International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-07, Oct 2018

Some Applications of Pair Resolving Sets

[1]G. Ramesh, [2]A. Rajasekaran

[1,2]PG and Research Department of Mathematics, Government Arts College (Autonomous), Kumbakonam-612002, Tamil Nadu, South India.

[2]Email: arsekar.math@gmail.com

Abstract **In this paper we determine the metric dimension problems associated with pair resolving sets for the graphs of certain crystal structures and chemical structures like the basic chemical unit of silicates is the (SiO_4) tetrahedron and bismuth tri-iodide, lead chloride. Also we study some applications of pair resolving set in network theories and we study the importance to avoid the overlapping between the robots in a network.**

Keywords —Minimum pair resolving set, Metric dimension, silicate structure, Crystal Structures, Bismuth, Tri-Iodide, Lead Chloride, Overlapping, Cardinal number.

I. INTRODUCTORY CONCEPTS

The distance between u and v in G, denoted by $d(u,v)$, is the lenght of the shortest path u to v in G. Let $W = \{w_1, w_2, \ldots \ldots w_k\}$ be an ordered subset of $V(G)$.

For a vertex $v \in V(G)$, a representation of v with respect to W is k-tuple $r(v/W) = (d(v,w_1), d(v,w_2), \ldots \ldots d(v,w_k))$. Metric dimension was initially introduced in 1970s, by Harary and Melter [3], and independently by Slater [2]. The set W is called a *pair resolving set* [10] for G if $u \in V(G)$ then $r(u/W) = r(v/W)$ for at most one v such that $v \in V(G)$. The minimum cardinality of a pair resolving set of G is called the metric dimension of G, denoted by $dim_{pr}(G)$.

Silicates are obtained by fusing metal oxides or metal carbonates with sand. Essentially all the silicates contain SiO_4 tetrahedra. In chemistry, the corner vertices of SiO_4 tetrahedran represent oxygen ions and the center vertex represents the silicon ion. In graph theory, we call the corner vertices as *oxygen nodes* and the center vertex as *silicon node*. The minerals are obtained by successively fusing oxygen nodes of two tetrahedra of different silicates. See Figure 1.

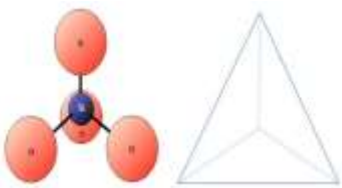

Figure 1

The physical structure of solid materials of engineering importance depends mainly on the arrangements of the atoms, ions, or molecules that make up the solid and the bonding forces between them. If the atoms or ions of a solid are arranged in a pattern that repeats itself in three dimensions, they form a solid that is said to have a crystal structure and is referred to as a crystalline solid or crystalline material. The atomic arrangement or crystalline structure of a material is important in determining the behavior and properties of a solid material. Examples of crystalline materials are metals, alloys, and some ceramic materials. The unit cell is the smallest structural unit or building block that can describe the crystal structure. Repetition of the unit cell generates the entire crystal. In this section, we investigate the metric dimension problems for the graphs bismuth tri-iodide, lead chloride.

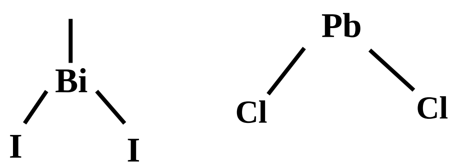

Figure 2

Metric dimension in graph theory has many applications in the real world. It has been applied to the optimization problems in complex networks, analyzing electrical networks; show the business relations, robotics, control of production processes etc. In this section, we study the applications of pair resolving set in various network theories. Also we study the importance to avoid the overlapping between the robots in a network.

 DOI : 10.18231/2454-9150.2018.0905

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-07, Oct 2018

II. PAIR RESOLVING SETS IN CHEMICAL STRUCTURES

2.1 Silicate Networks

We describe the construction of a silicate network from a honeycomb network. A honeycomb network can be built from a hexagon in various ways. The honeycomb network *HC*(1) is a hexagon. The honeycomb network *HC*(2) is obtained by adding six hexagons to the boundary edges of *HC*(1).

