

Study on MHD Boundary Layer Flow of A Visco-Elastic and Dissipative Fluid The Presence of Thermal Radiation

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Abstract-In this manuscript a theoretical study has been performed to analyze various properties of an electrically conducting visco-elastic and dissipative fluid past a vertical porous plate bounded by a porous medium in the presence of thermal radiation and variable permeability. The basic concepts like magnetohydrodynamics, visco-elasticity, heat transfer, skin friction and rate of heat transfer are presented. A magnetic field of uniform strength is applied perpendicular to the plate and the presence of heat source is also considered. The non-linear partial differential equations which govern the flow are solved numerically by finite difference scheme. The presence of thermal radiation decreases the temperature. The changes in skin friction and Nusselt number are also observed.

Keywords: MHD, thermal radiation, variable suction, visco-elasticity, vertical porous plate, heat transfer.

I. INTRODUCTION

In MHD generator, the hot fluid is passing through transverse magnetic field, and then electric field will be produced. The liquid metal provides electrical conductivity and inert gas is a convenient carrier to the liquid. The carrier gas is pressurized and heated by passage through the heat exchanger within the combustion chamber. The hot gas is incorporated into the liquid metal to form a working fluid. The liquid metal consists of gas bubbles uniformly dispersed in an approximately equal volume. The studies on MHD visco-elastic fluids with radiation effect past a porous media plays significant role in many scientific, industrial and engineering applications. To recover the water for drinking and irrigation purposes the principles of this flow are followed. Many researchers identified the importance of this flows and contributed in studying the application of visco-elastic fluid flow of several types past porous medium in the presence of thermal radiation. Makinde and Mishra [1] studied chemically reacting MHD mixed convection variable viscosity Blasius flow embedded in a porous medium. Mishra et al. [2] made an attempt on a hybrid computational approach for steady flow of 'Walters' B fluid in a vertical channel with porous wall. Chandra Reddy et al. [3, 4] analyzed magnetohydrodynamic convective double diffusive laminar boundary layer flow past an accelerated vertical plate as well as Soret and Dufour effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate. Dash et al. [5] pointed on numerical approach to boundary layer stagnation-point flow past a stretching/shrinking sheet. Umamaheswar et al. [6] examined on unsteady MHD free convective double diffusive visco-elastic fluid flow past an inclined permeable plate in the presence of viscous dissipation and heat absorption. Srinivas et al. [7]

considered Thermal-diffusion and diffusion-thermo effects on MHD flow of viscous fluid between expanding and contracting rotating porous disks with viscous dissipation. Rashidi et al. [8] established numerical investigation of magnetic field effect on mixed convection heat transfer of nanofluid in a channel with sinusoidal walls. Chandra Reddy et al. [9] explained free convective magneto-nanofluid flow past a moving vertical plate in the presence of radiation and thermal diffusion. Yabo et al. [11] explained Combined Effects of Thermal Diffusion and Diffusion-Thermo Effects on Transient MHD Natural Convection and Mass Transfer Flow in a Vertical Channel with Thermal Radiation. Sidda Reddy et al [12] discussed Thermal diffusion and Joule heating effects on MHD radiating fluid embedded in porous medium. Chandra Reddy et al. [10] explained Diffusion thermo and thermal diffusion effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate.

Motivated by the above studies, in this article a viscous incompressible electrically conducting fluid past a vertical porous plate bounded by a porous medium in the presence of thermal radiation, variable suction and variable permeability is analyzed.

II. FORMULATION OF THE PROBLEM

The unsteady Walters B visco-elastic fluid flow past an infinite vertical porous plate with heat and mass transfer embedded in a porous medium in the presence of thermal radiation, oscillatory suction as well as variable permeability is considered. A uniform magnetic field of strength B_0 is applied perpendicular to the plate. As shown in Fig. 1 let x^1 axis be taken along with the plate in the direction of the flow and y^1 axis is normal to it.

