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Application of Mohand Transform for Solving Partial Differential Equations

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Abstract : **Partial differential equations are used in modeling various phenomena in science, engineering and social sciences.** The new integral transform **Mohand transform of partial derivatives are derived and its applicability demonstrated using five different partial differential equations . In this paper we have to find the particular solution and we discuss some relationship between Laplace transform and the Mohand transform. We solve first and second order partial differential equations using both transform, and show that Mohand transform are closely connected with the Laplace transform.**

Keywords — **Mohand Transform, Partial Differential Equations.**

INTRODUCTION

The differential equation[1,2] have played a central role in every aspect of applied mathematics for every long time and with the advent of the computer, their importance has increased further. Thus investigation and analysis of differential equations cruising in application oriented mathematical problems; therefore, there are so many different techniques in order to solve differential equations. Fluid mechanics, heat and mass transfer, and electromagnetic theory are all modeled by partial differential equations and all have plenty of real life applications.The integral transform[3,4] were extensively used and thus there are several words on the theory and applications of integral transforms such as the Laplace, Fourier, Elzaki, Hankel and Sumudu, to name but a few. In this paper we discuss some relationship between Laplace transform and the transform called Mohand transform. We solve first and second order partial differential equations using both transforms, and show that Mohand transform are closely connected with the Laplace transform.

Recently, Mohand Mahgoub[5] introduced a new integral transform, named the Mohand transform, and further applied it to the solution of partial differential equations. In this paper we derive the formulate for Mohand transform of partial derivatives[6,7,8] and apply them in solving some types of initial value problems. Our purpose here is to show the applicability of this interesting new transform and its effecting in solving such problems.

The aim of this work is to establish exact solution for linear partial differential equation using Mohand transform without large computational work. The result reveals that the proposed method is very efficient, simple and can be applied to linear differential equations.

MOHAND TRANSFORM OF DERIVATIVES

Mohand transform of the function $f(t)$ is defined as

$$M[f(t)] = R(v) = v^2 \int_0^\infty f(t)\, e^{-vt} dt,\ t \geq 0,\quad k_1 \leq v \leq k_2 \qquad (1)$$

To obtain Mohand transform of partial derivatives , we use integration by parts as follows:

$$M\left[\frac{\partial}{\partial t} f(x,t)\right] = v^2 \int_0^\infty \frac{\partial}{\partial t} e^{-vt}\ dt$$

$$= \lim_{p\to\infty} \int_0^p v^2 e^{-vt} \frac{\partial f}{\partial t}\ dt$$

$$= \lim_{p\to\infty}\left[\left(v^2 e^{-vt} f(x,t)\right)_0^p - \int_0^p v^2(-v)e^{-vt} f(x,t)dt\right)\right]$$

$$= \left(v^2 e^{-vt} f(x,t)\right)_0^\infty + v^3 \int_0^\infty e^{-vt} f(x,t)dt$$

$$= vR(x,v) - v^2 f(x,0) \qquad (2)$$

We assume that , f is piecewise continuous and is of exponential order

Consider,

$$M\left[\frac{\partial}{\partial x} f(x,t)\right] = \int_0^\infty v^2 e^{-vt} \frac{\partial f(x,t)}{\partial x} dt$$

$$= \frac{\partial}{\partial x}\int_0^\infty v^2 e^{-vt} f(x,t) dt \quad \text{(using the}$$

Leibnitz' rule)

$$= \frac{\partial}{\partial x}[R(x,v)] \text{ and}$$

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$$M\left[\frac{\partial}{\partial x}f(x,t)\right]=\frac{d}{dx}[R(x,v)] \quad (3)$$

Also we can find $M\left[\frac{\partial^2 f}{\partial x^2}\right]=\frac{d^2}{dx^2}[R(x,v)]$ (4)

To find $M\left[\frac{\partial^2 f}{\partial t^2}\right]$

Let $\frac{\partial f}{\partial t}=g\ ,\ then$

From equation (2), we have

$$M\left[\frac{\partial^2 f}{\partial t^2}\right]=M\left[\frac{\partial g}{\partial t}\right]$$
$$=M[vg(x,t)-v^2g(x,0)]$$
$$=vM[g(x,t)]-v^2M[g(x,0)]$$

$$=v^2R(x,v)-v^3f(x,0)-v^2\frac{\partial}{\partial t}f(x,0) \quad (5)$$

We can easily extend this result to the n[th] partial derivative by using mathematical induction.

SOLUTION OF PARTIAL DIFFERENTIAL EQUATION

In this section we solve first order Partial differential Equations and the Second order partial differential equation, wave equation, heat equation, Laplace equation and Telegraphers equation w hich are known as four Fundamental equations in mathematical physics and occur in many branches of physics, in applied mathematics as well as in engineering.

Example 3. 1:

Find the solution of first order initial value

$\frac{\partial u}{\partial x}+u=\frac{\partial u}{\partial t},$ if $u=4e^{-3x}$ when $t=0.$ (6)

Taking Mohand transform of Eq. (6), we have

$$R'(x,v)+R(x,v)=vR(x,v)-v^2u(x,0)$$

Where R(x,v) is the Mohand transform of u(x,t).

