

Neoteric Techniques For Rough Neutrosophic Sets And Their Utilization In Medical Diagnosis

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Abstract - Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability etc., The concept of rough neutrosophic set is an essential tool for dealing with uncertainties free from the shortcomings that affect the existing methods. Innovative methods are devised in rough neutrosophic set and some of its properties are discussed herein. Execution of medical diagnosis is presented to find out the disease impacting the patient.

Keywords —Exponential measure, grade function ,logarithmic distance, medical diagnosis, rough neutrosophic set.

I. INTRODUCTION

In 1965, Fuzzy set theory was firstly given by Zadeh [8] which is applied in many real applications to handle uncertainty. Sometimes membership function itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed to capture the uncertainty of grade of membership. In 1986, Atanassov introduced the intuitionistic fuzzy sets which consider both truth-membership and falsity-membership. Later on, intuitionistic fuzzy sets were extended to the interval valued intuitionistic fuzzy sets. Intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems. So, Neutrosophic set (generalization of fuzzy sets, intuitionistic fuzzy sets and so on) defined by Florentin Smarandache [1] has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exists in real world from philosophical point of view. Wang et al [2] proposed the single valued neutrosophic set.

In 1982, Pawlak [4] introduced the concept of rough set (RS), as a formal tool for modeling and processing incomplete information in information systems. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of RSs. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Later on, Dubois and Prade introduced fuzzy rough sets as a fuzzy generalization of rough sets. Salehz Rizvi et al introduced rough intuitionistic fuzzy sets. Broumi et al [5] introduced rough neutrosophic sets.

In this paper, by using the notion of rough neutrosophic set, it is provided an exemplary for medical diagnosis. In order to make this, several types of methods are executed.

Rest of the article is structured as follows. Section 2, briefly presents the basic definitions. Section 3 deals with proposed definitions and some of its properties. Sections 4,5&6 contains methodology, algorithm and case study related to medical diagnosis respectively. Conclusion is given in Section 7.

II. PRELIMINARIES

2.1 Definition [6]

Let H be a universal space of points (objects) with a generic element of H denoted by x . A single valued neutrosophic set S is characterized by a truth membership function $T_N(x)$, a falsity membership function $F_N(x)$ and indeterminacy function $I_N(x)$ with $T_N(x)$, $F_N(x)$, $I_N(x) \in [0,1]$ for all x in H .

When H is continuous, a SVNS S can be written as follows:

$$S = \int_x \langle T_s(x), F_s(x), I_s(x) \rangle / x, \forall x \in H$$

and when H is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_s(x), F_s(x), I_s(x) \rangle / x, \forall x \in H$$

It should be observed that for SNVS S

$$0 \leq \sup T_s(x) + \sup I_s(x) + \sup F_s(x) \leq 3, \forall x \in H$$

2.2 Definition [7]

Let A be a fuzzy neutrosophic set in X . Let R be the relation from X to Y . Then max-min composition of fuzzy neutrosophic set with A is another fuzzy neutrosophic set

B of Y which is denoted by $R \circ A$. Then the membership function, indeterminate function and non-membership function of B is defined as

$$T_{R \circ A}(y) = \vee_x [T_A(x) \wedge T_A(x, y)]$$

$$I_{R \circ A}(y) = \vee_x [I_A(x) \wedge I_A(x, y)]$$

$$F_{R \circ A}(y) = \wedge_x [F_A(x) \vee F_A(x, y)]$$

2.3 Definition [5]

Let U be a non-null set and R be an equivalence relation on U . Let P be neutrosophic set in U with the membership function T_P , indeterminacy function I_P and non-membership function F_P . The lower and the upper approximations of P in the approximation (U, R) denoted by $\underline{N}(P)$ & $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \langle \langle x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) \rangle / y \in [x]_R, x \in U \rangle$$

$$\overline{N}(P) = \langle \langle x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) \rangle / y \in [x]_R, x \in U \rangle$$

where

$$T_{\underline{N}(P)}(x) = \bigwedge_{y \in [x]_R} T_P(y)$$

$$I_{\underline{N}(P)}(x) = \bigvee_{y \in [x]_R} I_P(y)$$

$$F_{\underline{N}(P)}(x) = \bigvee_{y \in [x]_R} F_P(y)$$

$$T_{\overline{N}(P)}(x) = \bigvee_{y \in [x]_R} T_P(y)$$

$$I_{\overline{N}(P)}(x) = \bigwedge_{y \in [x]_R} I_P(y)$$

$$F_{\overline{N}(P)}(x) = \bigwedge_{y \in [x]_R} F_P(y)$$

$$\text{So, } 0 \leq T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \leq 3 \text{ \&}$$