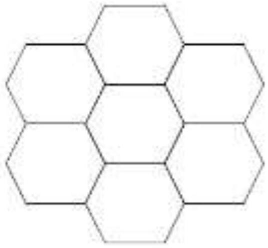

Figure 3: The honeycomb network *HC*(2)

Consider a honeycomb network *HC*(*2*) of dimension *2*. Place silicon ions on all the vertices of *HC*(*2*). Subdivide each edge of *HC*(*2*) once. Place oxygen ions on the new vertices. Introduce 6x2=12 new pendant edges one each at the 2-degree silicon ions of *HC*(*2*) and place oxygen ions at the pendent vertices. See Figure 4(a). With every silicon ion associate the three adjacent oxygen ions and form a tetrahedron as in Figure (b). The resulting network is a silicate network of dimension *2*, denoted *SL*(*2*). The diameter of *SL*(*2*) is 4*n*=4x2=8. The graph in Figure 4(b) is a silicate network of dimension two.

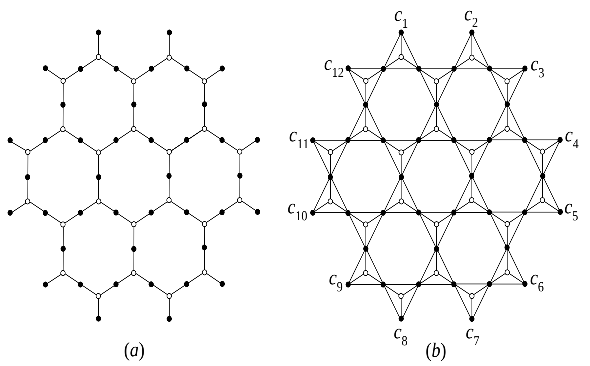

Figure 4: Silicate network construction with nodes

The 3-degree oxygen nodes of silicates are called *boundary nodes*. In Figure 4(b), c_1, c_2 … c_{12} are boundary nodes *SL*(2).

Observations

a) The number of nodes in *SL*(*n*) is $15n^2 + 3n$ and the number of edges of *SL*(*n*) is $36n^2$

b) In a Silicate network, minimum pair resolving set exists.

c) In a Silicate network, connected PR-set exists.

d) In a Silicate network, independent PR-set does not exists.

Theorem 2.1 $dim_{pr}(SL(2)) > 1$.

Proof In a silicate network SL(2), the number of vertices is even. By the definition of pair resolving set, a graph of even order does not contain any singleton pair resolving set. So that $dim_{pr}(SL(2)) > 1$.

Theorem 2.2 Metric dimension associated with pair resolving set of SL(2) greater than or equal to 4.

Proof By observation, the number of nodes in SL(n) is even for any value of *n*. Also by the definition of pair resolving set, a graph with even number of nodes does not contains a odd number of metric basis elements. Therefore the metric dimension associated with pair resolving set of SL(2) greater than 4. Now we have to verify metric dimension associated with pair resolving set of SL(2) greater than or equal to 4. Since every α line contain only even number of oxygen nodes in SL(*n*) and $\alpha = 0$ line in SL(2) contains four number of oxygen nodes.

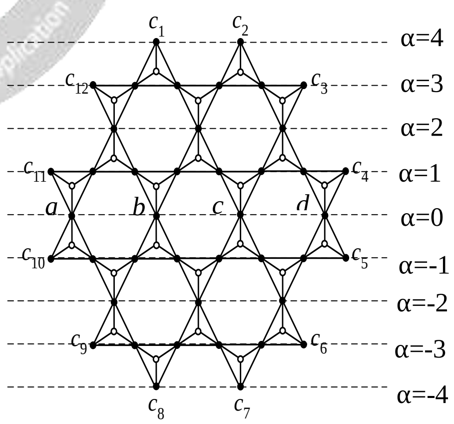

Figure 5

Now we claim that the set of all nodes in $\alpha = 0$ line is a metric basis. That is $dim_{pr}(SL(2)) = 4$. Therefore the metric basis of SL(2) is $W = \{a, b, c, d\}$ (see Figure 5). To prove W is pair resolving set of SL(2). By the definition pair resolving set, for each vertex u in SL(2), there exist a vertex v with $pr(u/W) = pr(v/W)$. In SL(2), for each vertex in the positive α −lines space (above $\alpha = 0$ line), there exists a vertex in the negative α −lines space (below $\alpha = 0$ line) with $pr(u/W) = pr(v/W)$. So that W is a pair

 DOI : 10.18231/2454-9150.2018.0905

resolving set of SL(2). Therefore every vertex in $\alpha = 0$ line is metric basis and is of even order. Hence metric dimension associated with pair resolving set of SL(2) greater than or equal to 4. This completes the proof.

