Three dimensional Heat and mass transfer analysis of Al_2O_3 nanofluid over a stretching sheet

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Abstract: This paper mainly focuses on the three dimensional heat and mass transfer analysis of Al_2O_3 nanofluid over a stretching sheet in view of enhancement of thermal conductivity and hence more heat transfer capability of nanofluid. The non-linear partial differential equations have been converted into strong non-linear ordinary differential equations by employing suitable transformations and these transformed equations are solved by Runga-Kutta method of fourth order along with Shooting technique. The results are presented through graphs for various parameters on velocity, temperature, concentration.

Key words: MHD, heat and mass transfer, Al_2O_3, Stretching sheet.

I. INTRODUCTION

It is well known that nanofluids are a new class of nanotechnology-based heat transfer fluids engineered by dispersing nanometer scale solid particles whose length scales is between 1 nm to 100 nm in traditional heat transfer fluids. Choi [1] was the first who introduced the term 'nanofluids'. Several industrial applications of nanofluids include improved heat transfer, chemical production, power generation in a power plant, automotives, microelectronics production, advanced nuclear systems, micro channel cooling. Therefore, a significant research interest has been carried out in recent years due to wide range of applications of nanofluids [2]-[4]. In the presence of spherical Au-Metallic Zubair et al.[5] analyzed and investigated the heat and mass transfer analysis of MHD nanofluid flow with radiative heat effects. In porous media over a permeable stretching/shrinking sheet Bhatti et al.[6] investigated and stuided new numerical simulation of MHD stagnation-point flow with heat transfer. Khan et al.[7] studied boundary layer flow of a nanofluid past a stretching sheet. Over a stretching sheet Sravan Kumar et al.[8] discussed a comparative study of thermal effects on MHD flow and heat transfer of nanofluids. On mass transfer Hayat et al.[9] studied three-dimensional flow of a visco elastic fluid. Nayak et al.[10] explained three-dimensional free convective MHD flow of nanofluid with thermal radiation over permeable linear stretching sheet.

From the above literature I noticed the scope of studying 3D heat and mass transfer analysis of Al_2O_3 nanofluid over a stretching sheet. The present work is the extension of the work of Nayak et al.[10] to analyze three dimensional heat and mass transfer analysis of Al_2O_3 nanofluid over a stretching sheet.

II. FORMULATION OF THE PROBLEM

Consider a steady three-dimensional incompressible electrical conducting free convective nanofluid flow past a permeable stretching sheet. The physcial representation of the problem is shown in Fig. 1.Assume that a transverse magnetic field of uniform strength B_0 is applied parallel to the z-axis. The magnetic Reynolds number is assumed to be small so that the induced magnetic field and impressed electric field are neglected.

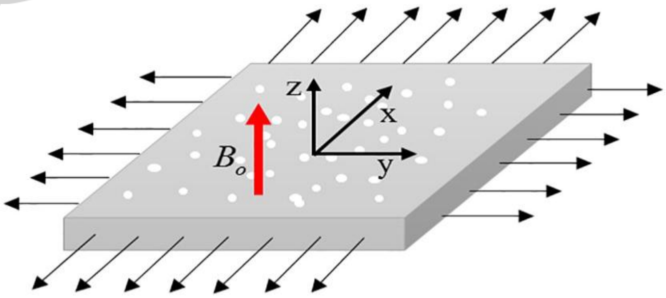

Fig.1 Physcial representation of the problem.

The governing equations (Nayak et al.[10]) based on the assumptions are as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\rho_{nf}}\left\{\mu_{nf}\frac{\partial^2 u}{\partial z^2}+(\rho\beta)_{nf}\,g(T-T_\infty)+(\rho\beta^*)_{nf}\,g(C-C_\infty)-\sigma B_0^2\bar{u}\right\} \tag{2}$$

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$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{1}{\rho_{nf}}\left\{\mu_{nf}\frac{\partial^2 v}{\partial z^2}+(\rho\beta)_{nf}g(T-T_\infty)-(\rho\beta^*)_{nf}g(C-C_\infty)-\sigma B_0^2\bar{u}\right\} \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial z^2}-\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial z} \quad (4)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\frac{\partial^2 C}{\partial z^2} \quad (5)$$

The boundary conditions are

$$u=U_w(x)=ax, v=V_w(x)=bx\ ,\ w=0,\ T=T_w,\ C=C_w \text{ at z=0}$$

$$u\to 0,\ v\to 0,\ w\to 0,\ T\to T_\infty,\ C\to C_\infty \quad \text{as } z\to\infty \quad (6)$$

where a > 0 and b > 0 for stretching sheet.