$$0 \leq T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \leq 3,$$

where \vee and \wedge mean “max” and “min” operators respectively,

$T_P(y), I_P(y)$ & $F_P(y)$ are the membership, indeterminacy and non-membership of y with respect to P . It is easy to see that $\underline{N}(P)$ & $\overline{N}(P)$ are two neutrosophic sets in U , thus the neutrosophic set mappings $\underline{N}, \overline{N}: N(U) \rightarrow N(U)$ are respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (U, R) .

2.4 Definition [5]

Let $N(P_1)$ & $N(P_2)$ be two rough neutrosophic sets of the neutrosophic sets P_1 & P_2 respectively in U , then $N(P_1) \subseteq N(P_2)$ if and only if $\underline{N}(P_1) \subseteq \underline{N}(P_2)$ and $\overline{N}(P_1) \subseteq \overline{N}(P_2)$

III. PROPOSED DEFINITIONS

The proposed definitions are as follows

3.1 Definition

Let $A = (a, b, c)$ be a single valued neutrosophic number, a grade function E of a single valued neutrosophic value, based on the truth-membership degree, indeterminacy-membership degree and falsity-membership degree is defined by

$$E(A) = \frac{((1-a)+b-c)^2}{4} \tag{1}$$

3.1.1 Proposition

$$E(A) \geq 0$$

Proof

The proof is straightforward

3.1.2 Theorem

Let $A = (a_1, b_1, c_1)$ and $B = (a_2, b_2, c_2)$ be two single valued neutrosophic numbers. If $A \subseteq B$ then $E(A) \geq E(B)$

Proof

By Eq. (1),

$$E(A) = \frac{((1-a_1)+b_1-c_1)^2}{4} \text{ \& } E(B) = \frac{((1-a_2)+b_2-c_2)^2}{4}$$

Since $A \subseteq B, a_1 \leq a_2, b_1 \geq b_2$ & $c_1 \geq c_2$.

$$\therefore (a_2 - a_1) \geq 0, (b_1 - b_2) \geq 0 \text{ \&}$$

$$(c_1 - c_2) \geq 0. \text{ Hence } E(A) - E(B) \geq 0.$$

3.2 Definition

Let $A = (a, b, c)$ be a single valued neutrosophic number, a similarity grade function N of a single valued neutrosophic value, based on the truth-membership degree, indeterminacy-membership degree and falsity-membership degree is defined by

$$N(A) = 1 - \frac{(1-a)+b-c}{3} \tag{2}$$

3.2.1 Proposition

$$N(A) \geq 0$$

Proof

The proof is straightforward

3.2.2 Theorem

Let $A = (a_1, b_1, c_1)$ and $B = (a_2, b_2, c_2)$ be two single valued neutrosophic numbers. If $A \subseteq B$ then $N(A) \leq N(B)$

Proof

By Eq. (2),

$$N(A) = 1 - \frac{(1-a_1)+b_1-c_1}{3} \text{ \& } N(B) = 1 - \frac{(1-a_2)+b_2-c_2}{3}$$

Since $A \subseteq B, a_1 \leq a_2, b_1 \geq b_2$ & $c_1 \geq c_2$.

$$\therefore (a_2 - a_1) \geq 0, (b_1 - b_2) \geq 0 \text{ \& } (c_1 - c_2) \geq 0.$$

Hence $N(A) - N(B) \leq 0$.

3.3 Definition

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$

and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$

be two rough neutrosophic sets, then the logarithmic distance

$$LD_{RNS}(A, B) =$$

$$\frac{1}{n} \left[\sum_{i=1}^n \log_2 \left[\frac{\pi}{2} + \frac{\sqrt{\frac{|\underline{T}_A(x_i) - \underline{T}_B(x_i)| + |\underline{I}_A(x_i) - \underline{I}_B(x_i)| + |\underline{F}_A(x_i) - \underline{F}_B(x_i)| + |\overline{T}_A(x_i) - \overline{T}_B(x_i)| + |\overline{I}_A(x_i) - \overline{I}_B(x_i)| + |\overline{F}_A(x_i) - \overline{F}_B(x_i)|}{3}}}{\pi} \right] \right] \quad (3)$$