Remark 2.3 From above theorem, we conclude that the metric dimension of SL(n) is the number of nodes in $\alpha = 0$ line. Also $\alpha = 0$ line contains only oxygen nodes.

- If n=1, then the $\alpha = 0$ line contains 2 oxygen nodes and $dim_{pr}(SL(n)) = 2$.
- If n=2, then the $\alpha = 0$ line contains 4 oxygen nodes and $dim_{pr}(SL(n)) = 4$.
- Therefore Metric dimension associated with pair resolving set of SL(n) greater than or equal to 2n.

Now we prove this conclusion as follows.

Theorem 2.4 The metric dimension of SL(n) is at least 2n.

Proof For $i = j$, each pair of vertices (a_i , b_j) are at equal distance from all the vertices in $\alpha = 0$ line of $SL(n)$. And there are at least 2n number of such vertices exist in $\alpha = 0$ line of $SL(n)$, Therefore that all 2n number of such vertices must present in the basis. Hence the cardinality of basis must be greater than or equal to 2n. Hence the metric dimension of $SL(n)$ is greater than or equal to 2n.

2.2 Crystal Structures

The Graph of Bismuth Tri-Iodide

Bismuth tri-iodide ($BiI3$) is an inorganic compound. It is the product of the reaction of bismuth and iodine, which once was of interest in qualitative inorganic analysis. Layered $BiI3$ crystal is considered to be a three-layered stacking structure, where bismuth atom planes are sandwiched between iodide atom planes, which form the sequence I-Bi-I planes. The periodic stacking of three layers forms rhombohedral $BiI3$ crystal with R-3 symmetry. The successive stacking of one I-Bi-I layer forms hexagonal structure with symmetry. Figure 6 shows one unit of bismuth tri-iodide. The graph of a single unit of bismuth tri-iodide contains six 4-cycles of which three are on the left, other three are in the right. The unit cells of bismuth tri-iodide can be arranged either linearly or in a sheet form. A linear arrangement with m unit cells is called an m-bismuth chain; mn unit cells arranged into m rows and n columns is called an $m \times n$ bismuth sheet.

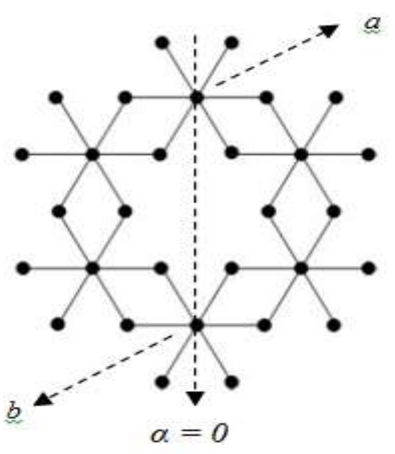

Figure 6

Observations

a) The number of vertices in one unit bismuth tri-iodide is 30.
b) The bismuth atoms in $\alpha = 0$ line of bismuth tri-iodide is the minimum pair resolving set for bismuth tri-iodide.
c) Independent PR-set exists for bismuth tri-iodide.
d) Connected PR-set does not exists for bismuth tri-iodide.

Theorem 2.5 Let G be an 1-bismuth chain. Then the dimension of G associated with pair resolving set is 2.

Proof We claim that the set all bismuth atoms in $\alpha = 0$ line are the metric basis (see Figure 6.10). That is $W = \{a, b\}$. Now we prove W is a pair resolving set. For that we have to prove G contains only pairwise equidistance vertices. Since each vertex in left part of $\alpha = 0$ line, there exists a equidistance vertex in right part of $\alpha = 0$ line. Therefore we obtained 13 pair of vertices with equal distance from W. Then W is a pair resolving set, which contains minimum number vertices. Hence a and b are metric basis for 1-bismuth chain. Thus dimension of G associated with pair resolving set is 2.

