Algorithm For Pareto-Type III Software Reliability Growth Model

[1]V.S.Akilandeswari1, [2]V.Saavithri

[1]Assistant professor, Saranathan College of Engineering, Trichy,akilaharish22@gmail.com

[2]Assistant professor, Nehru Memorial College, Trichy.

Abstract - In this paper, Pareto-Type III Order Statistics distribution is created and its parameters are estimated by unconstrained optimization technique. An algorithm for Software reliability growth Model is developed based on this distribution. A dataset is assumed to follow Pareto-Type III Order Statistics distribution. Maximum likelihood values at all orders are estimated and for the maximum out of it, software failure detection is done for the corresponding dataset.

Keywords - Pareto-Type III distribution, Order Statistics, Unconstrained Optimization Technique, Non-Homogeneous Poisson Process (NHPP), Statistics Process Control (SPC).

I. INTRODUCTION

Software Reliability [6] is defined as the probability that a software system operates with no failure occurring for a specified time on specified operating conditions. Assessing software reliability and thereby maintaining software quality during software development and software usage is most important. Software Reliability Growth Models (SRGM) can be used to test software reliability. These models detect the software failure which can be eradicated and hence increasing the life time of the software which in turn increases the reliability of the software too.

Let X be the random variable denoting the cumulative time between failures. The probability density function of Pareto – Type III is $f(x)=\dfrac{\alpha s^{\alpha}}{x^{\alpha+1}}$ where $x \geq 0$, $s > 0$ is the scale parameter, $\alpha > 0$ is the shape parameter. The cumulative distribution function is $F(x)=1-\left(\dfrac{s}{x}\right)^{\alpha}$.

Let us suppose that (X_1, ..., X_n) are n jointly distributed random variables. The X_i's are arranged in increasing order is its corresponding order statistics. Thus $X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n}$. An attractive expression for the joint density of the order statistics [5] corresponding to independent and identically distributed sample from an absolutely continuous distribution with density $f(x)$ is

$$f_{X_{1:n},\ldots,X_{n:n}}(x_1,x_2,\ldots,x_n)=n!\prod_{i=1}^{n} f(x_i), -\infty < x_1 < x_2 < \ldots < x_n < \infty$$

Non-Homogeneous Poisson Process (NHPP) models are also termed as fault counting models. Depending on the specification, the models can be categorized as finite and infinite failure models. In this model, the number of failures follows NHPP distribution. Based on the NHPP assumptions, the failure intensity function $\lambda(x)$ is defined as $\lambda(x)=af(x)$ where 'a' is the expected number of failures and $f(x)$ is the probability density function of X. Mean value function is $m(x) = aF(x)$ where $F(x)$ is the cumulative distribution function of X and $a=\dfrac{n}{F(x_n)}$.

Monitoring the failure occurrence process using the time chart is straightforward [7].To calculate the control limits of the X_r-chart, the exact probability limits will be used. If α is the accepted false alarm risk then the upper control limit, UCL_r, the central line, CL_r and lower control limit, LCL_r can be easily calculated using

$$F(UCL_r,r,\lambda)=1-\alpha/2$$
$$F(CL_r,r,\lambda)=0.5$$
$$F(LCL_r,r,\lambda)=\alpha/2$$

if the random variable is taken as representing inter failure time of a device, a control chart for such a data would be based on 0.9973 probability limits of the times between failures. These limits and the central line are respectively the solutions of

$$F(UCL_r,r,\lambda)=0.99865$$
$$F(CL_r,r,\lambda)=0.5$$
$$F(LCL_r,r,\lambda)=0.00135$$

If the plotted point falls below the LCL, it indicates that the process average or the failure occurrence that may have increased which results in a decrease in the failure time. This means that process may have deteriorated and thus actions should be taken to identify the causes, which may be removed.