3.3.1 Proposition

(i) $LD_{RNS}(A, B) \in [0, 1]$

(ii) $LD_{RNS}(A, B) = LD_{RNS}(B, A)$

(iii) If $A \subseteq B \subseteq C$ then $LD_{RNS}(A, C) \geq LD_{RNS}(A, B)$ and $LD_{RNS}(A, C) \geq LD_{RNS}(B, C)$

Proof

(i) The proof is straightforward

(ii) The proof is straightforward

(iii) By definition 2.4,

$$\underline{T}_A(x_i) \leq \underline{T}_B(x_i) \leq \underline{T}_C(x_i)$$

$$\overline{T}_A(x_i) \leq \overline{T}_B(x_i) \leq \overline{T}_C(x_i)$$

$$\underline{I}_A(x_i) \geq \underline{I}_B(x_i) \geq \underline{I}_C(x_i)$$

$$\overline{I}_A(x_i) \geq \overline{I}_B(x_i) \geq \overline{I}_C(x_i)$$

$$\underline{F}_A(x_i) \geq \underline{F}_B(x_i) \geq \underline{F}_C(x_i)$$

$$\overline{F}_A(x_i) \geq \overline{F}_B(x_i) \geq \overline{F}_C(x_i)$$

$$[\because A \subseteq B \subseteq C]$$

Hence,

$$|\underline{T}_A(x_i) - \underline{T}_B(x_i)| \leq |\underline{T}_A(x_i) - \underline{T}_C(x_i)|$$

$$|\overline{T}_A(x_i) - \overline{T}_B(x_i)| \leq |\overline{T}_A(x_i) - \overline{T}_C(x_i)|$$

$$|\underline{I}_A(x_i) - \underline{I}_B(x_i)| \leq |\underline{I}_A(x_i) - \underline{I}_C(x_i)|$$

$$|\overline{I}_A(x_i) - \overline{I}_B(x_i)| \leq |\overline{I}_A(x_i) - \overline{I}_C(x_i)|$$

$$|\underline{F}_A(x_i) - \underline{F}_B(x_i)| \leq |\underline{F}_A(x_i) - \underline{F}_C(x_i)|$$

$$|\overline{F}_A(x_i) - \overline{F}_B(x_i)| \leq |\overline{F}_A(x_i) - \overline{F}_C(x_i)|$$

Here, the logarithmic distance is an increasing function

$$\therefore LM_{RNS}(A, C) \geq LM_{RNS}(A, B) \text{ \& } LM_{RNS}(A, C) \geq LM_{RNS}(B, C)$$

3.4 Definition

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$ and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$ be two rough neutrosophic sets then the exponential measure.

$$EM_{RNS}(A, B) =$$

$$\frac{1}{2n} \left[\sum_{i=1}^n \frac{\pi}{4} + e^{-\left[\frac{|\underline{T}_A(x_i) - \underline{T}_B(x_i)| + |\underline{I}_A(x_i) - \underline{I}_B(x_i)| + |\underline{F}_A(x_i) - \underline{F}_B(x_i)| + |\overline{T}_A(x_i) - \overline{T}_B(x_i)| + |\overline{I}_A(x_i) - \overline{I}_B(x_i)| + |\overline{F}_A(x_i) - \overline{F}_B(x_i)|}{n} \right]} \right] \quad (4)$$

3.4.1 Proposition

(i) $EM_{RNS}(A, B) \geq 0$

(ii) $EM_{RNS}(A, B) = EM_{RNS}(B, A)$

(iii) If $A \subseteq B \subseteq C$ then $EM_{RNS}(A, C) \leq EM_{RNS}(A, B)$ and $EM_{RNS}(A, C) \leq EM_{RNS}(B, C)$