Remark 2.6 From the above theorem, We conclude that the dimension of m-bismuth chain associated with pair resolving set is *2(m+1)*.

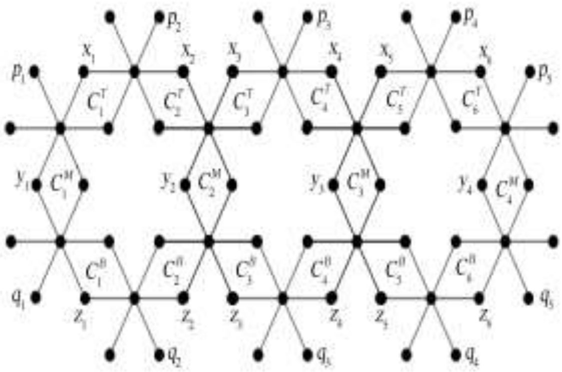

Figure 7: Three unit of bismuth tri-iodide

 DOI : 10.18231/2454-9150.2018.0905

The Graph of Lead Chloride

Lead chloride is a halide crystal which occurs naturally in the form of mineral cotunnite. It is used in the production of infra red transmitting glass and basic chloride of lead known as patteson's white lead, perry, ornamental glass called aurene glass, stained glass. It is also used as an intermediate in refining bismuth (Bi) ore, it is used in the synthesis of organometallic, lead titanate and barium titanate. The structure of lead chloride is orthorhombic dipyramidal. The graph of a single unit of lead chloride is obtained from that of bismuth tri-iodide by joining just one 2-degree vertex of each of the 4-cycles to a new vertex. As in the case of bismuth tri-iodide, chains and sheets of lead chloride are defined.

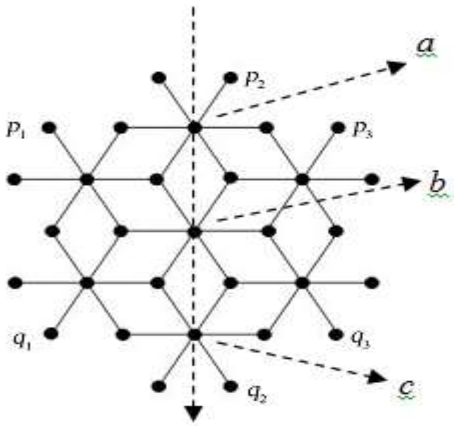

Figure 8: One unit of lead chloride

Observations

a) The number of vertices in one unit of lead chloride is 31.
b) The bismuth atoms in $\alpha = 0$ line of lead chloride is the minimum pair resolving set for lead chloride.
c) Independent PR-set exists for lead chloride.
d) Connected PR-set does not exists for lead chloride.

Theorem 2.7 Let G be an one unit of lead chloride. Then the dimension of G associated with pair resolving set is 3.

Proof We claim that the set all lead atoms in $\alpha = 0$ line are the metric basis (see Figure 8). That is $W = \{a, b, c\}$. Now we prove W is a pair resolving set. For that we have to prove G contains only pairwise equidistance vertices. Since each vertex in left part of $\alpha = 0$ line, there exists a equidistance vertex in right part of $\alpha = 0$ line. Therefore we obtained 14 pair of vertices with equal distance from W. Then W is a pair resolving set, which contains minimum number vertices. Hence a, b and c are metric basis for one unit of lead chloride. Thus dimension of G associated with pair resolving set is 3.

Remark 2.8 From the above theorem, We conclude that the dimension of *m*-lead chloride associated with pair resolving set is 2*m*+1.

III. PAIR RESOLVING SETS IN ROBOTIC NETWORK

Definition 3.9 Let W be the minimum pair resolving set for a graph G. The cardinality number of a basis element $u_i \in W$ is the number of vertices of G identified by u_i and it is denoted by $Card\,(u_i)$.

