1 - R) + b\gamma_{j}(1 - R + mR)} \right) A_{i}^{m} \\ T_{i} = \left| + \left[\frac{(1 - B_{i})(\gamma_{j}d_{j+1} - d_{j})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1 - R)(1 + b\gamma_{j})^{-m}}{(1 - R) + b\gamma_{j}(1 + mR)} \right) A_{i}^{m} \\ & + \left[\frac{(1 - B_{i})(d_{j} - d_{j+1}\gamma_{j+1})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j+1}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1 - R)(1 + b\gamma_{j+1})^{1 - m}}{(1 - R) + b\gamma_{j+1}(1 - R + mR)} \right) A_{i}^{m} \\ & + \left[\frac{(1 - B_{i})(d_{j} - d_{j+1}\gamma_{j+1})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j+1}^{2}} \right]_{m=0}^{\infty} \left(\frac{(1 - R)(1 + b\gamma_{j+1})^{1 - m}}{(1 - R) + b\gamma_{j+1}(1 - R + mR)} \right) A_{i}^{m} \end{split}$$

The second moment of the random variable T is derived by differentiating twice the Laplace transform of T with respect to s and at s = 0. From these results the variance of time to recruitment for the present model is determined. *Model-II*:

In Model-II, inter-policy decision times are assumed to form a geometric process with a positive constant a > 0 and independent and identically distributed hyper-exponential distribution representing the time between (i-1)th and ith policy decisions with mean $(p_1/\lambda_1) + ((1-p_1)/\lambda_2)$, $0 < p_1 < 1$, where p_1 is the proportion of policy decisions having high attrition rate $\lambda_1 > 0$ and $(1-p_1)$ is the proportion of policy decisions decisions having high attrition rate $\lambda_2 > 0$. Taking derivative for the Laplace transform of the random variable T with respect to s and at s = 0, the mean time to recruitment for the present case is derived. In Eq.(3), we substitute

$$\begin{aligned} & \left[\frac{\psi_{1}\psi_{2} - (1-F)f_{1}}{\psi_{1}\psi_{2}} \right]_{m=0}^{\infty} \left(\left\{ -\left[\frac{p_{1}}{\lambda_{1}} + \frac{(1-p_{1})}{\lambda_{2}} \right] \left[\frac{a^{m}-1}{a^{m-1}(a-1)} \right] \right\} \right)_{m=0}^{m} \right. \\ & \left. -\left\{ -\left[\frac{p_{1}}{\lambda_{1}} + \frac{(1-p_{1})}{\lambda_{2}} \right] \left[\frac{a^{m+1}-1}{a^{m}(a-1)} \right] \right\} \right)_{m=0}^{m} \right. \\ & \left. -\left[\frac{(1-F)(\psi_{1}f_{2} - f_{1})}{(\psi_{2} - \psi_{1})\psi_{1}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\psi_{1}}{a^{r-1}} \right) \right) E^{m} \right. \\ & \left. +\left[\frac{(1-F)(\psi_{1}f_{2} - f_{1})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\psi_{2}}{a^{r-1}} \right) \right) E^{m} \right. \\ & \left. +\left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\psi_{2}}{a^{r-1}} \right) \right) E^{m} \right. \\ & \left. +\left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\psi_{2}}{a^{r-1}} \right) \right) E^{m} \right. \\ & \left. + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\psi_{2}}{a^{r-1}} \right) \right) E^{m} \right] \right] \\ & \left. + \left[\frac{(1-F)(f_{1} - f_{2}\psi_{2})}{(\psi_{2} - \psi_{1})\psi_{2}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\psi_{2}}{a^{r-1}} \right) \right) E^{m} \right] \\ & \left. + \left[\sum_{r=1}^{m} \sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \right] \right] \\ & \left. + \left[\sum_{r=1}^{m} \sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right] \right] \right] \right] \\ & \left[\sum_{r=1}^{m} \sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \right] \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right] \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \left(\sum_{r=1}^{m} \overline{w}_{T}^{(r)} \right) \right] \left[\sum_{r=1}^{m} \overline{w}_{T}^{($$