Vamsidhar. Y., Srinivas. Y., Achanta Brahmini., [8] presented a Pareto-Type III SRGM based on Non-Homogeneous Poisson Process (NHPP). Akilandeswari V.S., Poornima R. and Saavithri V. developed a software reliability growth model based on Lehmann-type Laplace

 DOI : 10.18231/2454-9150.2018.0785

distribution- Type I[2]. Akilandeswari V.S., Poornima R. and Saavithri V. used Lehmann-Type Laplace distribution Type II (LLD-II)[1] SRGM to test Software Reliability which had a better fit for software failure data than Goel-okumoto, Weibull, Exponential Geometric, Pareto III, Lehmann-Type Laplace distribution Type I (LLD-I)distributions. Akilandeswari V.S., Poornima R. and Saavithri V developed Lehmann-Type Laplace distributions –Type I and Type II [3,4] software reliability growth models too.

In this paper, Pareto-Type III order statistics distribution is framed in section 2 with its parameter estimation. The SRGM based on this distribution is framed in section 3. In section 4, software failure data analysis is performed and paper is concluded in section 5.

II. PARETO-TYPE III ORDER STATISTICS DISTRIBUTION

Let $X_1, X_2, ..., X_n$ be the random variables representing a sample of cumulative time between failures with size n. Let $X_{1:n}, X_{2:n}, ... X_{n:n}$ be the original random variable so that $X_{1:n} \le X_{2:n} \le .... \le X_{n:n}$.

The probability density function of r^{th} order statistics of Pareto – Type III is

$$f_{r:n}(x) = r\binom{n}{r}\frac{\alpha}{x}\left[1-\left(\frac{s}{x}\right)^{\alpha}\right]^{r-1}\left(\frac{s}{x}\right)^{(n-r+1)\alpha} \quad ... (2.1)$$

where $x \ge 0$, $s > 0$, $\alpha > 0$.

The cumulative distribution function is

$$F_{r:n}(x) = \sum_{i=r}^{n}\binom{n}{i}\left[1-\left(\frac{s}{x}\right)^{\alpha}\right]^{i}\left(\frac{s}{x}\right)^{(n-i)\alpha} \quad ... (2.2)$$

2.1 Parameter estimation

Method of Maximum likelihood is used to estimate s and α.

The likelihood function of Pareto-Type III order statistics is

$$l = \prod_{i=1}^{n} r\binom{n}{r}\frac{\alpha}{x_i}\left[1-\left(\frac{s}{x_i}\right)^{\alpha}\right]^{r-1}\left(\frac{s}{x_i}\right)^{(n-r+1)\alpha} \quad ... (2.3)$$

The log likelihood function is

log l =

$$\log\left[\prod_{i=1}^{n} r\binom{n}{r}\frac{\alpha}{x_i}\left[1-\left(\frac{s}{x_i}\right)^{\alpha}\right]^{r-1}\left(\frac{s}{x_i}\right)^{(n-r+1)\alpha}\right] \quad ... (2.4)$$

Using unconstrained optimization technique, the maximum of 'log l' is found.

III. SOFTWARE RELIABILITY GROWTH MODEL

3.1 NHPP model for Pareto – Type III Order Statistics SRGM

The mean value function for this SRGM, using (2.2), is

$$m(x) = a\left[\sum_{i=r}^{n}\binom{n}{i}\left[1-\left(\frac{s}{x}\right)^{\alpha}\right]^{i}\left(\frac{s}{x}\right)^{(n-i)\alpha}\right] \quad ... (3.1)$$

The intensity function, using (2.1), is

$$\lambda(x) = a\frac{\alpha}{x}r\binom{n}{r}\left[1-\left(\frac{s}{x}\right)^{\alpha}\right]^{r-1}\left(\frac{s}{x}\right)^{(n-r+1)\alpha} \quad ... (3.2)$$

The expected number of failures, a in Pareto-type III order statistics SRGM, using (2.2), is

$$a = \frac{n}{\sum_{i=r}^{n}\binom{n}{i}\left[1-\left(\frac{s}{x_n}\right)^{\alpha}\right]^{i}\left(\frac{s}{x_n}\right)^{(n-i)\alpha}} \quad ... (3.3)$$

3.2 ALGORITHM FOR PARETO – TYPE III ORDER STATISTICS SRGM

Step 1: Find the cumulative data of time between failures

Step 2: Choose the value of r

Step 3: Using minimization technique of non-linear unconstrained objective function, find the minimum of –log l in (2.4)

Step 4: *Max f(z)* = *–Min – f(z)*, using this, find the maximum of log l multiplying the value by (-1). The values of α and s that gives the maximum of log l are the optimum values of α and s.