By applying the initial condition, we get

$$R'(x,v)+(1-v)R(x,v)=-4v^2e^{-3x}$$

This is the linear ordinary differential equation, it has the integration factor

$$F=e^{\int(1-v)\,dx}=e^{(1-v)x}$$

Therefore, $R(x,v)=\frac{4v^2}{v+4}e^{-3x}+ce^{-(1-v)x}$ (7)

Since R(x,v) is bounded, c should be zero, if we take the inverse Mohand transform to

Eq. (7), then the solution of Eq. (6) is,

$$u(x,t)=4e^{-3x-4t}$$

Example 3. 2:

Find the solution of the first - order initial value problem:

$y_x(x,t)-2y_t(x,t)=y(x,t)\ ,\ x>0,\ t>0,$ (8)

$y(x,0)=e^{-x}$

Taking Mohand transform of Eq. (8), we have

$$R'(x,v)-2vR(x,v)+2v^2y(x,0)=R(x,v)$$

Where R(x,v) is the Mohand transform of y(x,t).

By applying the initial condition, we get

$$R'(x,v)-(2v+1)R(x,v)=-2v^2e^{-x}$$

This is the linear ordinary differential equation, it has the integration factor

$$F=e^{-\int(2v+1)dx}=e^{-(2v+1)x}$$

Therefore, $R(x,v)=\frac{v^2}{v+1}e^{-x}+ce^{(2v+1)x}$ (9)

Since R(x,v) is bounded, c Should be zero, if we take the inverse Mohand transform to

Eq. (9), then the solution of Eq. (8) is,

$$y(x,t)=e^{-x-t}$$

Example 3. 3:

Consider the Laplace equation:

$u_{xx}+u_{tt}=0,\ u(x,0)=0,\ \ u_t(x,0)=\cos x,$ (10)

where $x,t>0$

Applying Mohand transform to both sides of this equation and using the differential property of Mohand transform, Eq. (10) can be written as:

$$R''(x,v)+v^2R(x,v)-v^3u(x,0)-v^2u_t(x,0)=0$$

And, $R''(x,v)+v^2R(x,v)-v^2\cos x=0$

This is the second – order ordinary differential equation have the particular solution in the form

$$R(x,v)=\frac{v^2\cos x}{D^2+v^2}=\frac{v^2\cos x}{v^2-1} \quad (11)$$

If we take the inverse Mohand transform for Eq. (11), we obtain the solution of Eq. (10) in the form

$$u(x,t)=\cos x\sinh t$$

Example 3. 4:

Let's consider the homogeneous heat equation in one dimension in a normalized form: $u_t=u_{xx}$, $u(x,0)=\sin\frac{\pi}{l}x,$ $u(0,t)=u(l,t)=0$ (12)

Applying Mohand transform to both sides of this equation and using the differential property of Mohand transform, Eq. (12) can be written as:

$$vR(x,v)-v^2u(x,0)=R''(x,v)$$

And, $R''(x,v)-vR(x,v)=-v^2\sin\frac{\pi}{l}x$

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This is the second – order ordinary differential equation have the particular, solution in the form

$$R(x,v) = \frac{-v^2 \sin\frac{\pi}{l}x}{D^2 - v} = \frac{-v^2 \sin\frac{\pi}{l}x}{-\left(\frac{\pi}{l}\right)^2 - v} = \frac{v^2 \sin\frac{\pi}{l}x}{\left[\left(\frac{\pi}{l}\right)^2 + v\right]^2} \quad (13)$$

If we take the inverse Mohand transform for Eq. (13), we obtain the solution of Eq. (12) in the form

$$u(x,t) = \sin\frac{\pi}{l}x\, e^{-\left(\frac{\pi}{l}\right)^2 t}$$

Example 3. 5:

Consider the telegraphers equation:

$$u_{tt}(x,t) + 2\alpha u_t(x,t) = \alpha^2 u_{xx}(x,t),\ 0 < x < 1,\ t > 0 \quad (14)$$

with the initial condition $u(x,0) = cosx$, $u_t(x,0) = 0$

Applying Mohand transform to both sides of this equation and using the differential property of Mohand transform, Eq. (14) can be written as:

$$v^2R(x,v) - v^3u(x,0) - v^2u_t(x,0) + 2\alpha vR(x,v) - 2\alpha v^2u(x,0) = \alpha^2R''(x,v)$$

And,

$$\alpha^2R''(x,v) - (v^2 + 2\alpha v)R(x,v) = -(v^3 + 2\alpha v^2)\cos x$$

This is the second – order ordinary differential equation have the particular, solution in the form

$$R(x,v) = \frac{-(v^3 + 2\alpha v^2)\cos x}{\alpha^2D^2 - (v^2 + 2\alpha v)} = \frac{(v^3 + 2\alpha v^2)\cos x}{\alpha^2 + (v^2 + 2\alpha v)} = \frac{(v^3 + 2\alpha v^2)\cos x}{(v + \alpha)^2} \quad (15)$$

If we take the inverse Mohand transform for Eq. (15), we obtain the solution of Eq. (14) in the form

$$u(x,t) = \cos x\, M^{-1}\left[\frac{2\alpha v^2}{(v+\alpha)^2} + \frac{v^3}{v+\alpha}\right] = cosx(1 + \alpha t)e^{-\alpha t}$$

IV . CONCLUSION

The definition and application of the new transform " Mohand transform" to the solution of partial differential equations has been demonstrated. The methods of Mohand Transform are successfully used to solve a general linear PDE's. Finally, we get exact solutions of such PDE after a few steps of calculations. The main advantage of Mohand transform (its like Elzaki transform) is reveals that it may be used to solve problems without resorting to a new frequency domain because it preserves scales and unit properties. Like Elzaki transform, the Mohand transform may be used to solve intricate problems in engineering, mathematics and applied science without resorting to a new frequency domain.

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