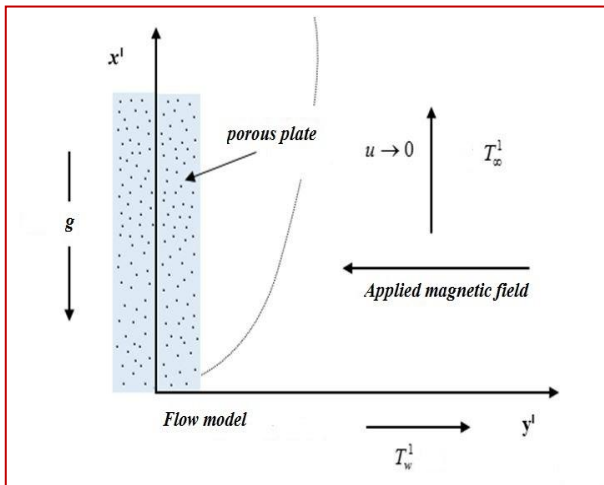


Fig.1: Physical model of the fluid flow

Let us consider the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t^1 \leq 0$, the plate and fluids are at the same temperature and concentration. When $t^1 > 0$, the temperature of the plate is instantaneously raised to T_w^1 . Under the above assumption with usual Boussinesq's approximation, the governing equations and boundary conditions are given by

$$\frac{\partial u'}{\partial t'} + v \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty) - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'(t')} - \frac{k_0}{\rho} \left[\frac{\partial^3 u'}{\partial t' \partial y'^2} + v \frac{\partial^3 u'}{\partial y'^3} \right] \quad (1)$$

$$\frac{\partial T'}{\partial t'} + v \frac{\partial T'}{\partial y'} = K \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_\infty) + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 - \frac{\partial q_r'}{\partial y'} \quad (2)$$

with the boundary conditions

$$u = 0, T' = T_w + \varepsilon(T_w - T_\infty)e^{n't'}, \quad \text{at } y' = 0 \quad (3)$$

$$u \rightarrow 0, T' \rightarrow T_\infty, \quad \text{as } y' \rightarrow \infty$$

The fluid considered here is optically thin with relatively low density, therefore the radiative heat flux is defined as

$$\frac{\partial q_r'}{\partial y} = 4(T' - T_\infty)I. \quad \text{Let the permeability of the porous medium and the suction velocity be of the form}$$

$$K'(t') = K_p(1 + \varepsilon e^{n't'}) \quad (4)$$

$$v(t') = -v_0(1 + \varepsilon e^{n't'}) \quad (5)$$

where $v_0 > 0$ and $\varepsilon \ll 1$ are positive constants.

Introducing the non-dimensional quantities

$$y = \frac{v_0 y'}{v}, \quad t = \frac{v_0^2 t'}{4\nu}, \quad w = \frac{4\nu w'}{v_0^2}, \quad u = \frac{u'}{v_0}, \quad T = \frac{T' - T_\infty}{T_w - T_\infty}, \quad (6)$$

$$S = \frac{\nu S'}{v_0^2}, \quad Kp = \frac{v_0^2 K_p}{\nu^2}, \quad Pr = \frac{\nu}{K}, \quad M^2 = \frac{\sigma B_0^2 \nu}{\rho v_0^2},$$

$$Rc = \frac{k_0 v_0^2}{\sigma \nu^2}, \quad n = \frac{4\nu n'}{v_0^2}, \quad Gr = \frac{\nu g \beta (T_w - T_\infty)}{v_0^3},$$

$$\frac{\partial q_r'}{\partial y'} = 4(T' - T_\infty)I', \quad F = \frac{4\nu I'}{\rho C_p U_0^2}, \quad E = \frac{\mu U_0^2}{\nu \rho C_p (T_w - T_\infty)}$$

(7)

