Combined effect of MHD, couple stress and surface roughness on curved annular plates

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Abstract: **The squeeze film characteristics between rough curved annular plates lubricated with conducting non-Newtonian fluid in the presence of an external magnetic field is investigated in the present study. Based upon the Magneto hydrodynamic flow theory together with the Stokes micro-continuum theory and Christensen theory, the modified Reynold's equation is derived and applied to predict the squeeze film characteristics. The expressions for mean squeeze film pressure, mean load-carrying capacity and squeeze film time are obtained. The results are presented both numerically and graphically and compared with conducting smooth surface case. It is found that the squeeze film characteristics are more pronounced for rough curved annular plates with increasing values of Hartmann number and couple stress parameter.**

Keywords — Squeeze film, MHD, Couple stress, Surface roughness, curved annular plates, Non-Newtonian fluid.

I. Introduction

Squeeze film characteristics plays a significant role in many areas of engineering and most of the engineering techniques are purely based on the squeeze film process. Studies of squeeze film behaviour are of practical significance in lubrication of machine tools, automotive engines, aircraft engines, turbo machinery and skeletal joints. The analysis of squeeze film performance assumes that the lubricant behaves essentially as a Newtonian viscous fluid. Thus to stabilize the flow properties and to enhance the lubricating qualities, the use of different additives has been considered. In the classical theory of fluids, micro continuum theory derived by Stokes[1] is the simplest theory which describes the couple stress concepts. Many investigators [2-4] have used the Stoke's model to study the various problems of hydrodynamic lubrication.

Measure of texture in a surface is taken as surface roughness in most of the bearings. Surface roughness effect is seen in many fields such as science, engineering and industrial applications. Since bearing surfaces are rough to some extent due to the manufacturing process, wear and impulsive damage. So, to enhance the performance of hydrodynamic lubrication in various bearings, it becomes important to evaluate the influence of surface roughness. Several theories have been proposed to study the effect of surface roughness on the bearing performances such as Davies[5] modelled used saw-tooth curve, Burton[6] used Fourier series type approximation and Mitchell[7] modelled used high frequency sine curve. Due to random character of surface roughness, the stochastic method developed by Christensen[8] is considered to study surface roughness in hydrodynamic lubrication. This model assumes that the probability density function for the random variable characterizing the roughness is symmetric with the mean of the random variable equal to zero. Based on this model, there are two types of roughness patterns which are of much interest in the roughness theory; one is radial roughness and other one is azimuthal roughness. Several authors [9-12] used this model to study the roughness effects on bearing performances.

Magneto hydrodynamic (MHD) is the study of the interaction of conducting fluids with electromagnetic phenomena. The squeeze film action in bearings is improved by using electrically conducting lubricants. Recently, the study of magneto hydrodynamics (MHD) have given more importance by many researcher in the field of lubrication of bearings, since it prevents the unexpected difference of viscosity for lubricant with temperature under sever operating conditions.

In the literature, quite a good number of authors [13-15] have studied the effect of MHD and surface roughness. All above authors noticed that roughness pattern along with MHD play an important role to improve the performance of squeeze film characteristics of the bearings system. Recently, Hanumagowda et.al [16-17] have analyzed the effect of surface roughness with MHD for various bearing configurations and concluded that the mean film pressure, mean load supporting capacity and squeeze film time are more proclaimed for azimuthal roughness than radial roughness.

The objectives of present work is to study the squeeze film characteristics of rough curved annular plates with MHD and couple stress which is not discussed so far and obtained numerical findings are compared with smooth case studied by Hanumagowda.et.al[20]

II. Mathematical formulation

A pictorial presentation of the bearing system in which the lower plate is rough and separated by central thickness h_m of fluid film in the presence of external magnetic field B_0 which is perpendicular to plates is shown in Figure 1

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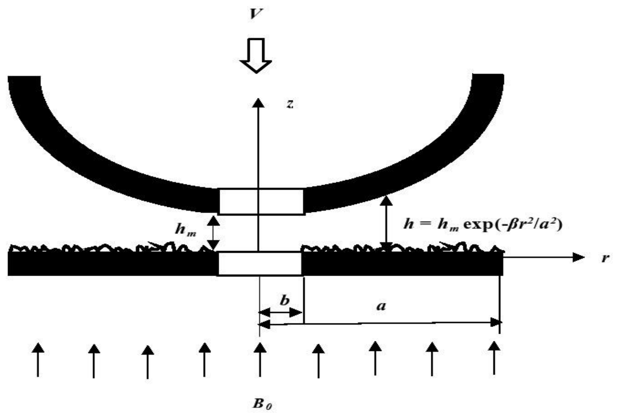

Figure 1: Pictorial presentation of rough curved annular plates.