Proof

(i) The proof is straightforward

(ii) The proof is straightforward

(iii) By definition 2.4,

$$\underline{T}_A(x_i) \leq \underline{T}_B(x_i) \leq \underline{T}_C(x_i)$$

$$\overline{T}_A(x_i) \leq \overline{T}_B(x_i) \leq \overline{T}_C(x_i)$$

$$\underline{I}_A(x_i) \geq \underline{I}_B(x_i) \geq \underline{I}_C(x_i)$$

$$\overline{I}_A(x_i) \geq \overline{I}_B(x_i) \geq \overline{I}_C(x_i)$$

$$\underline{F}_A(x_i) \geq \underline{F}_B(x_i) \geq \underline{F}_C(x_i)$$

$$\overline{F}_A(x_i) \geq \overline{F}_B(x_i) \geq \overline{F}_C(x_i)$$

$$[\because A \subseteq B \subseteq C]$$

Hence,

$$|\underline{T}_A(x_i) - \underline{T}_B(x_i)| \leq |\underline{T}_A(x_i) - \underline{T}_C(x_i)|$$

$$|\overline{T}_A(x_i) - \overline{T}_B(x_i)| \leq |\overline{T}_A(x_i) - \overline{T}_C(x_i)|$$

$$|\underline{I}_A(x_i) - \underline{I}_B(x_i)| \leq |\underline{I}_A(x_i) - \underline{I}_C(x_i)|$$

$$|\overline{I}_A(x_i) - \overline{I}_B(x_i)| \leq |\overline{I}_A(x_i) - \overline{I}_C(x_i)|$$

$$|\underline{F}_A(x_i) - \underline{F}_B(x_i)| \leq |\underline{F}_A(x_i) - \underline{F}_C(x_i)|$$

$$|\overline{F}_A(x_i) - \overline{F}_B(x_i)| \leq |\overline{F}_A(x_i) - \overline{F}_C(x_i)|$$

Here, the exponential measure is a decreasing function.

$$\therefore EM_{RNS}(A, C) \leq EM_{RNS}(A, B) \text{ \& } EM_{RNS}(A, C) \leq EM_{RNS}(B, C)$$

3.5 Definition

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$ and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$ be two rough neutrosophic sets then the similarity measure

$$SM_{RNS}(A, B) = \frac{1}{n} \left[\sum_{i=1}^n 1 - \frac{\sqrt{\frac{|\underline{T}_A(x_i) - \underline{T}_B(x_i)| + |\underline{I}_A(x_i) - \underline{I}_B(x_i)| + |\underline{F}_A(x_i) - \underline{F}_B(x_i)| + |\overline{T}_A(x_i) - \overline{T}_B(x_i)| + |\overline{I}_A(x_i) - \overline{I}_B(x_i)| + |\overline{F}_A(x_i) - \overline{F}_B(x_i)|}{3}}}{\pi} \right] \quad (5)$$

3.5.1 Proposition

(i) $SM_{RNS}(A, B) \in [0, 1]$

(ii) $SM_{RNS}(A, B) = SM_{RNS}(B, A)$

(iii) If $A \subseteq B \subseteq C$ then $SM_{RNS}(A, C) \leq SM_{RNS}(A, B)$ and $SM_{RNS}(A, C) \leq SM_{RNS}(B, C)$

Proof

(i) The proof is straightforward

(ii) The proof is straightforward
(iii) By definition 2.4,

$$\begin{aligned} \underline{T}_A(x_i) &\leq \underline{T}_B(x_i) \leq \underline{T}_C(x_i) \\ \overline{T}_A(x_i) &\leq \overline{T}_B(x_i) \leq \overline{T}_C(x_i) \\ \underline{I}_A(x_i) &\geq \underline{I}_B(x_i) \geq \underline{I}_C(x_i) \\ \overline{I}_A(x_i) &\geq \overline{I}_B(x_i) \geq \overline{I}_C(x_i) \\ \underline{F}_A(x_i) &\geq \underline{F}_B(x_i) \geq \underline{F}_C(x_i) \\ \overline{F}_A(x_i) &\geq \overline{F}_B(x_i) \geq \overline{F}_C(x_i) \\ [\because A \subseteq B \subseteq C] \end{aligned}$$

Hence,

$$\begin{aligned} |\underline{T}_A(x_i) - \underline{T}_B(x_i)| &\leq |\underline{T}_A(x_i) - \underline{T}_C(x_i)| \\ |\overline{T}_A(x_i) - \overline{T}_B(x_i)| &\leq |\overline{T}_A(x_i) - \overline{T}_C(x_i)| \\ |\underline{I}_A(x_i) - \underline{I}_B(x_i)| &\leq |\underline{I}_A(x_i) - \underline{I}_C(x_i)| \\ |\overline{I}_A(x_i) - \overline{I}_B(x_i)| &\leq |\overline{I}_A(x_i) - \overline{I}_C(x_i)| \\ |\underline{F}_A(x_i) - \underline{F}_B(x_i)| &\leq |\underline{F}_A(x_i) - \underline{F}_C(x_i)| \\ |\overline{F}_A(x_i) - \overline{F}_B(x_i)| &\leq |\overline{F}_A(x_i) - \overline{F}_C(x_i)| \end{aligned}$$