The properties of nanofluid are given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\ (\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s,\ (\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s,$$

$$K_{nf}=Kf\left\{\frac{K_s+2K_f-2\phi(K_f-K_s)}{K_s+2K_f+2\phi(K_f-K_s)}\right\} \quad (7)$$

The effective dynamic viscosity of the nanofluid is

$$\mu_{nf}=\mu_f(1+39.11\phi+533.9\phi^2) \quad (8)$$

The Rosseland approximation is

$$q_r=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z},\ T^4=4T_\infty^3T-3T_\infty^4,\ \frac{\partial q_r}{\partial z}=-16\frac{T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial z^2} \quad (9)$$

From (7), (8) and (4) energy equation is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial z^2}-\frac{1}{(\rho C_p)_{nf}}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial z^2} \quad (10)$$

The following are the dimensionless variables

$$u=axf'(\eta),\quad v=ayf'(\eta),\quad w=-(a\nu_f)^{1/2}(f(\eta)+g(\eta)),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad C(\eta)=\frac{C-C_\infty}{C_w-C_\infty},$$

$$\eta=\left(\frac{a}{\nu_f}\right)^{1/2}z \quad (11)$$

Using (7)-(11) Eqs. (2),(3),(8) and (5)

$$f'''+\varepsilon\{\varepsilon_1((f+g)f''-(f')^2)+(\varepsilon_2\gamma_1\theta+\varepsilon_4\gamma_2C)f'-Mf'\}=0 \quad (12)$$

$$g'''+\varepsilon\{\varepsilon_1((f+g)g''-(g')^2)+(\varepsilon_2\gamma_3\theta+\varepsilon_4\gamma_4C)g'+Mg'\}=0 \quad (13)$$

$$(A+R)\theta''+\varepsilon_3\Pr(f+g)\theta'=0 \quad (14)$$

$$C''+Sc(f+g)C'=0 \quad (15)$$

with the boundary conditions

$$f'(\eta)=1,\ g'(\eta)=\lambda,\ f(\eta)=0,\ g(\eta)=0,\quad \theta(\eta)=1,\ C(\eta)=1 \text{ at } \eta=0$$

$$f'(\eta)\to 0,\ g'(\eta)\to 0,\ \theta(\eta)\to 0,,\ C(\eta)\to 0 \qquad \text{as } \eta\to\infty \quad (16)$$

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where $\varepsilon = (1-\phi)^{2.5}$, $\varepsilon_1 = 1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)$, $\varepsilon_2 = 1-\phi+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)$, $\varepsilon_3 = 1-\phi+\phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)$,

$$\varepsilon_4 = 1-\phi+\phi\left(\frac{(\rho\beta^*)_s}{(\rho\beta^*)_f}\right) \text{ and } M = \frac{\sigma B_0^2}{a\rho_f},\ R = \frac{16\sigma^* T_\infty^3}{3k^* k_f},\ S = \frac{W}{\sqrt{a\nu_f}},\ \lambda = \frac{b}{a},\ \gamma_1 = \frac{g\beta_f(T_w - T_\infty)}{au},$$

$$\gamma_2 = \frac{g\beta_f^*(C_w - C_\infty)}{au},\ \gamma_3 = \frac{g\beta_f(T_w - T_\infty)}{av},\ \gamma_4 = \frac{g\beta_f^*(C_w - C_\infty)}{av},\ \Pr = \frac{\nu_f}{\alpha_f},\ A = \frac{k_{nf}}{k_f} \quad (17)$$

III. RESULTS AND DISCUSSION

The numerical solutions of the governing ordinary differential equations (12) to (16) with the boundary conditions equation (17) are obtained by using Runge-Kutta fourth order method along with shooting technique using MAT lab. We have converted the boundary value problem into initial value problem and assumed a suitable finite value for the far field boundary condition.