The film shape h is taken to be an exponential type as Lin. et.al[18]

$$h = h_m \exp(-\beta r^2/a^2), \qquad b \le r \le a \qquad (1)$$

Where β is curvature parameter and h_m is the minimum film thickness.

The modified Reynolds equation for MHD squeeze film with couple stress between smooth surfaces derived by Hanumagowda.et.al [19] is,

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ rG(h,l,M_0)\frac{\partial p}{\partial r}\right\} = \mu V \qquad (2)$$

Where,

$$G(h,l,M_0) = \begin{cases} \frac{h_0^2}{M_0^2}\{\xi_1 + h\}, & \text{for } M_0^2 l^2/h_0^2 < 1 \\ \frac{h_0^2}{M_0^2}\{\xi_2 + h\}, & \text{for } M_0^2 l^2/h_0^2 = 1 \\ \frac{h_0^2}{M_0^2}\{\xi_3 + h\}, & \text{for } M_0^2 l^2/h_0^2 > 1 \end{cases}$$

$$\xi_1 = \frac{2l}{(A^2 - B^2)}\left(\frac{B^2}{A}\tanh\frac{Ah}{2l} - \frac{A^2}{B}\tanh\frac{Bh}{2l}\right)$$

$$\xi_2 = \frac{h}{2}\sec h^2\left(\frac{h}{2\sqrt{2l}}\right) - 3\sqrt{2l}\tanh\left(\frac{h}{2\sqrt{2l}}\right)$$

$$\xi_3 = \frac{2lh_0}{M_0}\left(\frac{(A_2\cot\theta - B_2)\sin B_2 h - (B_2\cot\theta + A_2)\sin A_2 h}{\cos B_2 h + \cosh A_2 h}\right)$$

To study surface roughness, the stochastic model is considered in which film thickness is divided into two parts and given by

$$H = h + h_s(r,\theta,\xi) \qquad (3)$$

The probability distribution function is given by $f(h_s)$,

$$f(h_s) = \begin{cases} \frac{35}{32c^7}(c^2 - h_s^2)^3, & -c < h_s < c \\ 0, & \text{elsewhere} \end{cases}$$

Where h_s is the stochastic film thickness, $\bar{\sigma}$ is standard deviation and $c = 3\bar{\sigma}$.

The modified stochastic Reynolds equation is found by taking the stochastic average of (2) with respect to $f(h_s)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ rE(G(H,l,M_0))\frac{\partial E(p)}{\partial r}\right\} = \mu V \qquad (4)$$

Where, $E(\bullet) = \int_{-\infty}^{\infty}(\bullet) f(h_s)dh_s$

As per stochastic theory by Christensen [8] surface roughness consists of two parts, namely radial roughness and azimuthal roughness pattern in one dimension.

Radial roughness pattern.

In 1-D radial roughness pattern, the surface roughness is in the form of long, narrow ridges and valleys running in *r*-direction and the film thickness is given by

$$H = h + h_s(\theta,\xi) \qquad (5)$$

Thus modified-stochastic Reynold's equation (4) is written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ rE(G(H,l,M_0))\frac{\partial E(p)}{\partial r}\right\} = \mu V \qquad (6)$$

Azimuthal roughness pattern.

In 1-D Azimuthal roughness pattern, the surface roughness is in the form of long, narrow ridges and valleys running in z- direction and the film thickness is given by

$$H = h + h_s(r,\xi) \qquad (7)$$

Thus modified-stochastic Reynold's equation (4) is written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ r\frac{1}{E(1/(G(H,l,M_0))}\frac{\partial E(p)}{\partial r}\right\} = \mu V \qquad (8)$$

Combining equations (6) and (8), the resultant expression is written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{ K(H,l,M_0,C)r\frac{\partial E(p)}{\partial r}\right\} = \mu V \qquad (9)$$

Where

$$K(H,l,M_0,C) = \begin{cases} E(G(H,l,M_0)), & \text{Radial Roughness} \\ E(1/G(H,l,M_0)), & \text{Azimuthal Roughness} \end{cases}$$

For Radial Roughness:

$$E(G(H,l,M_0)) = \frac{35}{32c^7}\int_{-c}^{c} G(H,l,M_0)(c^2 - h_s^2)^3 dh_s \qquad (10)$$