The equations (1), (2), (3) reduce to the following non-dimensional form:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon e^{nt}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + GrT - M^2 u - \frac{u}{Kp(1 + \varepsilon e^{nt})} - \frac{Rc}{4} \frac{\partial^3 u}{\partial t \partial y^2}$$

$$\frac{1}{4} \frac{\partial T}{\partial t} - (1 + \varepsilon e^{nt}) \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + E \left(\frac{\partial u}{\partial y} \right)^2 + ST - FT \quad (8)$$

with the boundary conditions

$$u = 0, T = 1 + \varepsilon e^{nt}, \text{ at } y = 0$$

$$u \rightarrow 0, T = 0, \text{ as } y \rightarrow \infty$$

(9)

III. SOLUTION OF THE PROBLEM

Equations (7)-(8) are coupled non-linear partial differential equations which are complicated to solve with usual analytical methods and so they are solved by using finite difference method. The finite difference schemes of partial derivatives are written by using finite difference space grids. The expressions for velocity $u(i,j+1)$, temperature $T(i,j+1)$ and concentration $C(i,j+1)$ written and then $j+1^{\text{th}}$ level values are calculated by using j^{th} level values. By taking the range appropriately for i values we have plotted the graphs. The equivalent finite difference schemes of equations for (7)-(8) are as follows:

$$\left. \begin{aligned} \frac{1}{4} \frac{u_{i,j+1} - u_{i,j}}{\Delta t} - (1 + \varepsilon e^{nt}) \frac{u_{i+1,j} - u_{i,j}}{\Delta y} &= Gr T_{i,j} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \\ - \frac{Rc}{4} \left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1} - u_{i-1,j} + 2u_{i,j} - u_{i+1,j}}{\Delta t (\Delta y)^2} \right) \\ - M^2 u_{i,j} - \frac{1}{K_p (1 + \varepsilon e^{nt})} u_{i,j} \end{aligned} \right\} \quad (10)$$

$$\frac{1}{4} \left(\frac{T_{i,j+1} - T_{i,j}}{\Delta t} \right) - (1 + \varepsilon e^{nt}) \left(\frac{T_{i,j+1} - T_{i,j}}{\Delta y} \right) = \frac{1}{Pr} \left(\frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{(\Delta y)^2} \right) + E \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right)^2 + ST_{i,j} - FT_{i,j} \quad (11)$$

The equations 10 & 11 are simplified and the expressions for velocity and temperature are written in terms of $u_{i,j+1}$ and $T_{i,j+1}$ respectively. The numerical results are obtained by entering in MATLAB programme. Here, the index i refer to y and j to time. The mesh system is divided by taking $\Delta y = 0.1$. From equation (11), we have the following equivalent initial condition

$$u(i, 0) = 0, T(i, 0) = 0, \text{ for all } i$$

The boundary conditions from (11) are expressed in finite-difference form as follows

$$u(0, j) = 1, T(0, j) = 1, \text{ for all } j$$

$$u(i_{\max}, j) = 0, T(i_{\max}, j) = 0, \text{ for all } j$$

(Here i_{\max} was taken as 20)

First the velocity at the end of time step viz, $u(i,j+1)(i=1,20)$ is calculated from (10) in terms of velocity and temperature at points on the earlier time-step. Then $T(i, j+1)$ is computed from (11). The procedure is repeated until $t = 0.5$ (i.e. $j = 500$). During computation Δt was chosen as 0.001.

Skin-friction:

The skin-friction in non-dimensional form is given by the relation

$$\tau = - \left(\frac{du}{dy} \right)_{y=0}, \text{ where } \tau = \frac{\tau^1}{\rho U_0^2}$$

Rate of heat transfer:

The dimensionless rate of heat transfer in terms of Nusselt number is given by

$$Nu = - \left(\frac{d\theta}{dy} \right)_{y=0}$$

IV. PHYSICAL INTERPRETATION

The influence of physical parameters like Grashof number, magnetic parameter, thermal radiation, Prandtl number on velocity and temperature is discussed with the help of graphs.