Here, the similarity measure is a decreasing function

$$\therefore SM_{RNS}(A, C) \leq SM_{RNS}(A, B) \& SM_{RNS}(A, C) \leq SM_{RNS}(B, C)$$

3.6 Definition

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$ and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$ be two rough neutrosophic sets then the logarithmic function based on similarity measure

$$l_{RNS}(A, B) = \frac{1}{2} \left[\log \frac{2 + SM_{RNS}(A, B)}{2 - SM_{RNS}(A, B)} \right] \quad (6)$$

3.6.1 Proposition

- (i) $l_{RNS}(A, B) \in [0, 1]$
 - (ii) $l_{RNS}(A, B) = l_{RNS}(B, A)$
- If $A \subseteq B \subseteq C$ then

$$(ii) l_{RNS}(A, C) \leq l_{RNS}(A, B) \text{ and } l_{RNS}(A, C) \leq l_{RNS}(B, C)$$

Proof

- (i) The proof is straightforward
- (ii) The proof is straightforward
- (iii) By definition 2.4,

$$\begin{aligned} \underline{T}_A(x_i) &\leq \underline{T}_B(x_i) \leq \underline{T}_C(x_i) \\ \overline{T}_A(x_i) &\leq \overline{T}_B(x_i) \leq \overline{T}_C(x_i) \\ \underline{I}_A(x_i) &\geq \underline{I}_B(x_i) \geq \underline{I}_C(x_i) \\ \overline{I}_A(x_i) &\geq \overline{I}_B(x_i) \geq \overline{I}_C(x_i) \\ \underline{F}_A(x_i) &\geq \underline{F}_B(x_i) \geq \underline{F}_C(x_i) \\ \overline{F}_A(x_i) &\geq \overline{F}_B(x_i) \geq \overline{F}_C(x_i) \\ [\because A \subseteq B \subseteq C] \end{aligned}$$

Hence,

$$|\underline{T}_A(x_i) - \underline{T}_B(x_i)| \leq |\underline{T}_A(x_i) - \underline{T}_C(x_i)|$$

$$\begin{aligned} |\overline{T}_A(x_i) - \overline{T}_B(x_i)| &\leq |\overline{T}_A(x_i) - \overline{T}_C(x_i)| \\ |\underline{I}_A(x_i) - \underline{I}_B(x_i)| &\leq |\underline{I}_A(x_i) - \underline{I}_C(x_i)| \\ |\overline{I}_A(x_i) - \overline{I}_B(x_i)| &\leq |\overline{I}_A(x_i) - \overline{I}_C(x_i)| \\ |\underline{F}_A(x_i) - \underline{F}_B(x_i)| &\leq |\underline{F}_A(x_i) - \underline{F}_C(x_i)| \\ |\overline{F}_A(x_i) - \overline{F}_B(x_i)| &\leq |\overline{F}_A(x_i) - \overline{F}_C(x_i)| \end{aligned}$$

Here, the logarithmic function is a decreasing function

$$\therefore l_{RNS}(A, C) \leq l_{RNS}(A, B) \& l_{RNS}(A, C) \leq l_{RNS}(B, C)$$

3.7 Definition

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$ and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$ be two rough neutrosophic sets then the exponential function based on similarity measure

$$e_{RNS}(A, B) = \frac{1}{2} \left[e^{\left[\frac{2 + SM_{RNS}(A, B)}{2n(2 - SM_{RNS}(A, B))} \right]} \right] \quad (7)$$

3.7.1 Proposition

- (i) $e_{RNS}(A, B) > 0$
- (ii) $e_{RNS}(A, B) = e_{RNS}(B, A)$
- (iii) If $A \subseteq B \subseteq C$ then $e_{RNS}(A, C) \leq e_{RNS}(A, B)$ and $e_{RNS}(A, C) \leq e_{RNS}(B, C)$