For Azimuthal Roughness:

$$E\left(\frac{1}{G(H,l,M_0)}\right) = \frac{35}{32c^7}\int_{-c}^{c}\frac{(c^2 - h_s^2)^3}{G(H,l,M_0)}dh_s \qquad (11)$$

Introducing the following non-dimensional parameters.

$$r^* = \frac{r}{a},\ h_m^* = \frac{h_m}{h_0},\ H^* = \frac{h}{h_0} + \frac{h_s}{h_0} = h^* + h_s^*$$

$$l^* = \frac{2l}{h_0},\ P^* = -\frac{h_m{}^3 E(p)}{\mu a^2 V},\ C = \frac{c}{h_0}$$

The modified Reynolds equation (9) takes the form

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left\{ K^*(H^*,l^*,M_0,C)r^*\frac{\partial P^*}{\partial r^*}\right\} = -1 \qquad (12)$$

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Where,

$$K^*(H^*, l^*, M_0, C) = \begin{cases} E(G^*(h_m^*, l^*, M_0)), & \text{Radial Roughness} \\ E\left(1/G^*(h_m^*, l^*, M_0)\right), & \text{Azimuthal Roughness} \end{cases}$$

and

$$G^*(h_m^*, l^*, M_0) = \begin{cases} \frac{1}{M_0^2}\{\xi_1^* + h_m^*\}, & \text{for } M_0^2 l^{*2} < 1 \\ \frac{1}{M_0^2}\{\xi_2^* + h_m^*\}, & \text{for } M_0^2 l^{*2} = 1 \\ \frac{1}{M_0^2}\{\xi_3^* + h_m^*\}, & \text{for } M_0^2 l^{*2} > 1 \end{cases}$$

$$\xi_1^* = \frac{l^*}{(A^{*2} - B^{*2})}\left(\frac{B^{*2}}{A^*}\tanh\frac{A^* h_m^*}{l^*} - \frac{A^{*2}}{B^*}\tanh\frac{B^* h_m^*}{l^*}\right)$$

$$\xi_2^* = \frac{h_m^*}{2}\sec h^2\left(\frac{h_m^*}{\sqrt{2l^*}}\right) - \frac{3l^*}{\sqrt{2}}\tanh\left(\frac{h_m^*}{\sqrt{2l^*}}\right)$$

$$\xi_3^* = \frac{l^*\left(A_2^*\cot\theta^* - B_2^*\right)\sin B_2^* h_m^* - l^*\left(B_2^*\cot\theta^* + A_2^*\right)\sinh A_2^* h_m^*}{M_0(\cos B_2^* h_m^* + \cosh A_2^* h_m^*)}$$

The boundary conditions to present the squeeze problem is,

$$P^* = 0 \quad \text{at} \quad r^* = \delta = b/a \tag{13a}$$

$$P^* = 0 \quad \text{at} \quad r^* = 1 \tag{13b}$$

The non-dimensional film pressure P^* is obtained by solving equation (12) using the boundary conditions (13a) and (13b) by integration and is,

$$P^* = \frac{f_2(r^*)\, f_1(1) - f_1(r^*)\, f_2(1)}{2\, f_2(1)} \tag{14}$$

$$\text{Where, } f_1(r^*) = \int_{r^*=\delta}^{r^*} \frac{r^*}{K^*} dr^* \quad \text{and} \quad f_2(r^*) = \int_{r^*=\delta}^{r^*} \frac{1}{r^* K^*} dr^* \tag{15}$$

The load supporting capacity is derived by solving pressure field over the film region by integration and is,

$$W = \int_{r=b}^{a} 2\pi r p\, dr \tag{16}$$

The non-dimensional load supporting capacity W^* is given by

$$W^* = \frac{E(W) h_m^3}{2\pi\mu a^4(-dh_m/dt)} =$$

$$= \frac{-1}{2}\int_{r^*=\delta}^{1} f_1(r^*) r^* dr^* + \frac{1}{2}\frac{f_1(1)}{f_2(1)}\int_{r^*=\delta}^{1} f_2(r^*) r^* dr^* \tag{17}$$

The non-dimensional squeeze film time for the film thickness is given by

e non-dimensional squeeze film time for the film thickness is given b

$$T^* = \frac{E(W) h_m^2}{\pi\mu a^4} t =$$

$$\int_{h_m^*}^{1}\left(\frac{2\, f_2(1)}{f_2(1)\int_{r^*=\delta}^{1} f_1(r^*) r^* dr^* - f_1(1)\int_{r^*=\delta}^{1} f_2(r^*) r^* dr^*}\right) dh_m^* \tag{18}$$

III. RESULTS AND DISCUSSION

In the current investigation, the behavior of squeeze film between rough annular plates is analyzed. The results are presented graphically for different values of operating parameters namely Roughness parameter C = 0, 0.1, 0.2, Hartmann number M_0 = 0, 2, 4, Couple stress parameter l^* = 0, 0.2, 0.4, Curvature parameter β = -0.5, 0, 0.5 and radius ratio δ = 0.2, 0.4, 0.6.

