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Radio Geometric Mean Number of Shadow of Star And Bistar

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Abstract: A radio Geometric Mean Labeling of a connected graph G is a one to one map f from the vertex set V(G) to the set of natural numbers N such that for two distinct vertices u and v of G, $d(u,v)+\left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1+diam(G)$. The radio geometric mean number of f, $r_{gmn}(f)$ is the maximum number assigned to any vertex of G. The radio geometric mean number of G, $r_{gmn}(G)$ is the minimum value of $r_{gmn}(f)$ taken over all radio geometric mean labeling f of G. In this paper, we determine the radio geometric mean number of shadow graph of star and bistar.

Keywords — Radio Geometric Mean labeling, Star, Bistar, Diameter.

I. INTRODUCTION

We consider finite, simple, undirected graphs only. Let V(G) and E(G) respectively denote vertex set and edge set of G. Chartand et al.[1] defined the concept radio labeling of G in 2001. Radio labeling of graphs is applied in channel assignment problem [1]. Radio number of several graphs determined [2,7,5,9]. In this sequence Ponraj et al.[8] introduced the radio mean labeling in G. Here we introduce a new type of labeling, a radio geometric mean labeling is a one to one mapping f from V(G) to N satisfying the condition

$$d(u,v)+\left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 1+diam(G)$$

for every $u,v \in V(G)$.

The span of a labeling f is the maximum integer that f maps to a vertex of graph G. The radio geometric mean number of G, r_{gmn}(G) is the lowest span taken over all radio geometric mean labeling of the graph G. In this paper we determine the radio geometric mean number of some star like graphs. Let *x* be any real number. Then $\lceil x \rceil$ stands for smallest integer greater than or equal to *x*. Terms and definitions not defined here are followed from Harary [12] and Gallian [13].

The channel assignment to radio transmitters is one of the main objectives in setup of wireless communication system. A proper channel assignment to radio transmitters which satisfies interference constraints with maximum use of spectrum is a need of wireless communication system. The interference constraints between a pair of transmitters is closely related with separation of channels and distance between transmitters. In a network, if two transmitters are closer then higher the interference between them and large separation.

Definition 1.1 A **Star** is the complete bipartite graph $K_{1,n}$.

Definition 1.2 The graph **Bistar** $B_{n,n}$ obtained by joining the center vertices of two copies of $K_{1,n}$ with an edge.

Definition 1.3 [14] The Shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G as G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

II. MAIN RESULTS

Theorem 2.1 Radio Geometric Mean number of Shadow of star, $r_{gmn}\left(D_2\left(K_{1,n}\right)\right)=2(n+1)$

Proof: Consider $K_{1,n}$ with the vertex set $\{u,u',u_i,u_i':1\leq i\leq n\}$ where u_i,u_i' are the pendant vertices. $|V(G)|=2(n+1)$ and $|E(G)|=4n$.

The diameter of the shadow of star is 2.

We define the labeling *f* as follows,

Assign the labels of the vertices u,u',u_i,u_i' be

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$$f(u)=2n+1$$
$$f(u')=2(n+1)$$
$$f(u_i)=2i-1 \quad ;1\le i\le n$$
$$f(u_i')=2i \quad ;1\le i\le n$$

Now we check the radio geometric mean condition for any two vertices, it should satisfy

$$d(u,v)+\left\lceil\sqrt{f(u)f(v)}\right\rceil\ge 1+diam(G)=1+2=3$$

Case (1): Check the pair (u,u')

$$d(u,u')+\left\lceil\sqrt{f(u)f(u')}\right\rceil\ge 1+\left\lceil\sqrt{(2n+1).2(n+1)}\right\rceil\ge 5$$

Case (2): Check the pair (u,u_i)

$$d(u,u_i)+\left\lceil\sqrt{f(u)f(u_i)}\right\rceil\ge 1+\left\lceil\sqrt{(2n+1).(1)}\right\rceil\ge 3$$

Case (3): Check the pair (u,u_i')

$$d(u,u_i')+\left\lceil\sqrt{f(u)f(u_i')}\right\rceil\ge 1+\left\lceil\sqrt{(2n+1).(2)}\right\rceil\ge 4$$