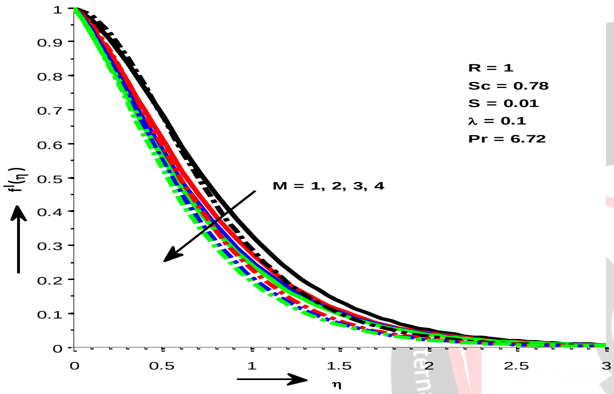

Fig.2 Effect of Primary velocity for various M values using Al_2O_3 nanofluid.

The effect of primary velocity is shown in Fig.2 and effect of secondary velocity is shown in Fig.3 using Al_2O_3 nanofluid for various magnetic parameter values. It is clear that for increasing values of M the primary velocity decreases and secondary velocity increases.

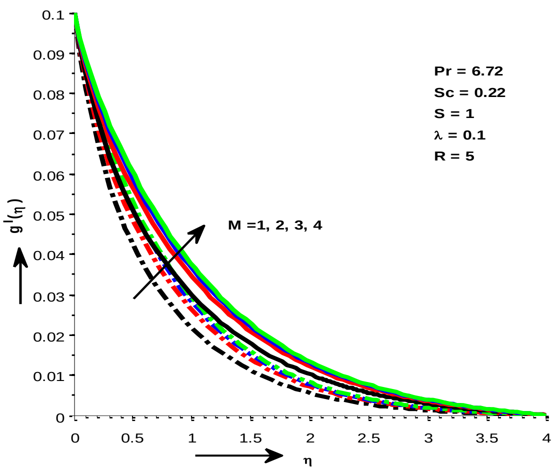

Fig.3 Effect of secondary velocity for various M values using Al_2O_3 nanofluid.

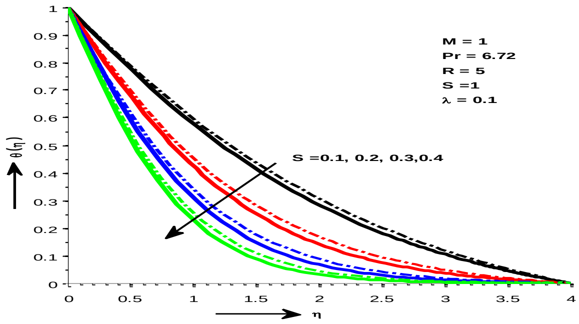

Fig.4 Effect of temperature for various S values using Al_2O_3 nanofluid.

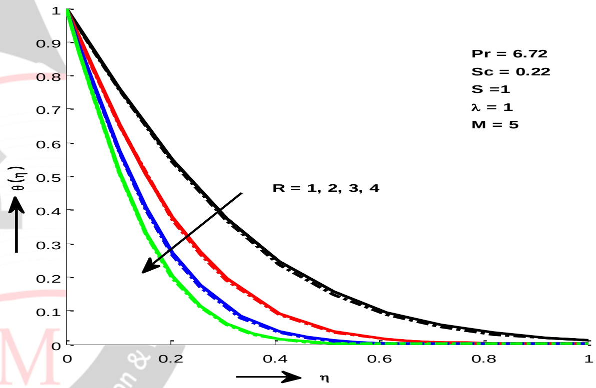

Fig.5 Effect of temperature for various R values using Al_2O_3 nanofluid.

The effect of temperature using Al_2O_3 nanofluid for various values of suction parameter S is displayed in Fig.4. It stated that with the increase in suction parameter, temperature decreases. The effect of temperature using Al_2O_3 nanofluid for various values of radiation parameter R is displayed in Fig.5. It depicts that the temperature decelerates as R accelerates.