The viscosity of a viscoelastic substance gives the substance a strain rate dependence on time. Viscoelastic substance loses energy when a load is applied and then removed. Hysteresis is observed in the stress-strain curve, with the area of the loop being equal to the energy lost during the loading cycle. Since viscosity is the resistance to thermally activated plastic deformation, a viscous material will lose energy through a loading cycle. Visco-elasticity is a molecular rearrangement, when a stress is applied to a viscoelastic material. This movement or rearrangement is called creep. Figure 2 represents the changes in velocity under the influence of thermal Grashof number. It is observed that velocity increases when the values of thermal Grashof number are increased. The increased velocity of the fluid increases the boundary wall temperature. By increasing the Grashof number, buoyancy force increases, the bonds between the fluid particles become weaker and

strength of internal friction will be decreased. This makes the gravity to become stronger enough for immediate fluid layer adjacent to the boundary wall. Figure 3 reveals that velocity decreases for raising values of magnetic field parameter because the magnetic parameter is inversely proportional to the velocity of the fluid at constant applied magnetic field. Thermal diffusion is the process of movement particles from one place to other place by temperature gradient. Figure 4 illustrates that the temperature decreases under the effects of Prandtl number. The prandtl number is increased from 0.7 to 7.1. If the prandtl number is less than one, thermal diffusivity dominates where as the Prandtl number is greater than one, momentum diffusivity predominates. Resultantly the temperature of the fluid decreases by increasing the prandtl number. Figure 5 illustrates that the temperature decreases under the effects of increased thermal radiation. This happens due to the heat transfer created by thermal radiation which causes retardation of temperature.

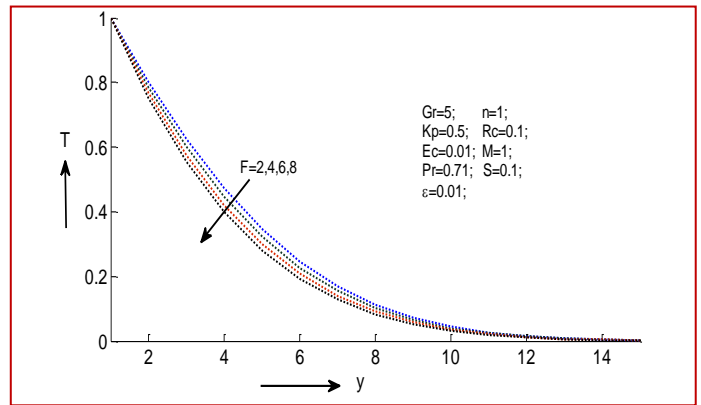


Figure 5: Temperature in the presence of thermal radiation

Table 1. Effect of various physical parameters on skin friction, Nusselt number and Sherwood number

| Ec | F | M | τ | Nu |
|-----|---|-----|--------|--------|
| 0.1 | 1 | 5 | 2.0463 | 0.6742 |
| 0.3 | 1 | 5 | 1.9897 | 0.6544 |
| 0.5 | 1 | 5 | 1.5417 | 0.5834 |
| 0.2 | 2 | 5 | 2.0364 | 0.6733 |
| 0.2 | 4 | 5 | 2.0860 | 0.7352 |
| 0.2 | 6 | 5 | 2.1583 | 0.7929 |
| 0.2 | 1 | 1 | 6.9674 | 0.6742 |
| 0.2 | 1 | 1.4 | 7.9649 | 0.6742 |
| 0.2 | 1 | 1.8 | 9.3246 | 0.6742 |