Case (4): Verify the pair (u',u_i')

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil\ge 1+\left\lceil\sqrt{2(n+1)(2)}\right\rceil\ge 4$$

Case (5): Verify the pair (u',u_i)

$$d(u',u_i)+\left\lceil\sqrt{f(u')f(u_i)}\right\rceil\ge 2+\left\lceil\sqrt{2(n+1)(1)}\right\rceil\ge 4$$

Case (6): Verify the pair $(u_i,u_j),\ i\ne j$

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil\ge 2+\left\lceil\sqrt{(1)(3)}\right\rceil\ge 4$$

Case (7): Verify the pair $(u_i',u_j'),\ i\ne j$

$$d(u_i',u_j')+\left\lceil\sqrt{f(u_i')f(u_j')}\right\rceil\ge 2+\left\lceil\sqrt{(2)(4)}\right\rceil\ge 5$$

Case (8): Verify the pair (u_i,u_j')

Subcase (i): If $i=j$

$$d(u_i,u_j')+\left\lceil\sqrt{f(u_i)f(u_j')}\right\rceil\ge 2+\left\lceil\sqrt{(1)(2)}\right\rceil\ge 4$$

Subcase (ii): If $i\ne j$

$$d(u_i,u_j')+\left\lceil\sqrt{f(u_i)f(u_j')}\right\rceil\ge 2+\left\lceil\sqrt{(3)(2)}\right\rceil\ge 5$$

Hence every pair of vertices satisfies the radio geometric mean condition.

Thus $r_{gmn}(D_2(K_{1,n}))=2(n+1)$.

Theorem 2.2 Radio Geometric Mean number of Shadow of bistar, $r_{gmn}(D_2(B_{n,n}))=4(n+1)$.

Proof: Consider $B_{n,n}$ with the vertex set $\{u,v,u_i,v_i:1\le i\le n\}$ where u_i,v_i are the pendant vertices. In order to obtain $Spl(B_{n,n})$ add u',v',u_i',v_i' vertices corresponding to u,v,u_i,v_i where $1\le i\le n$.

$|V(G)|=4(n+1)$ and $|E(G)|=4(2n+1)$.

The diameter of the splitting of bistar is 3.

We define the labeling *f* as follows,

Assign the labels of the vertices u,v,u_i,v_i be

$$f(u)=4n+3$$
$$f(u')=4(n+1)$$
$$f(u_i)=2i-1 \quad ;1\le i\le n$$
$$f(u_i')=2i \quad ;1\le i\le n$$

and the labels of the vertices u',v',u_i',v_i' be

$$f(v)=2n+1$$
$$f(v')=2(n+1)$$
$$f(v_i')=2(n+1)+2i \quad ;1\le i\le n$$
$$f(v_i)=2n+2i-1 \quad ;1\le i\le n$$

Now we check the radio geometric mean condition for any two vertices, it should satisfy

$$d(u,v)+\left\lceil\sqrt{f(u)f(v)}\right\rceil\ge 1+diam(G)=1+3=4$$

Case (1): Check the pair (u,u')

$$d(u,u')+\left\lceil\sqrt{f(u)f(u')}\right\rceil\ge 1+\left\lceil\sqrt{(4n+3).4(n+1)}\right\rceil\ge 5$$

Case (2): Check the pair (u,u_i)

$$d(u,u_i)+\left\lceil\sqrt{f(u)f(u_i)}\right\rceil\ge 1+\left\lceil\sqrt{(4n+3).(1)}\right\rceil\ge 4$$

Case (3): Check the pair (u,u_i')

$$d(u,u_i')+\left\lceil\sqrt{f(u)f(u_i')}\right\rceil\ge 1+\left\lceil\sqrt{(4n+3).(2)}\right\rceil\ge 4$$

Case (4): Verify the pair (u',u_i')

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil\ge 1+\left\lceil\sqrt{4(n+1)(2)}\right\rceil\ge 5$$

Case (5): Verify the pair (u',u_i)

$$d(u',u_i)+\left\lceil\sqrt{f(u')f(u_i)}\right\rceil\ge 2+\left\lceil\sqrt{4(n+1)(1)}\right\rceil\ge 4$$