Step 5: Calculate the expected number of failures in (3.3) using these parameters.

Step 6: Find the control limits UCL, LCL and CL.

Step 7: Estimate the mean value function in (3.1) at all failure numbers.

Step 8: Then find the successive differences of mean value functions.

Step 9: Plot the mean value chart taking failure numbers along X-axis and successive differences along Y-axis.

Step10: The failure numbers at which mean value function is below LCL, detects the failure of the software.

IV. DATA ANALYSIS

The following results were obtained for the dataset when tested using Pareto-Type III Order Statistics SRGM.

Dataset

 DOI : 10.18231/2454-9150.2018.0785

ISSN : 2454-9150 Vol-04, Issue-06, Sep 2018

4.1 Cumulative time between failures

Failure Number	Time between failure times in CPU units	Cumulative time between failures
1	5.5	5.5
2	1.83	7.33
3	2.75	10.08
4	70.89	80.97
5	3.94	84.91
6	14.98	99.89
7	3.47	103.36
8	9.96	113.32
9	11.39	124.71
10	19.88	144.59
11	7.81	152.4
12	14.59	166.99
13	11.42	178.41
14	18.94	197.35
15	65.3	262.65
16	0.04	262.69
17	125.67	388.36
18	82.69	471.05
19	0.45	471.5
20	31.61	503.11
21	129.31	632.42
22	47.6	680.02

Table 4.2 gives the maximum likelihood values of dataset when algorithm 3.2 is executed for all possible values of *r*.

Table 4.2 Maximum likelihood values for Pareto-Type III Order Statistics distribution at all possible values of *r*

r	Maximum Likelihood Value
1	-150.8491
2	-147.9171
3	-148.1881
4	-149.8380
5	-152.2310
6	-155.0824
7	-37.4045
8	-161.6030
9	-165.1118
10	-168.7147
11	-172.3697
12	-176.0372
13	-179.6763
14	-183.2399
15	-186.6694
16	-189.8855
17	-192.7728
18	-195.1496
19	-196.6996
20	-196.7892
21	-193.8122
22	-180.7045

From Table 4.2 it is found that maximum likelihood value for dataset is obtained at VII Order statistics of Pareto-Type III. Thus the SRGM is developed at $r = 7$ to detect the failures.

Table 4.1 gives the cumulative data between failures.

Parameters,

$s = 76.3291$

$\alpha = 0.6951$

Expected number of failures, $a = 22$

Table 4.3 gives the mean value function and its successive differences.

Table 4.3 Successive differences of mean value function

Failure Number	Mean value function $m(x)$	Successive differences of $m(x)$
1	-1.1607×10^{23}	1.1498×10^{23}
2	-1.0924×10^{21}	1.0868×10^{21}
3	-5.6333×10^{18}	5.6333×10^{18}
4	3.7258×10^{-4}	**0.0132**
5	0.0136	1.4639
6	1.4775	0.8894
7	2.3669	3.4225
8	5.7894	4.3424
9	10.1318	5.8493
10	15.9811	1.5153
11	17.4964	1.9163
12	19.4127	0.9140
13	20.3267	0.8527
14	21.1794	0.7351
15	21.9145	$\mathbf{1.0955\times10^{-4}}$
16	21.9146	0.0828
17	21.9974	**0.0022**
18	21.9996	$\mathbf{3.8502\times10^{-6}}$
19	21.9996	$\mathbf{1.9434\times10^{-4}}$
20	21.9998	$\mathbf{1.9921\times10^{-4}}$
21	22	$\mathbf{1.2075\times10^{-5}}$
22	22	-

Control Limits

$$UCL = 21.9703$$

$$LCL = 0.0297$$

$$CL = 11$$

Figure 1 gives the mean value chart for Order Statistics distribution of Pareto-Type III SRGM at VII Order. It is found from the graph that the failures are detected at failure points 4, 15, 17, 18, 19, 20 and 21.