Skin friction arises from the interaction between the fluid and the skin of the body, and is directly related to the wetted surface, the area of the surface of the body that is in contact with the fluid. Air in contact with a body will stick to the body's surface and that layer will tend to stick to the next layer of air and that in turn to further layers, hence the body is dragging some amount of air with it. The force required to drag an "attached" layer of air with the body is called skin friction drag. Skin friction drag imparts some momentum to a mass of air as it passes through it and that air applies a retarding force on the body. The present work is extended to observe the changes in skin friction and Nusselt number under the influence of thermal radiation, Eckert number and magnetic parameter. Table 1 show that the skin friction coefficient reduces for increasing values of Eckert number. The Eckert number increases by decreasing the advective transport of fluid, then convective heat transfer reduces and Nusselt number decreases. A reverse trend is shown in the case of radiation parameter and magnetic parameter. Nusselt number (Nu) is the ratio of convective to conductive heat transfer across (normal to) the boundary. The convection and conduction heat flows are parallel to each other and to the surface normal of the boundary surface, and are all perpendicular to the mean fluid flow in the simple case. The rate of heat transfer increases under the influence of thermal radiation whereas it decreases in the case of Eckert number. These results are significant in MHD power generation process.

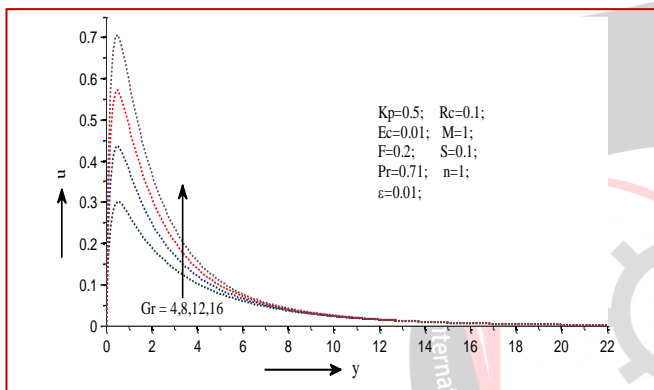


Figure 2: Velocity under the influence of Grashof number

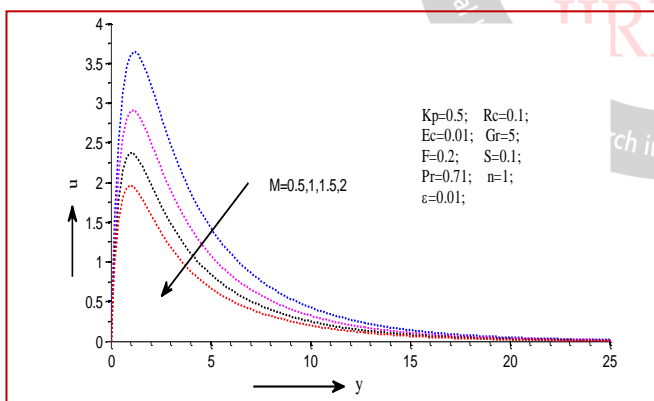


Figure 3: Velocity under the influence of magnetic parameter

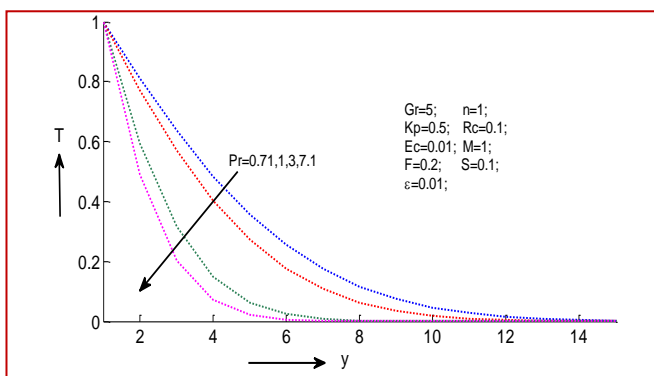


Figure 4: Temperature under the influence of Prandtl number

V. CONCLUSION

The governing equations for the velocity field, temperature and concentration by finite difference method. The radiation effect is included, that causes decreasing of temperature. The main points observed from this study are as follows:

- Velocity decreases for increasing values of Prandtl number and magnetic parameter.
- The temperature of the fluid decreases for rising values of Prandtl number and radiation parameter.
- Skin friction decreases with an increase of Eckert number and but a reverse effect is noticed in the case of magnetic parameter.
- Nusselt number increases as radiation parameter increases but in the case of Eckert number it decreases.