Case (6): Verify the pair $(u_i,u_j),\ i\ne j$

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil\ge 2+\left\lceil\sqrt{(1)(3)}\right\rceil\ge 4$$

Case (7): Verify the pair $(u_i',u_j'),\ i\ne j$

$$d(u_i',u_j')+\left\lceil\sqrt{f(u_i')f(u_j')}\right\rceil\ge 2+\left\lceil\sqrt{(2)(4)}\right\rceil\ge 5$$

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Case (8): Verify the pair (u_i, u_j')

Subcase (i): If $i = j$

$d(u_i, u_j') + \lceil \sqrt{f(u_i) f(u_j')} \rceil \geq 2 + \lceil \sqrt{(1)(2)} \rceil \geq 4$

Subcase (ii): If $i \neq j$

$d(u_i, u_j') + \lceil \sqrt{f(u_i) f(u_j')} \rceil \geq 2 + \lceil \sqrt{(3)(2)} \rceil \geq 5$

Case (9): Check the pair (v, v')

$d(v, v') + \lceil \sqrt{f(v) f(v')} \rceil \geq 1 + \lceil \sqrt{(2n+1).2(n+1)} \rceil \geq 5$

Case (10): Check the pair (v, v_i')

$d(v, v_i') + \lceil \sqrt{f(v) f(v_i')} \rceil \geq 1 + \lceil \sqrt{(2n+1).(2n+4)} \rceil \geq 6$

Case (11): Check the pair (v, v_i)

$d(v, v_i) + \lceil \sqrt{f(v) f(v_i)} \rceil \geq 1 + \lceil \sqrt{(2n+1).(2n+3)} \rceil \geq 5$

Case (12): Verify the pair (v', v_i')

$d(v', v_i') + \lceil \sqrt{f(v_i) f(v_j)} \rceil \geq 1 + \lceil \sqrt{2(n+1)(2n+4)} \rceil \geq 5$

Case (13): Verify the pair (v', v_i)

$d(v', v_i) + \lceil \sqrt{f(v') f(v_i)} \rceil \geq 2 + \lceil \sqrt{2(n+1)(2n+3)} \rceil \geq 7$

Case (14): Verify the pair (v_i, v_j), $i \neq j$

$d(v_i, v_j) + \lceil \sqrt{f(v_i) f(v_j)} \rceil \geq 2 + \lceil \sqrt{(2n+3)(2n+5)} \rceil \geq 10$

Case (15): Verify the pair (v_i', v_j'), $i \neq j$

$d(v_i', v_j') + \lceil \sqrt{f(v_i') f(v_j')} \rceil \geq 2 + \lceil \sqrt{(2n+4)(2n+6)} \rceil \geq 11$

Case (16): Verify the pair (v_i, v_j')

Subcase (i): If $i = j$

$d(v_i, v_j') + \lceil \sqrt{f(v_i) f(v_j')} \rceil \geq 2 + \lceil \sqrt{(2n+3)(2n+4)} \rceil \geq 10$

Subcase (ii): If $i \neq j$

$d(v_i, v_j') + \lceil \sqrt{f(v_i) f(v_j')} \rceil \geq 2 + \lceil \sqrt{(2n+3)(2n+6)} \rceil \geq 11$

Case (17): Check the pair (u_i', v_j')

Subcase (i): If $i = j$

$d(u_i', v_j') + \lceil \sqrt{f(u_i') f(v_j')} \rceil \geq 3 + \lceil \sqrt{(2)(2n+4)} \rceil \geq 7$

Subcase (ii): If $i \neq j$

$d(u_i', v_j') + \lceil \sqrt{f(u_i') f(v_j')} \rceil \geq 3 + \lceil \sqrt{(4)(2n+4)} \rceil \geq 9$

Case (18): Check the pair (u_i', v')

$d(u_i', v') + \lceil \sqrt{f(u_i') f(v')} \rceil \geq 2 + \lceil \sqrt{(2).2(n+1)} \rceil \geq 5$

Case (19): Check the pair (u_i', v)

$d(u_i', v) + \lceil \sqrt{f(u_i') f(v)} \rceil \geq 2 + \lceil \sqrt{(2).(2n+1)} \rceil \geq 6$