Example 3.10 Consider the following graph G,

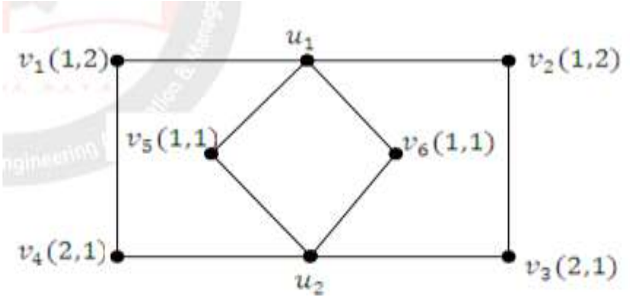

Figure 9

Then the minimum pair resolving set is $W = \{u_1, u_2\}$ and the $Card\,(u_1) = 4$, $Card\,(u_2) = 4$.

Definition 3.11 Two vertices u_1 and u_2 are called Overlapping with each other if $pr(u_1, v) = pr(u_2, v) = a$ for any vertex $v \in G$.

Example 3.12 In Figure 9, the basis elements u_1 and u_2 are Overlapping with respect to $pr(u_1, v_5) = pr(u_2, v_5) = 1$.

Definition 3.13 Let W be the set of all basis elements of G and $V(G)$ be the set of all vertices of G.Consider every basis element as a robotic elements. The Robotic Assignment is defined as, for a vertex $v \in V(G)$,we can assign the Robotic (basis) element $u \in W$, if $pr(u, v) =$ Minimum of the coordinate values.

Example 3.14 In Figure 9, the coordinate of vertex v_1 is $(1,2)$ with respect to the basis elements $\{u_1, u_2\}$. Therefore $min\ (1,2) = 1$ and the robotic element u_1 is assigned to v_1.

Remark 3.15 Suppose a graph contains overlapping robotic elements, then some vertex of G having possibility to assign by two robotic elements. In Figure 9, we can assign u_1 or u_2 to the vertex v_5 and similarly we can assign u_1 or u_2 to the vertex v_6. Suppose $pr(u_1, v) = pr(u_2, v)$ and $Card\,(u_1) < Card\,(u_2)$, then we can assign u_1 to v.

Definition 3.16 The Robotic Assignment spanning Subgraph (RASS) of the robotic network G is the subgraph obtained from the robotic assignment of G.

Example 3.17 Consider the graph G in Figure 9 as a Robotic network, then we can establish the Robotic Assignment spanning Subgraphs (RASS) as follows,

 DOI : 10.18231/2454-9150.2018.0905

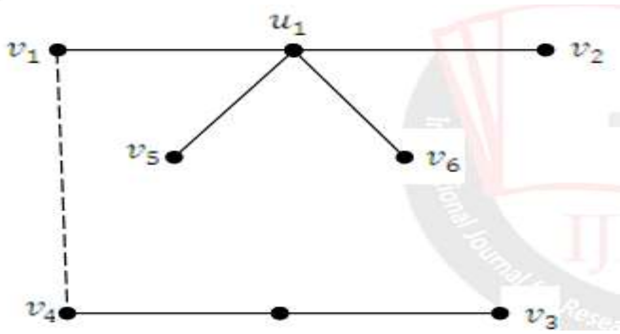

Figure 10: RASS

Remark 3.18 For large scale computations, network models consist of several nodes and can place uniquely a minimum number of Robots to identify them. But in the case of optimization we cannot assign two machines (Robots) to the same node. So we should avoid the overlapping between the machines.

Result 3.19 If the Robotic assignment subgraph is not connected then adjoin the edge or path between any two vertices which are not in W to make it a connected spanning subgraph. This is possible since the graph is connected. Hence we obtain a spanning tree for *G*.

The following figure represents the Robotic spanning tree *S* of the graph in Figure 9.