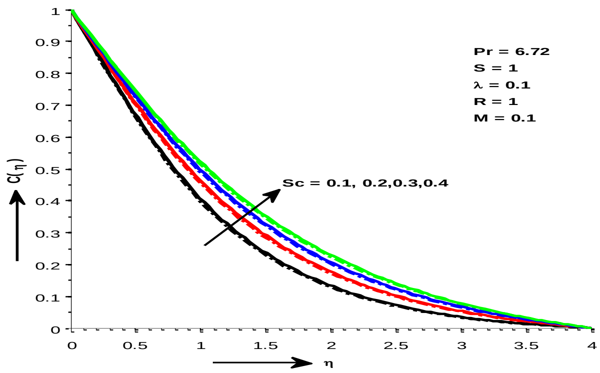

Fig.6 Effect of concentration for various Sc values using Al_2O_3 nanofluid.

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From fig.6 it is clear that concentration increases for the increasing values of Schmidt number.

IV. CONCLUSION

The three dimensional heat and mass transfer analysis of Al_2O_3 nanofluid over a stretching sheet is analyzed by solving the governing ordinary differential equations with the boundary conditions equation using Runge-Kutta fourth order method along with shooting technique using MAT lab. The following conclusions are obtained from the present study.

1. For increasing values of M the primary velocity decreases and secondary velocity increases.
2. Temperature decreases with the increase in suction parameter and radiation parameter.
3. Concentration increases with the increase in Schmidt number.

Appendix:

$$\varepsilon = \frac{1}{1+39.11\phi+533.9\phi^2}$$

$$\varepsilon_1 = 1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)$$

$$\varepsilon_2 = 1-\phi+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)$$

$$\varepsilon_3 = 1-\phi+\phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)$$

$$\varepsilon_4 = 1-\phi+\phi\left(\frac{(\rho\beta^*)_s}{(\rho\beta^*)_f}\right)$$

REFERENCES

[1] Choi SUS (1995) Enhancing Thermal Conductivity of Fluids with Nanoparticles, Developments and Applications of Non-Newtonian Flows. FED-231/MD-vol (66) 99–1053.

[2] Eastman J.A, Choi S.L.S.S, Yu.W, Thompson.L.J, Anomalously increased effective thermal conductivity of ethylene glycol-based nanofluids containing copper nanoparticles, Appl. Phys. Lett. 78 (6) (2001) 718–720.

[3] Y. Xuan, Q. Li, Investigation on convective heat transfer and flow features of nanofluids, J. Heat Transf. 125 (2003) 151–155

[4] Buongiorno J, Hu L.W , Nanofluid coolants for advanced nuclear power plants , Proceedings of ICAPP, Seoul, May 15-19 (2005).

[5] Zubair Akbar Qureshi M , Qammar Rubbab, Saadia Irshad, Salman Ahmad and Aqeel M, Heat and Mass Transfer Analysis of MHD Nanofluid Flow with Radiative Heat Effects in the Presence of Spherical Au-Metallic, Nanoscale Research Letters (2016) 11:472.DOI 10.1186/s11671-016-1692-2.

[6] Bhatti MM, Abbas T, Rashidi MM. A new numerical simulation of MHD stagnation-point flow over a permeable stretching/shrinking sheet in porous media with heat transfer. Iran J Sci Techno Trans A Sci. 2016:1–7. http://doi.org/10.1007/s40995-016-0027-6.

[7] Khan, W. A., & Pop, I. Boundary layer flow of a nanofluid past a stretching sheet. International Journal of Heat and Mass Transfer,(2010) 53, 2477–2483.

[8] Sravan Kumar T, Rushi Kumar B, A comparative study of thermal effects on MHD flow of nanofluids and heat transfer over a stretching sheet, Frontiers in Heat and Mass Transfer (FHMT), 9, 13 (2017), DOI: 10.5098/hmt.9.13.

[9] Hayat T, Mustafa M, Sajid M On mass transfer in three-dimensional flow of a viscoelastic fluid. Num Meth Partial Diff Eq, (2011) 27, 915–936.

[10] Nayak M .K, Noreen Sher Akbar, Pandey V.S, Zafar Hayat Khan, Dharmendra Tripathi, 3D free convective MHD flow of nanofluid over permeable linear stretching sheet with thermal radiation, Powder Technology.315(2017)205-215, http://dx.doi.org/10.1016/j.powtec.2017.04.017.

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