Proof

- (i) The proof is straightforward
- (ii) The proof is straightforward
- (iii) By definition 2.4,

$$\begin{aligned} \underline{T}_A(x_i) &\leq \underline{T}_B(x_i) \leq \underline{T}_C(x_i) \\ \overline{T}_A(x_i) &\leq \overline{T}_B(x_i) \leq \overline{T}_C(x_i) \\ \underline{I}_A(x_i) &\geq \underline{I}_B(x_i) \geq \underline{I}_C(x_i) \\ \overline{I}_A(x_i) &\geq \overline{I}_B(x_i) \geq \overline{I}_C(x_i) \\ \underline{F}_A(x_i) &\geq \underline{F}_B(x_i) \geq \underline{F}_C(x_i) \\ \overline{F}_A(x_i) &\geq \overline{F}_B(x_i) \geq \overline{F}_C(x_i) \\ [\because A \subseteq B \subseteq C] \end{aligned}$$

Hence,

$$\begin{aligned} |\underline{T}_A(x_i) - \underline{T}_B(x_i)| &\leq |\underline{T}_A(x_i) - \underline{T}_C(x_i)| \\ |\overline{T}_A(x_i) - \overline{T}_B(x_i)| &\leq |\overline{T}_A(x_i) - \overline{T}_C(x_i)| \\ |\underline{I}_A(x_i) - \underline{I}_B(x_i)| &\leq |\underline{I}_A(x_i) - \underline{I}_C(x_i)| \\ |\overline{I}_A(x_i) - \overline{I}_B(x_i)| &\leq |\overline{I}_A(x_i) - \overline{I}_C(x_i)| \\ |\underline{F}_A(x_i) - \underline{F}_B(x_i)| &\leq |\underline{F}_A(x_i) - \underline{F}_C(x_i)| \\ |\overline{F}_A(x_i) - \overline{F}_B(x_i)| &\leq |\overline{F}_A(x_i) - \overline{F}_C(x_i)| \end{aligned}$$

Here, the exponential function is a decreasing function

$$\therefore e_{RNS}(A, C) \leq e_{RNS}(A, B) \& e_{RNS}(A, C) \leq e_{RNS}(B, C)$$

IV. METHODOLOGY

In this section, application of rough neutrosophic set in medical diagnosis is presented. In a given pathology,

suppose S is a set of symptoms, D is a set of diseases and P is a set of patients and let Q be a rough neutrosophic relation from the set of patients to the symptoms i.e., $Q(P \rightarrow S)$ and R be a rough neutrosophic relation from the set of symptoms to the diseases i.e., $R(S \rightarrow D)$ and then the methodology involves three main jobs:

1. Determination of symptoms.
2. Formulation of medical knowledge based on rough neutrosophic sets.
3. Determination of diagnosis on the basis of various computation techniques of rough neutrosophic sets.

V. ALGORITHM

Step 1: The symptoms of the patients are given to obtain the patient - symptom relation Q and are noted in Table 1.

Step 2 : The medical knowledge relating the symptoms with the set of diseases under consideration are given to obtain the symptom - disease relation R and are noted in Table 2.

Step 3 : Table 3 is obtained by calculating average values for Table 1.

Step 4 : Table 4 is obtained by calculating average values for Table 2.

Step 5 : Table 5 is obtained by applying definition 2.2 between Table 3 & Table 4.

Step 6 : The Computation T of the relation of patients and diseases is found using definitions 3.1 & 3.2 in Table 5 and are noted in Table 6 & Table 7 respectively.

Step 7 : The Computation T of the relation of patients and diseases is found using definitions 3.3, 3.4, 3.5, 3.6 & 3.7 and are noted in Table 8 to 12 respectively.

Step 8 : Finally, the minimum value from Table 6 & 8 and maximum value from Table 7, 9, 10, 11 & 12 of each row are selected to find the possibility of the patient affected with the respective disease and then it is concluded that the patient $P_k(k=1,2&3)$ is suffering from the disease $D_r(r=1,2,3&4)$

VI. CASE STUDY [6]

Let there be three patients $P = \{P_1, P_2, P_3\}$ and the set of symptoms $S = \{S_1 = \text{Temperature}, S_2 = \text{Headache}, S_3 =$

Stomach pain, $S_4 = \text{Cough}, S_5 = \text{Chest pain}\}$. The Rough Neutrosophic Relation $Q(P \rightarrow S)$ is given as in Table 1. Let the set of diseases $D = \{D_1 = \text{Viral fever}, D_2 = \text{Malaria}, D_3 = \text{Stomach problem}, D_4 = \text{Chest problem}\}$. The Rough Neutrosophic Relation $R(S \rightarrow D)$ is given as in Table 2.