Limiting cases of the present study:

(a) As $C \to 0$, present analysis reduces to smooth case discussed by Hanumagowda.et.al [20],

(b) As $C \to 0, l^* \to 0$, present analysis reduces to Magnetic case studied by Lin.et.al [18].

(c) As $C \to 0, M_0 \to 0$ and $l^* \to 0$ present analysis reduces to Newtonian case studied by Gupta and Vora [21]

The squeeze film characteristics of present analysis are compared with Hanumagowda.et.al [20] analysis and shown in the Table 1 excellent agreements of results were found.

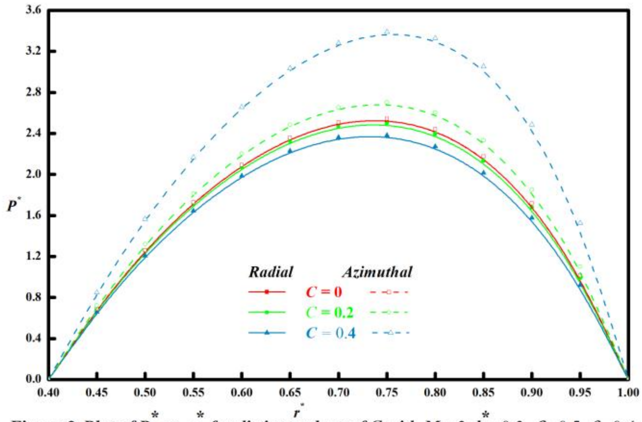

Figure 2: *Plot of P^* vs r^* for distinct values of C with M_0=3, l^*=0.3, β=0.5, δ=0.4.*

Non-Dimensional Film Pressure:

The variation of non-dimensional film pressure P^* with the dimensionless co-ordinate r^* for distinct values of C is elaborated in Figure 2 with M_0 = 3, l^*= 0.3, β = 0.5 and δ = 0.4 for both roughness patterns. It is noticed that for increasing values of C, pressure P^* increases (decreases) for azimuthal (radial) roughness patterns and also it is interesting to note that at C = *0* it corresponds to smooth case (both roughness patters coincides). The variation of P^* versus r^* for distinct values of M_0 with C = 0.2, l^* = 0.3, β = 0.5 and δ = 0.4 is illustrated in Figure 3 and it is observed that the effect of Hartmann number M_0, enhances the film pressure P^*.

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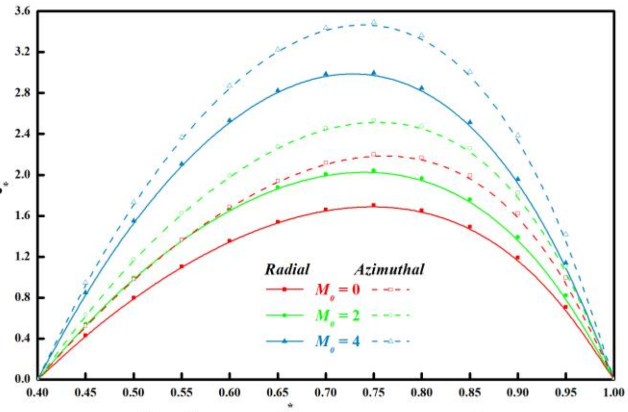

Figure 3:*Plot of* P^* *vs* r^* *for distinct values of* M_0 *with C=0.2,* l^**=0.3,* β*=0.5,* δ*=0.4.*

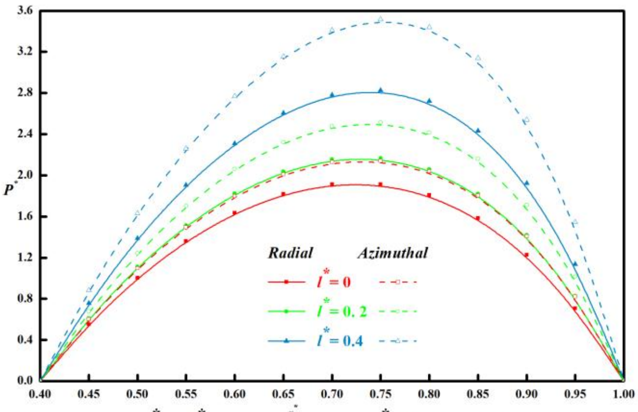

Figure 4:*Plot of* P^* *vs* r^* *for distinct values of* l^* *with* M_0*=3, C=0.3,* β*=0.5,* δ*=0.4.*