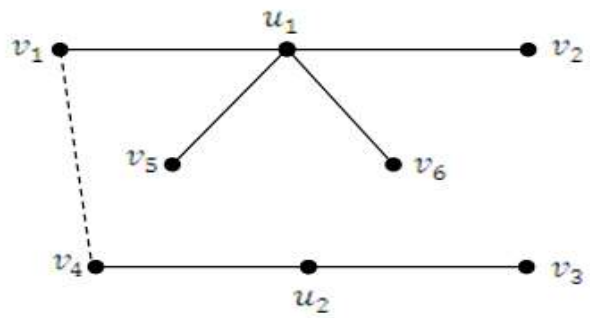

Figure 11

Robotic spanning tree is one of the main concepts that have wide applications in various fields. These concepts are highly utilized by computer science applications. Especially in research areas of computer science such data mining, image segmentation, clustering, image capturing, networking etc., For example a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using these concepts. In the same it is utilized in resource allocation, scheduling, traveling salesman problem, database design concepts, resource networking.

IV. Conclusion

The metric dimension problems associated with pair resolving set of crystal structure referred to as a crystalline solid or crystalline material are important in determining the behavior and properties of a solid material. Using the concept of pair resolving set, we have also obtained Robotic Assignment spanning subgraph in a complex network and the problem of avoiding overlapping are helps in solving some complicated networks. Our future work is to concentrate metric dimension associated with pair resolving set on other interconnection networks which are worth considering for Cayley and Non-Cayley graphs.

References

[1] J. Caceres, C. Hernado, M. Mora, I. M. Pelayo, M. L. Puertas, C. Seara and D. R. Wood, "On the Metric Dimension of Some Families of Graphs," Electronic Notes in Discrete Mathematics, Vol. 22, 2005, pp. 129-133.

[2] P. J. Slater. Leaves of trees. In: Proc. 6th Southeastern Conf. on Combinatorics, Graph Theory and Computing, 14, pp 549-559, 1975.

[3] F. Harary and R. A. Melter, "On the Metric Dimension of a Graph," ArsCombinatorica, Vol. 2, 1976, pp. 191-195.

[4] I. Javaid, M. T. Rahim and K. Ali, " Families of Regular Graphs with constant Metric Dimension," UtilitasMathematica, Vol. 75, 2008, pp. 21-33.

[5] S. Khuller, B. Raghavachari and A. Rosenfeld, "Localization in Graphs," Technical Report CS-Tr-3326, University of Maryland at College Park, 1994.

[6] S. Khuller, B. Raghavachari and A. Rosenfeld. Landmarks in Graphs. Discrete Appl.Math, 70(3), pp. 217-229, 1996.

[7] G. Chartrand, L. Eroh, M. A. Johnson, and O. R. Oellermann. Resolvability in graphs and Metric Dimension of Graph. Discrete Appl. Math., 105(1-3), pp. 99-113, 2000.

[8] D. Nason and L. Keller, The Growth and Crystallography of Bismuth Tri-Iodide Crystals Grown by Vapor Transport. *Journal of Crystal Growth*, **156**, (1995) 221. http://dx.doi.org/ 10.1016/0022- 0248(95)00291-X.

[9] C. Poisson and P. Zhang. The metric dimension of unicyclic graphs. J. Combin. Math.Combin.Compute. 40, pp. 17-32, 2002.

[10] G. Ramesh and A. Rajasekaran, "On pair resolving sets in graphs", *Elixir Appl. Math.* 107 (2017) 46995-46999.

[11] B Sooryanaranyana, B. Shanmuka, A Note on metric dimension, Far. East Journal of Applied Mathematics, 5, 331-339, 2001.

[12] B Sooryanaranyana, B. Shanmuka, Metric dimension of a wheel, Far. East Journal of Applied Mathematics, 6, 8(3), 217-229, 2002.

[13] P. Erdos, "On sets of distances of 'n' points in Euclidean space," Publ. Math. Inst. Hung.Acad. Sci, 5 (1960), 165-169.

[14] A. Sebo, E. Tannier, "On metric generators of Graphs, Mathematics of Operation Research , 29(2) (2004) 383-393.

[15] J. Peters-Fransen and O. R. Oellermann. The metric dimension of Cartesian products of Graphs. Util.Math., 69, pp 33-41, 2006.

[16] Douglas B. West, "Introduction to Graph Theory, Second Edition, University of Illions-Urbana.

[17] H. Yorikawa and S. Muramatsu, Theoretical Study of Crystal and Electronic Structures of BiI3. *Journal of Physics*: *Condensed Matter*, **20**, (2008) 325220.

 DOI : 10.18231/2454-9150.2018.0905