Case (20): Check the pair (u_i', v_j)

Subcase (i): If $i = j$

$d(u_i', v_j) + \lceil \sqrt{f(u_i') f(v_j)} \rceil \geq 3 + \lceil \sqrt{(2)(2n+3)} \rceil \geq 7$

Subcase (ii): If $i \neq j$

$d(u_i', v_j) + \lceil \sqrt{f(u_i') f(v_j)} \rceil \geq 3 + \lceil \sqrt{(4)(2n+3)} \rceil \geq 9$

Case (21): Verify the pair (u', v_i')

$d(u', v_i') + \lceil \sqrt{f(u') f(v_i')} \rceil \geq 2 + \lceil \sqrt{4(n+1).(2n+4)} \rceil \geq 9$

Case (22): Check the pair (u', v')

$d(u', v') + \lceil \sqrt{f(u') f(v')} \rceil \geq 1 + \lceil \sqrt{4(n+1).2(n+1)} \rceil \geq 7$

Case (23): Verify the pair (u', v)

$d(u', v) + \lceil \sqrt{f(u') f(v)} \rceil \geq 1 + \lceil \sqrt{4(n+1)(2n+1)} \rceil \geq 6$

Case (24): Check the pair (u', v_i)

$d(u', v_i) + \lceil \sqrt{f(u') f(v_i)} \rceil \geq 2 + \lceil \sqrt{4(n+1).(2n+3)} \rceil \geq 9$

Case (25): Check the pair (u, v_i')

$d(u, v_i') + \lceil \sqrt{f(u) f(v_i')} \rceil \geq 2 + \lceil \sqrt{(4n+3).(2n+4)} \rceil \geq 9$

Case (26): Verify the pair (u, v')

$d(u, v') + \lceil \sqrt{f(u) f(v')} \rceil \geq 2 + \lceil \sqrt{(4n+3)2(n+1)} \rceil \geq 7$

Case (27): Verify the pair (u, v)

$d(u, v) + \lceil \sqrt{f(u) f(v)} \rceil \geq 1 + \lceil \sqrt{(4n+3)(2n+1)} \rceil \geq 6$

Case (28): Check the pair (u, v_i)

$d(u, v_i) + \lceil \sqrt{f(u) f(v_i)} \rceil \geq 2 + \lceil \sqrt{(4n+3).(2n+3)} \rceil \geq 8$

Case (29): Verify the pair (u_i, v_j')

Subcase (i): If $i = j$

$d(u_i, v_j') + \lceil \sqrt{f(u_i) f(v_j')} \rceil \geq 3 + \lceil \sqrt{(1)(2n+4)} \rceil \geq 6$

Subcase (ii): If $i \neq j$

$d(u_i, v_j') + \lceil \sqrt{f(u_i) f(v_j')} \rceil \geq 3 + \lceil \sqrt{(3)(2n+4)} \rceil \geq 8$

Case (30): Verify the pair (u_i, v')

$d(u_i, v') + \lceil \sqrt{f(u_i) f(v')} \rceil \geq 2 + \lceil \sqrt{(1).2(n+1)} \rceil \geq 4$

Case (31): Verify the pair (u_i, v)

$d(u_i, v) + \lceil \sqrt{f(u_i) f(v)} \rceil \geq 2 + \lceil \sqrt{(1)(2n+1)} \rceil \geq 4$

Case (32): Verify the pair (u_i, v_j)

Subcase (i): If $i = j$

$d(u_i, v_j) + \lceil \sqrt{f(u_i) f(v_j)} \rceil \geq 3 + \lceil \sqrt{(1)(2n+3)} \rceil \geq 6$

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Subcase (ii): If $i \neq j$

$$d(u_i, v_j) + \left\lceil \sqrt{f(u_i) f(v_j)} \right\rceil \geq 3 + \left\lceil \sqrt{(3)(2n+3)} \right\rceil \geq 8$$

Hence every pair of vertices satisfies the radio geometric mean condition.

Thus $r_{gmn}\left(D_2\left(B_{n,n}\right)\right) = 4(n+1)$.

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