$$\begin{cases} \left[\frac{\gamma_{j}\gamma_{j+1} - (1-B_{i})d_{j}}{\gamma_{j}\gamma_{j+1}} \right]_{m=0}^{\infty} \left\{ \left\{ -\left[\frac{P_{1}}{\lambda_{1}} + \frac{(1-P_{1})}{\lambda_{2}} \right] \right] \frac{a^{m} - 1}{a^{m} - (a-1)} \right] \right\} \\ -\left\{ -\left[\frac{P_{1}}{\lambda_{1}} + \frac{(1-P_{1})}{\lambda_{2}} \right] \right] \frac{a^{m+1} - 1}{a^{m} (a-1)} \right] \right\} \\ -\left[\frac{(1-B_{i})(\gamma_{j}d_{j+1} - d_{j})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m} \overline{w}_{U}^{(r)} \left(\frac{\gamma_{j}}{a^{r-1}} \right) \right) A_{i}^{m} \\ +\left[\frac{(1-B_{i})(q_{j}d_{j+1} - d_{j})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m+1} \overline{w}_{U}^{(r)} \left(\frac{\gamma_{j}}{a^{r-1}} \right) \right) A_{i}^{m} \\ +\left[\frac{(1-B_{i})(d_{j} - d_{j+1}\gamma_{j+1})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j+1}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m+1} \overline{w}_{U}^{(r)} \left(\frac{\gamma_{j+1}}{a^{r-1}} \right) \right) A_{i}^{m} \\ +\left[\frac{(1-B_{i})(d_{j} - d_{j+1}\gamma_{j+1})}{(\gamma_{j+1} - \gamma_{j})\gamma_{j+1}^{2}} \right] \sum_{m=0}^{\infty} \left(\prod_{r=1}^{m+1} \overline{w}_{U}^{(r)} \left(\frac{\gamma_{j+1}}{a^{r-1}} \right) \right) A_{i}^{m} \end{cases}$$

The second moment of the random variable T is derived by differentiating twice the Laplace transform of T with respect to s and at s = 0. From these results the variance of time to recruitment for the present model is determined. In all the above cases, the following notations are used

$$\begin{split} &C_{1} = \frac{\theta_{B}\theta_{C}\theta_{D}(\theta_{B} - \theta_{C})}{(\theta_{A} - \theta_{B})(\theta_{B} - \theta_{C})(\theta_{A} - \theta_{C})(\theta_{A} - \theta_{D})};\\ &C_{2} = \frac{\theta_{A}\theta_{C}\theta_{D}(\theta_{A} - \theta_{C})}{(\theta_{A} - \theta_{B})(\theta_{B} - \theta_{C})(\theta_{A} - \theta_{C})(\theta_{B} - \theta_{D})};\\ &C_{3} = \frac{\theta_{A}\theta_{B}\theta_{D}(\theta_{A} - \theta_{B})}{(\theta_{A} - \theta_{B})(\theta_{B} - \theta_{C})(\theta_{A} - \theta_{C})(\theta_{C} - \theta_{D})};\\ &E = \prod_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{D}}{a_{1}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{D}}{a_{2}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{D}}{a_{3}^{r-1}}\right);\\ &A_{1} = \prod_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{A}}{a_{1}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{A}}{a_{2}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{A}}{a_{3}^{r-1}}\right);\\ &A_{2} = \prod_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{C}}{a_{1}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{B}}{a_{2}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{B}}{a_{3}^{r-1}}\right);\\ &A_{3} = \prod_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{C}}{a_{1}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{C}}{a_{2}^{r-1}}\right)_{r=1}^{m} \overline{w}_{U}^{r} \left(\frac{\theta_{C}}{a_{3}^{r-1}}\right);\\ &F = \overline{w}_{Y_{A}}(\theta_{D})\overline{w}_{Y_{B}}(\theta_{D})\overline{w}_{Y_{C}}(\theta_{D});\\ &B_{2} = \overline{w}_{Y_{A}}(\theta_{B})\overline{w}_{Y_{B}}(\theta_{B})\overline{w}_{Y_{C}}(\theta_{B});\\ &B_{3} = \overline{w}_{Y_{A}}(\theta_{C})\overline{w}_{Y_{B}}(\theta_{C})\overline{w}_{Y_{C}}(\theta_{C});\\ &f_{1} = d_{1} = d_{3} = d_{5} = p_{2}\lambda_{3}\lambda_{4} + (1 - p_{2})\lambda_{3}\lambda_{4};\\ &f_{1}^{*} = \lambda_{3}+\lambda_{4} - Fp_{2}\lambda_{3} - F(1 - p_{2})\lambda_{4};\\ &f_{1}^{*} = \lambda_{3}\lambda_{4} - Fp_{2}\lambda_{3}\lambda_{4} - F(1 - p_{2})\lambda_{4};\\ &f_{2}^{*} = \lambda_{3}\lambda_{4} - Fp_{2}\lambda_{3}\lambda_{4} - F(1 - p_{2})\lambda_{3};\\ &\psi_{1} = \frac{f_{1}^{*} - \sqrt{f_{1}^{*2} - 4f_{2}^{*}}}{2};\\ &\psi_{2} = \frac{f_{1}^{*} + \sqrt{f_{1}^{*2} - 4f_{2}^{*}}}{2};\\ &\psi_{2} = \frac{f_{1}^{*} + \sqrt{f_{1}^{*2} - 4f_{2}^{*}}}{2};\\ \end{aligned}$$