Figure 4 represents the deviation of P^* versus r^* for various values of l^* with C = 0.2, M_0 = 3, β = 0.5 and δ = 0.4 and it is seen that for increasing value of l^*, there is a significant raise in film pressure. The deviation of P^* along r^* for various values of β with C = 0.2, M_0 = 3, l^* = 0.3 and δ = 0.4 is illustrated in Figure 5 for both roughness patterns and it is found that P^* increases with increasing values of β.

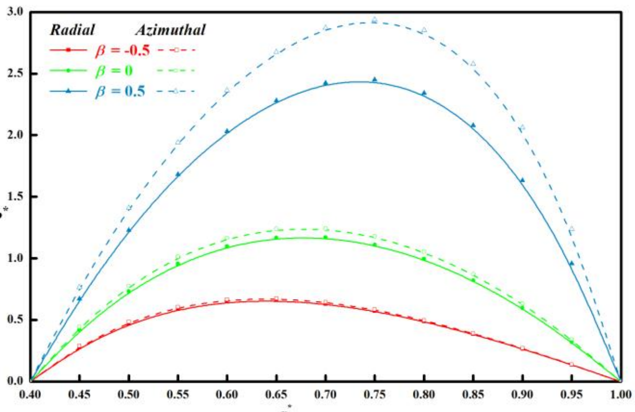

Figure 5: *Plot of* P^* *vs* r^* *for distinct values of* β *with* M_0*=3,* l^* *=0.3, C=0.3,* δ*=0.4.*

Non-Dimensional Load supporting capacity:

The deviation of non-dimensional load supporting capacity W^* with curvature parameter β as a function of roughness parameter C is plotted in Figure 6 with M_0 = 3, l^* = 0.3 and δ = 0.4. It is noticed that for increasing values of C, load carrying capacity W^* increases (decreases) for azimuthal (radial) roughness patterns Further it is noticed that W^* enhances for increasing values of β.

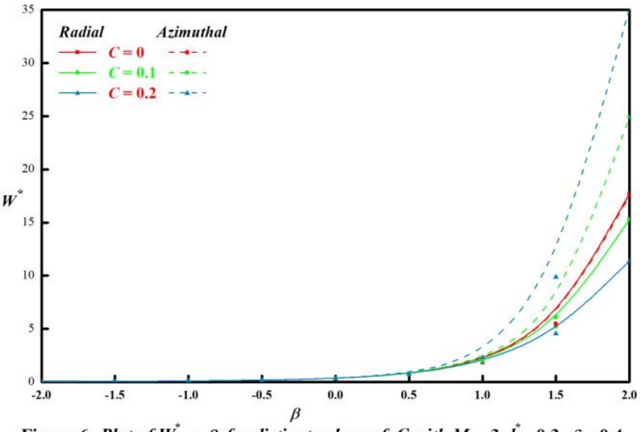

Figure 6: *Plot of* W^* *vs* β *for distinct values of C with* M_0 *= 3,* l^* *= 0.3,* δ *= 0.4.*

Figure 7 displays the variation of W^* along β for different values of M_0 with C = 0.2, l^* = 0.3 and δ = 0.4 and it is found that for increasing value of Hartmann number M_0, W^* increases when compared to Non-magnetic case. The deviation of W^* with β for various values of l^* with C = 0.2, M_0 = 3 and δ = 0.4 is depicted in Figure 8 and observed that due the effect of couple stress parameter there is an enhancement in the load supporting capacity as compared to Newtonian case.