REFERENCES

- [1] Makinde O.D.; Mishra, S.R.(2017); Chemically reacting MHD mixed convection variable viscosity blasius flow embedded in a porous medium, Defect and Diffusion Forum, 374, pp. 83-91.
- [2] Mishra, S.R.; Sharma, R.P.; Kumar, D.; Singh, J. (2017); A hybrid computational approach for steady flow of 'Walters' B fluid in a vertical channel with porous wall, Ganita, 67, (1), pp. 89-98.
- [3] Chandra Reddy, P.; Raju, M.C.; Raju, G.S.S. (2016); Magnetohydrodynamic convective double diffusive laminar boundary layer flow past an accelerated vertical plate, International Journal of Engineering Research in Africa, 20, pp. 80-92..
- [4] Chandra Reddy, P.; Raju, M.C.; Raju, G.S.S.(2016); Soret and Dufour effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate, Advances and Applications in Fluid Mechanics, 19, pp. 401-414.
- [5] Dash, G.C.; Tripathy, R.S.; Rashidi, M.M.; Mishra, S.R.(2016); Numerical approach to boundary layer stagnation-point flow past a stretching/shrinking sheet, Journal of Molecular Liquids, 22 (1), pp. 860-866.
- [6] Umamaheswar, M.; Varma, S.V.K.; Raju, M.C.; Chamkha, A.J.(2016); Unsteady MHD free convective double diffusive visco-elastic fluid flow past an inclined permeable plate in the presence of viscous dissipation and heat absorption, Special topics and Reviews in Porous media: An international Journal, 6 (4), pp. 333-342.
- [7] Srinivas, S.; Subramanyam Reddy, A.; Ramamohan, T.R.; KantShukla, A.(2016); Thermal-diffusion and diffusion-thermo effects on MHD flow of viscous fluid between expanding or contracting rotating porous disks with viscous dissipation, Journal of the Egyptian Mathematical Society, 24 (1), pp. 100-107.
- [8] Rashidi, M.M.; Nasiri, M.; Khezerloo, M.; Najib Laraqi (2016); Numerical investigation of magnetic field effect on mixed convection heat transfer of nanofluid in a channel with sinusoidal walls, Journal of Magnetism and Magnetic Materials, 401 (1), pp. 159-168.
- [9] P. Chandra Reddy, M.C. Raju, G.S.S. Raju, S.V.K. Varma, Free convective magneto-nanofluid flow past a moving vertical plate in the presence of radiation and thermal diffusion, Frontiers in Heat and Mass Transfer, Int. Journal. 7, 28 (2016) 1-7.
- [10] P. Chandra Reddy, M.C. Raju, G.S.S. Raju, C. Madhava Reddy, Diffusion thermo and thermal diffusion effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate, Bulletin of Pure and Applied Sciences. 36E(2) (2017) 266-284.
- [11] Yabo, I. , Jha, B. and Lin, J. (2016) Combined Effects of Thermal Diffusion and Diffusion-Thermo Effects on Transient MHD Natural Convection and Mass Transfer Flow in a Vertical Channel with Thermal Radiation. Applied Mathematics, 7, 2354-2373.
- [12] K. Sidda Reddy, P. Chandra Reddy, G.S.S. Raju, Thermal diffusion and Joule heating effects on MHD radiating fluid embedded in porous medium, International Journal for Research in Engineering Application & Management (IJREAM), 4(4) (2018) 206-212.