DOI : 10.18231/2454-9150.2018.0785

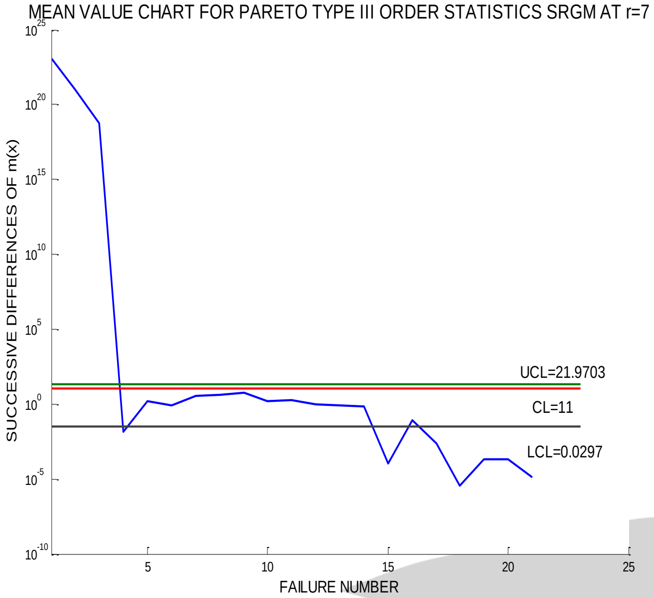

Figure 1

V. CONCLUSION

Here Pareto-Type III Order Statistics Software reliability growth model is developed and it is tested for a dataset using the algorithm framed. Parameters are estimated using unconstrained optimization technique and maximum likelihood values at all orders are evaluated. The maximum out of this is found to be at the VII order and hence the software failure detection is done for VII order statistics of Pareto-Type III distribution and it is detected at seven failure points 4, 15, 17, 18, 19, 20 and 21.

REFERENCES

[1] Akilandeswari V.S., Poornima R. and Saavithri V.,(2015), Software Relaibility Growth Model using Lehmann-Type Laplace Distribution-Type II, International Journal of Applied Engineering Research, 10(13), 32835-32841.

[2] Akilandeswari. V. S, Saavithri. V.,(2016), Lehmann-Type Laplace distribution Type-I Software Reliability Growth Model, DOI 10.1007/s12597-016-0281-6

[3] Akilandeswari. V. S., Saavithri. V., Software Reliability Growth Model based on Order Statistics of Lehmann-Type Laplace Distribution-Type II(2015), IJAER, 10(82), 73-78.

[4] Akilandeswari. V. S., Saavithri. V., Algorithm for Software Reliability Growth Model based on Order Statistics of Lehmann-Type Laplace Distribution-Type I(2016), Res. J. Mathematical and Statistical Sci., 4(8), 1-6.

[5] Balakrishnan. N., and Cohen,A.C., (1991), Order Statistics and Inference: Estimation methods, Academic, Bosten

[6] Pham H. (2006). System Software Reliability, Springer, London.

[7] Xie M., Goh T.N. and Ranjan P. (2002). Some effective control chart procedures for Reliability Monitoring, Reliability engineering and System Safety, 77(1), 143-50.

[8] Vamsidhar. Y., Srinivas. Y., Achanta Brahmini., (2013), Software reliability growth model based on Pareto type-III distribution, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, 2, pp. 2694-2698.

 DOI : 10.18231/2454-9150.2018.0785