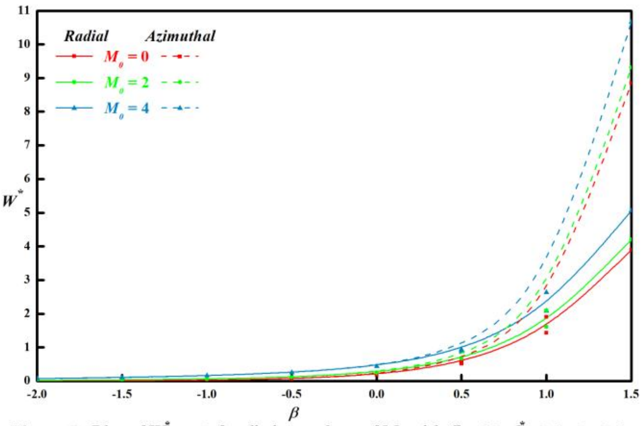

Figure 7: *Plot of* W^* *vs* β *for distinct values of* M_0 *with C = 0.2,* l^* *= 0.3,* δ *= 0.4.*

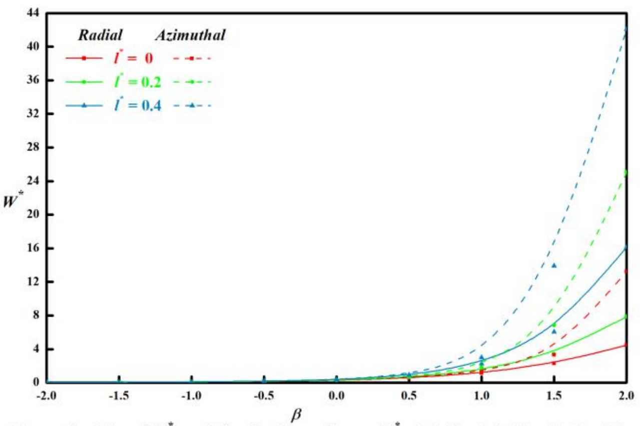

Figure 8 : *Plot of* W^* *vs* β *for distinct values of* l^* *with C = 0.2,* M_0 *= 3,* δ *= 0.4.*

Figure 9 represents the variation of W^* versus β for distinct values of radius ratio δ with C = 0.2, M_0 = 3 and l^* = 0.3 and it is observed that for increasing values of radius ratio δ, the load supporting capacity decreases considerably for both roughness patterns. Hence for better performance of bearing, the radius ratio should be minimum.

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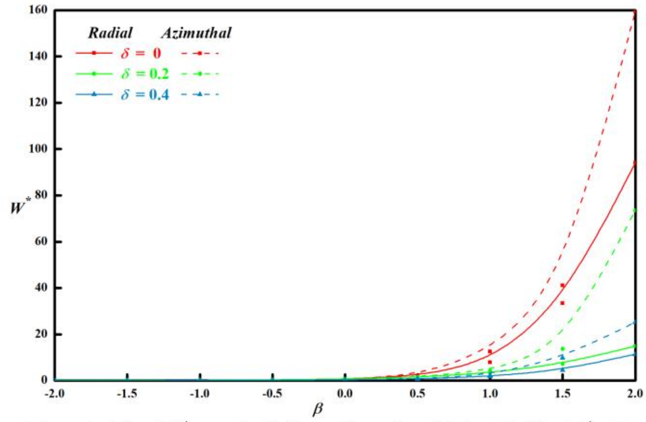

Figure 9: *Plot of W^* vs β for distinct values of δ with* C = 0.2, M_0 = 3, l^* = 0.3.

Non-Dimensional Squeeze film time:

Figure 10 represents, the variation of non-dimensional squeeze film time T^* against film height h_m^* for distinct of roughness parameter C with M_0 = 3, l^* = 0.3, β = 0.5 and δ = 0.4. From the figure it is clear that at $C = 0$ both the roughness patterns reduces to smooth case also it is observed that for increasing values of C, squeeze film time T^* increases (decreases) for azimuthal (radial) roughness patterns. Further it is observed T^* decreases with increasing values of h_m^*.

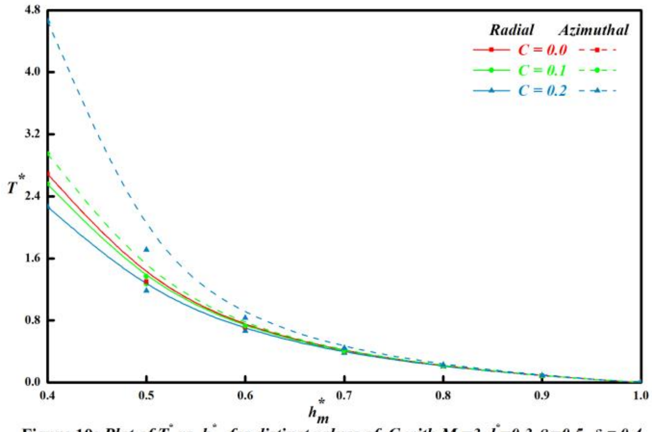

Figure 10: *Plot of T^* vs h_m^* for distinct values of C with M_0=3, l^*=0.3, β=0.5, δ = 0.4.*