For i=1,2,3 and j=1,3,5

$$d_j^* = \lambda_3 + \lambda_4 - B_i p_2 \lambda_3 - B_i (1 - p_2) \lambda_4;$$

 $d_{j+1}^* = \lambda_3 \lambda_4 - B_i p_2 \lambda_3 \lambda_4 - B_i (1 - p_2) \lambda_3 \lambda_4;$
 $\gamma_j = \frac{d_j^* - \sqrt{d_j^{*2} - 4d_{j+1}^*}}{2}; \gamma_{j+1} = \frac{d_j^* + \sqrt{d_j^{*2} - 4d_{j+1}^*}}{2};$
 $\overline{w}_U(s) = \frac{p_1 \lambda_1}{\lambda_1 + s} + \frac{(1 - p_1) \lambda_2}{\lambda_2 + s}; \overline{w}_U(0) = 1;$
 $E \neq F \neq A_i \neq B_i \neq 1, i = 1, 2, 3$
 $\psi_1 \neq \psi_2, \gamma_j \neq \gamma_{j+1}, j = 1, 3, 5$

IV. NUMERICAL ILLUSTRATION

The mean and variance of time to recruitment for all the models are numerically illustrated by using OCTAVE. *Model-I*:

The effect of the nodal parameters a_1, a_2, a_3 and R on the mean and variance of time to recruitment are shown in the following Table-1.

 $\begin{array}{c} \textbf{TABLE-1} \\ \alpha_{1A} = 0.9; \, \alpha_{1B} = 0.8; \, \alpha_{1C} = 0.8; \, \alpha_{2A} = 0.2; \, \alpha_{2B} = \\ 0.29; \, \alpha_{2C} = 0.7; p_1 = 0.6; \, p_2 = 0.5; \, \theta_A = 0.41; \, \theta_B = \\ 0.4; \, \theta_C = 0.3; \, \theta_D = 0.51; \, \lambda = 0.75; \, \lambda_2 = 0.75; \, \lambda_4 = 0.8 \end{array}$

-	a 2	<i>a</i> 3	R	Model-I		
<i>a</i> ₁				E(T)	V(T)	
3	4	4	0.15	4.6032	4.6131	
4	4	4	0.15	4.5986	4.6533	
0.1	4	4	0.15	4.4568	5.2554	
0.2	4	4	0.15	4. <mark>544</mark> 3	4.7567	
4	2	4	0.15	4.6250	4.1762	
4	3	4	0.15	4.6088	4.4893	
4	0.8	4	0.15	4.6510	2.9937	
4	0.9	4	0.15	4.6503	3.1875	
4	4	8	0.15	4.6167	4.5734	les al
4	4	9	0.15	4.6183	4.5676	
4	4	0.8	0.15	4.3073	6.2329	
4	4	0.9	0.15	4.3556	5.9702	
4	4	3	0.1	4.6296	4.2705	
4	4	3	0.2	4.5634	4.9481	
4	4	3	-0.02	4.8366	2.2672	
4	4	3	-0.01	4.8147	2.4804	