VII. CONCLUSION

The propounded techniques are most reliable to handle medical diagnosis problems quiet comfortably. The recommended methods can invade in other areas such as clustering, image processing etc., In future, these methods can be enhanced to other types of neutrosophic sets also.

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Table 1: Patient – Symptom (Using Step 1)

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
P ₁	$\langle\langle(0.6,0.4,0.3), (0.8,0.2,0.1)\rangle\rangle$	$\langle\langle(0.4,0.4,0.4), (0.6,0.2,0.2)\rangle\rangle$	$\langle\langle(0.5,0.3,0.2), (0.7,0.1,0.2)\rangle\rangle$	$\langle\langle(0.6,0.2,0.4), (0.8,0.0,0.2)\rangle\rangle$	$\langle\langle(0.4,0.4,0.4), (0.6,0.2,0.2)\rangle\rangle$
P ₂	$\langle\langle(0.5,0.3,0.4), (0.7,0.3,0.2)\rangle\rangle$	$\langle\langle(0.5,0.5,0.3), (0.7,0.3,0.3)\rangle\rangle$	$\langle\langle(0.5,0.3,0.4), (0.7,0.1,0.4)\rangle\rangle$	$\langle\langle(0.5,0.3,0.3), (0.9,0.1,0.3)\rangle\rangle$	$\langle\langle(0.5,0.3,0.3), (0.7,0.1,0.3)\rangle\rangle$
P ₃	$\langle\langle(0.6,0.4,0.4), (0.8,0.2,0.2)\rangle\rangle$	$\langle\langle(0.5,0.2,0.3), (0.7,0.0,0.1)\rangle\rangle$	$\langle\langle(0.4,0.3,0.4), (0.8,0.1,0.2)\rangle\rangle$	$\langle\langle(0.6,0.1,0.4), (0.8,0.1,0.2)\rangle\rangle$	$\langle\langle(0.5,0.3,0.3), (0.7,0.1,0.1)\rangle\rangle$

Table 2: Symptom – Disease (Using Step 2)

R	Viral fever	Malaria	Stomach problem	Chest problem
Temperature	$\langle\langle(0.6,0.5,0.4), (0.8,0.3,0.2)\rangle\rangle$	$\langle\langle(0.1,0.4,0.4), (0.5,0.2,0.2)\rangle\rangle$	$\langle\langle(0.3,0.4,0.4), (0.5,0.2,0.2)\rangle\rangle$	$\langle\langle(0.2,0.4,0.6), (0.4,0.4,0.4)\rangle\rangle$
Headache	$\langle\langle(0.5,0.3,0.4), (0.7,0.3,0.2)\rangle\rangle$	$\langle\langle(0.2,0.3,0.4), (0.6,0.3,0.2)\rangle\rangle$	$\langle\langle(0.2,0.3,0.3), (0.4,0.1,0.1)\rangle\rangle$	$\langle\langle(0.1,0.5,0.5), (0.5,0.3,0.3)\rangle\rangle$
Stomach pain	$\langle\langle(0.2,0.3,0.4), (0.4,0.3,0.2)\rangle\rangle$	$\langle\langle(0.1,0.4,0.4), (0.3,0.2,0.2)\rangle\rangle$	$\langle\langle(0.4,0.3,0.4), (0.6,0.1,0.2)\rangle\rangle$	$\langle\langle(0.1,0.4,0.6), (0.3,0.2,0.4)\rangle\rangle$
Cough	$\langle\langle(0.4,0.3,0.3), (0.6,0.1,0.1)\rangle\rangle$	$\langle\langle(0.3,0.3,0.3), (0.5,0.1,0.3)\rangle\rangle$	$\langle\langle(0.1,0.6,0.6), (0.3,0.4,0.4)\rangle\rangle$	$\langle\langle(0.5,0.3,0.4), (0.7,0.1,0.2)\rangle\rangle$
Chest pain	$\langle\langle(0.2,0.4,0.4), (0.4,0.2,0.2)\rangle\rangle$	$\langle\langle(0.1,0.3,0.3), (0.3,0.1,0.1)\rangle\rangle$	$\langle\langle(0.1,0.4,0.4), (0.3,0.2,0.2)\rangle\rangle$	$\langle\langle(0.4,0.4,0.4), (0.6,0.2,0.2)\rangle\rangle$