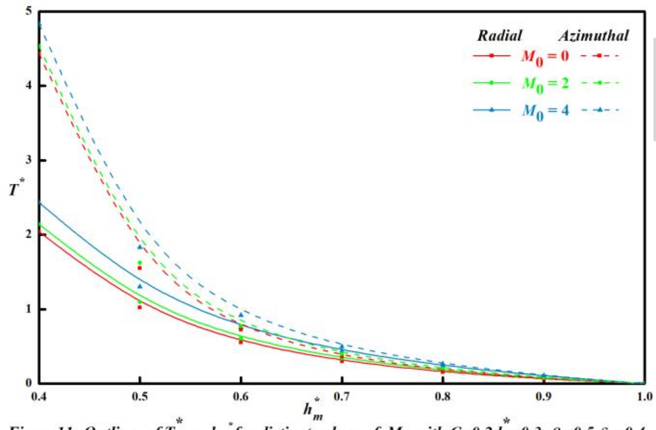

Figure11: Outline of T^* vs h_m^* for distinct values of M_0 with C=0.2, l^*=0.3, β=0.5, δ =0.4.

The variation of T^* against h_m^* for various values of M_0 with C = *0.2*, l^* = 0.3, β = 0.5 and δ = 0.4 is elaborated in Figure 11 and it is found that squeeze film time T^* increases with increasing values of Hartmann number M_0. Figure 12 shows the variation of T^* against h_m^* for different values of l^* with C = 0.2, M_0 = 3, β = 0.5 and δ = 0.4 and it is seen that T^* significantly increases for increasing values of l^* as compared to $l^* \to 0$.

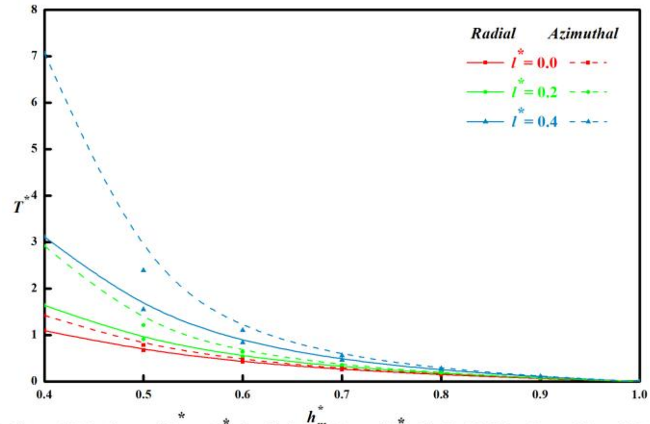

Figure12:Outline of T^* vs h_m^* for distinct values of l^* with C=0.2, M_0=3, β=0.5, δ=0.4.

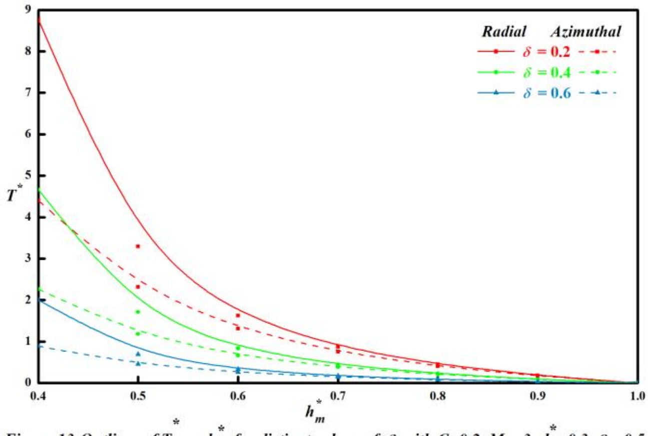

Figure 13:Outline of T^* vs h_m^* for distinct values of δ with C=0.2, M_0=3, l^*=0.3, β =0.5.

Figure 13 depicts, the variation of T^* with h_m^* for various values of δ with C = 0.2, M_0 = 3, l^* = 0.3 and β = 0.5 and observed that T^* decreases for increasing values of radius ratio δ. The variation of T^* along h_m^* for different values of β is illustrated in Figure 14 with C = 0.2, M_0 = 3, l^* = 0.3 and δ = 0.4 for both roughness patterns and it is noticed that the increasing values of β significantly increases squeeze film time T^*.