Model-II:

The effect of the nodal parameters a_1, a_2, a_3 and a on the mean and variance of time to recruitment are shown in the following Table-2.

TABLE-2

 $\begin{array}{l} \alpha_{1A} = 0.9; \, \alpha_{1B} = 0.8; \, \alpha_{1C} = 0.8; \, \alpha_{2A} = 0.2; \, \alpha_{2B} = \\ 0.5; \, \alpha_{2C} = 0.4; \, p_1 = 0.2; \, p_2 = 0.35; \, \theta_A = 0.6; \, \theta_B = \\ 0.7; \, \theta_C = 0.3; \, \theta_D = 0.51; \, \lambda_1 = 0.5; \, \lambda_2 = 0.05; \, \lambda_3 = \\ 0.6; \, \lambda_4 = 0.2 \end{array}$

	a 2			Model-II	
<i>a</i> ₁		a_3	a	E(T)	V(T)
3	4	8	2	1.6016	6.2894
4	4	8	2	1.6070	6.2350
0.1	4	8	2	1.1807	8.8351
0.2	4	8	2	1.3122	8.7986
4	2	8	2	1.5876	6.4414
4	3	8	2	1.6008	6.2963
4	0.8	8	2	1.5219	7.3017
4	0.9	8	2	1.5341	7.1370
4	8	5	2	1.6104	6.2020
4	8	6	2	1.6126	6.1816
4	8	0.1	2	1.1619	8.8189
4	8	0.2	2	1.2908	8.8105
2	4	8	2	1.5900	6.4185
2	4	8	3	1.9698	0.3734
2	4	8	0.8	1.3215	14.474
2	4	8	0.9	1.2949	14.061

V. CONCLUSION

In Table-1,

- 1. When $0 < a_1 < 1$ and a_1 increases, the mean increase and the variance of time to recruitment decrease for a_1 and when $a_1 > 1$ and a_1 increases, the mean decrease and the variance of time to recruitment increase for a_1 .
- 2. When $0 < a_2 < 1$ and a_2 increases, the mean decrease and the variance of time to recruitment increase for a_2 and when $a_2 > 1$ and a_2 increases, the mean decrease and the variance of time to recruitment increase for a_2 .
- 3. When $0 < a_3 < 1$ and a_3 increases, the mean increase and the variance of time to recruitment decrease for a_3 and when $a_3 > 1$ and a_3 increases, the mean increase and the variance of time to recruitment decrease for a_3 .
- 4. For the increasing negative values of R, the mean decrease and the variance of time to recruitment increase and for the increasing positive values of R, the mean decrease and the variance of time to recruitment increase.

In Table-2,

- 1. When $0 < a_1 < 1$ and a_1 increases, the mean increase and the variance of time to recruitment decrease for a_1 and when $a_1 > 1$ and a_1 increases, the mean increase and the variance of time to recruitment decrease for a_1 .
- 2. When $0 < a_2 < 1$ and a_2 increases, the mean increase and the variance of time to recruitment decrease for a_2 and when $a_2 > 1$ and a_2



increases, the mean increase and the variance of time to recruitment decrease for a_2 .

- 3. When 0 < a₃ < 1 and a₃ increases, the mean increase and the variance of time to recruitment decrease for a₃ and when a₃ > 1 and a₃ increases, the mean increase and the variance of time to recruitment decrease for a₃.
- 4. When 0 < a < 1 and a increases, the mean and the variance of time to recruitment decrease for a and when a > 1 and a increases, the mean increase and the variance of time to recruitment decrease for a.

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