Table 3: Average (using step 3 [3])

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
P ₁	[0.7 ,0.3 ,0.2]	[0.5 ,0.3 ,0.3]	[0.6 ,0.2 ,0.2]	[0.7 ,0.1 ,0.3]	[0.5 ,0.3 ,0.3]
P ₂	[0.6 ,0.3 ,0.3]	[0.6 ,0.4 ,0.3]	[0.6 ,0.2 ,0.4]	[0.7 ,0.2 ,0.3]	[0.6 ,0.2 ,0.3]
P ₃	[0.7 ,0.3 ,0.3]	[0.6 ,0.1 ,0.2]	[0.6 ,0.2 ,0.3]	[0.7 ,0.1 ,0.3]	[0.6 ,0.2 ,0.2]

Table 4: Average (Using step 4[3])

R	Viral fever	Malaria	Stomach problem	Chest problem
Temperature	[0.7 ,0.4 ,0.3]	[0.3 ,0.3 ,0.3]	[0.4 ,0.3,0.3]	[0.3 ,0.4 ,0.5]
Headache	[0.6 ,0.3 ,0.3]	[0.4 ,0.3 ,0.3]	[0.3 ,0.2 ,0.2]	[0.3 ,0.4 ,0.4]
Stomach pain	[0.3 ,0.3 ,0.3]	[0.2 ,0.3 ,0.3]	[0.5 ,0.2 ,0.3]	[0.2 ,0.3 ,0.5]
Cough	[0.5 ,0.2 ,0.2]	[0.4 ,0.2 ,0.3]	[0.2 ,0.5 ,0.5]	[0.6 ,0.2 ,0.3]
Chest pain	[0.3 ,0.3 ,0.3]	[0.2 ,0.2 ,0.2]	[0.2 ,0.3 ,0.3]	[0.5 ,0.3,0.3]

Table 5: Max-Min Composition (Using step 5)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	[0.7 ,0.3,0.3]	[0.4 ,0.3 ,0.3]	[0.5 ,0.3 ,0.3]	[0.6 ,0.3 ,0.3]
P ₂	[0.6 ,0.3 ,0.3]	[0.4 ,0.3 ,0.3]	[0.5 ,0.3 ,0.3]	[0.6 ,0.4 ,0.3]
P ₃	[0.7 ,0.3 ,0.3]	[0.4 ,0.3 ,0.2]	[0.5 ,0.3 ,0.2]	[0.6 ,0.3 ,0.3]

Table 6: Grade Function (Using step 6& step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.0225	0.0900	0.0625	0.0400
P ₂	0.0400	0.0900	0.0625	0.0625
P ₃	0.0225	0.1225	0.0900	0.0400

Table 7: Similarity Grade Function (Using step 6 & step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.9000	0.8000	0.8333	0.8666
P ₂	0.8666	0.8000	0.8333	0.8333
P ₃	0.9000	0.7666	0.8000	0.8666

Table 8: Logarithmic Distance (Using step 7 & step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.2605	0.2751	0.2740	0.2623
P ₂	0.2644	0.2742	0.2791	0.2714
P ₃	0.2639	0.2762	0.2740	0.2796

Table 9: Exponential Measure (Using step 7 & step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.8365	0.8090	0.8083	0.8169
P ₂	0.8295	0.8121	0.7978	0.8160

P ₃	0.8295	0.8085	0.8055	0.7972
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Table 10: Similarity Measure (Using step 7 & step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.7471	0.6847	0.6880	0.7333
P ₂	0.7306	0.6893	0.6658	0.7004
P ₃	0.7325	0.6806	0.6869	0.6640

Table 11: Logarithmic Function (Using step 7 & step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.1704	0.1549	0.1557	0.1670
P ₂	0.1663	0.1560	0.1503	0.1588
P ₃	0.1668	0.1539	0.1554	0.1498

Table 12: Exponential Function (Using step 7 & step 8)

T	Viral fever	Malaria	Stomach problem	Chest problem
P ₁	0.6225	0.6132	0.6136	0.6204
P ₂	0.6199	0.6138	0.6105	0.6154
P ₃	0.6202	0.6126	0.6135	0.6103

From Table 6 to 12, it is obvious that, if the doctor agrees, then P₁, P₂ & P₃ suffers from Viral fever.