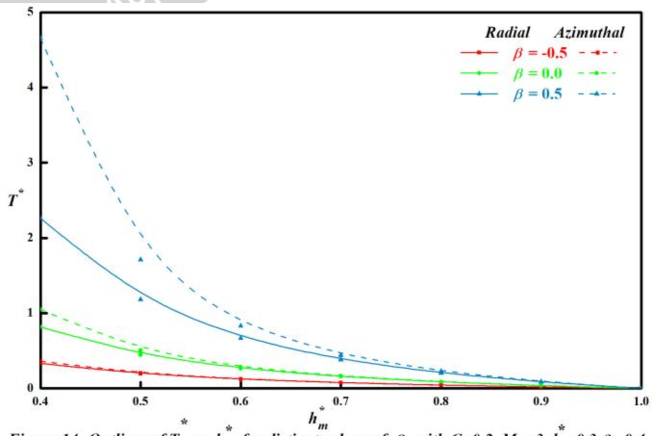

Figure 14: Outline of T^* vs h_m^* for distinct values of β with C=0.2, M_0=3, l^*=0.3,δ=0.4.

IV. CONCLUSION

A combined effect of surface roughness with MHD and couple stress fluid between curved annular plates based on Stoke's theory for couple stress fluids and Christensen

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stochastic model for the surfaces roughness is investigated in the present study. From the obtained results and discussion the following conclusion can be drawn.

The effect of azimuthal (radial) roughness patterns increases (decreases) the pressure, the load supporting capacity and the squeeze film time. Also when C = 0, both the patterns reduces to smooth case discussed by Hanumagowda.et.al [20].

The effect of Hartmann number enhances the pressure, load supporting capacity and squeeze film time as compared with the corresponding non-magnetic case. The squeeze film characteristics increases for increasing values of l^* as compared to Newtonian case. The mean load supporting capacity and squeeze film time decreasing for increasing values of radius ratio δ. The squeeze film characteristics increases for larger values of *β*.

NOMENCLATURE

a: Inner radius of the plate
b: Outer radius of the plate
B_0 : Applied magnetic field
M_0 : Hartmann number $(= B_0 h_0 (\sigma/\mu)^{1/2})$
h_0 : Initial film thickness
p : Pressure in the film region
P^*: Non-dimensional mean squeeze film pressure
l : Couple stress parameter $(\eta/\mu_0)^{1/2}$
l^* : Non-dimensional couple stress parameter $(2l/h_0)$
h_m : Minimum film thickness
h_m^* : Non-dimensional minimum film thickness
u, w : Velocity components in r and z directions
r, z : Radial and Axial coordinates
W : Load carrying capacity
W^* : Non-dimensional mean load carrying capacity
t : Response time
T^* : Non-dimensional response time
c : Maximum asperity deviation from the nominal film height
C : Dimensionless roughness parameter (c/h_0)
E : Expectancy operator defined by Eq. (12)
H : Film thickness $(h + h_s)$
H^* : Non-dimensional film thickness
β : Curvature parameter
η : Material constant responsible for couple stresses
μ : Lubricant viscosity
σ : Electrical conductivity
$\bar{\sigma}$: Standard deviation(c /3)
δ : Radius ratio

Table1: Numerical comparison of the Squeeze film characteristics W^* and T^* between Hanumagowda.et.al [20] and present analysis with fixed $h^*=0.6$, $\beta=0.5$, $\delta=0.4$.

	M_0	Hanumagowda et.al[20]		Present analysis					
				$C=0$		$C=0.2$, $l^*=0.2$		$C=0.2$, $l^*=0.4$	
		$l^*=0.2$	$l^*=0.4$	$l^*=0.2$	$l^*=0.4$	Radial	Azimuthal	Radial	Azimuthal
W^*	0	0.4471	0.6538	0.4473	0.6539	0.4362	0.4777	0.6303	0.7136
	2	0.5371	0.7482	0.5374	0.7484	0.5293	0.5694	0.7290	0.8103
	4	0.7980	1.0202	0.7983	1.0206	0.7961	0.8342	1.0101	1.0884
	6	1.2200	1.4566	1.2204	1.4570	1.2242	1.2614	1.4565	1.5322
T^*	0	0.9051	1.5896	0.9051	1.5897	0.8479	1.0564	1.4417	1.9615
	2	0.9990	1.6890	0.9990	1.6890	0.9469	1.1540	1.5489	2.0664
	4	1.2729	1.9788	1.2729	1.9788	1.2321	1.4378	1.8571	2.3719
	6	1.7161	2.4450	1.7161	2.4450	1.6877	1.8946	2.3457	2.8607

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