Response of a parallel Ł- C- $\mathcal{R}$ network connected to an excitation source providing a constant current by matrix method

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Abstract: In most of the branches of the engineering, the analysis of electric networks containing elements like inductor Ł, capacitor C, and resistor $\mathcal{R}$ is an essential course. The response of such networks is generally obtained by adopting the classical method or Laplace transform. In this paper, we discuss a matrix method for obtaining the response of a parallel electric network of an inductor, a capacitor, and a resistor, connected to an excitation source providing a constant current. The response obtained will be in the form an equation for the voltage across the parallel Ł- C- $\mathcal{R}$ network connected to an excitation source providing a constant current. The nature of this response (i.e. voltage) depends on the values of elements Ł, C, and $\mathcal{R}$ of the network.

Keywords: Constant current, Excitation source, Electric network, Response, Voltage.

I. INTRODUCTION

An electric circuit of parallel Ł- C- $\mathcal{R}$ network comprises of three basic elements namely an inductor having inductance Ł, a capacitor having capacitance C, and a resistor having resistance $\mathcal{R}$, connected to a to an excitation source providing a constant current. The electric elements like an inductor, a capacitor and a resistor are passive elements since these elements don't have the ability to transfer non – zero average power in an infinite time interval whereas, the elements like a current source and a voltage source are active elements since these elements have the ability to transfer non- zero average power in an infinite time interval. The electric circuit of parallel Ł- C- $\mathcal{R}$ network is widely used as a tuning circuit (i.e. a filtering circuit) in the analogue radios, and have many applications in oscillatory circuits[1, 2, 3].

II. EIGENVALUES AND EIGENVECTORS

Let e_{ij} be the elements of a matrix E of order n, then we can write the characteristic equation of E such that $|E - \Omega I|T = 0$, where T is a column matrix and Ω is a constant.

This characteristic equation of E on simplifying will provide n homogeneous linear equations which have a non – trivial solution if the determinant of the coefficients of the equations is zero i.e. If

$$\begin{vmatrix} (e_{11}-\Omega) & e_{12} & e_{13} \;\dots\dots & \dots .\, e_{1n} \\ e_{21} & (e_{22}-\Omega) & e_{23} \;\dots & \dots .\, e_{2n} \\ \dots\dots\dots . & \dots\dots\dots . & & \dots\dots\dots \\ e_{n1} & e_{n2} & e_{n3} \;\dots\dots & \dots .\,(e_{nn}-\Omega) \end{vmatrix} = 0,$$ then on expanding the determinant on left hand side of this equation,

we obtain n^{th} degree equation in Ω , which is the characteristic equation of the matrix E and its roots are known as Eigenvalues. Corresponding to each Eigenvalue there is a column matrix $T = [t_1 \quad t_2 \quad \dots\dots \quad t_n]'$ known as Eigenvector [4, 5].

III. FORMULATION

To find the governing differential equation:

Considering a parallel Ł - C- $\mathcal{R}$ network and connect a steady current source to it through a switch K as shown in figure 1.

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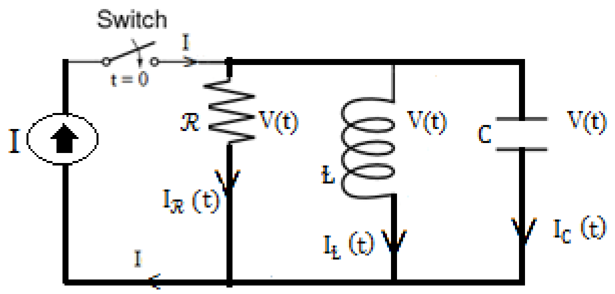

Figure 1: Parallel Ł - C- $\mathcal{R}$ network connected to a steady current source.

Let the switch is closed at the instant t =0, then the currents flowing in the elements Ł, C and $\mathcal{R}$ of the network are given by $I_{\mathcal{R}}(t) = \frac{V(t)}{\mathcal{R}}$, $I_{Ł}(t) = \frac{1}{Ł}\int V(t)dt$ and $I_C(t) = C\,Đ_t[V(t)]$, where V (t) is the electric potential across the network elements at any instant t [1, 3].

The application of Kirchhoff's current law as the switch is closed at the instant t = 0 gives

$$I_{\mathcal{R}}(t) + I_{Ł}(t) + I_C(t) = I$$

Or $\frac{V(t)}{\mathcal{R}} + \frac{1}{Ł}\int V(t)dt + C\,Đ_t[V(t)] = I$ ….. (1) $\qquad Đ_t \equiv \frac{d}{dt}$.

When we differentiate equation (1), we get a linear homogeneous differential equation of order 2 as given below:

$$\frac{1}{\mathcal{R}}Đ_t[V(t)] + \frac{1}{Ł}V(t) + CĐ_t^2[V(t)] = 0$$

Or $Đ_t^2[V(t)]\ \frac{1}{C\mathcal{R}}Đ_t[V(t)] + \frac{1}{ŁC}V(t) = 0$ …. (2)

To obtain the solution (response):

To find the solution of equation (2), we first write the necessary boundary conditions as follows:

(i) Since the potential across the plates of the capacitor and the current through the inductor cannot varies instantaneously [2, 6], therefore, at the instant t = 0, then $V(0) = 0$.

(ii) Since at t = 0, $V(0) = 0$, therefore, equation (1) gives $Đ_t[V(0)] = \frac{I}{C}$.

On putting $V(t) = V_1(t)$ … … … . (3)

And $Đ_t[V_1(t)] = V_2(t)$……. (4)

We can rewrite equation (2) as

$$Đ_t[V_2(t)] + \frac{1}{C\mathcal{R}}V_2(t) + \frac{1}{ŁC}V_1(t) = 0$$

Or $Đ_t[V_2(t)] = -\frac{1}{ŁC}V(t) - \frac{1}{C\mathcal{R}}V_2(t)$ …. (5)

We can write the differential equations (4) and (5) in a matrix form as

$$Đ_t\begin{bmatrix}V_1(t)\\V_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{ŁC} & -\frac{1}{C\mathcal{R}}\end{bmatrix}\begin{bmatrix}V_1(t)\\V_2(t)\end{bmatrix}$$

On equating the determinant of $\begin{bmatrix}0 & 1\\ -\frac{1}{ŁC} & -\frac{1}{C\mathcal{R}}\end{bmatrix}$ to zero, we obtain its characteristic equation as

$$\begin{vmatrix}0-\Omega & 1\\ -\frac{1}{ŁC} & -\frac{1}{C\mathcal{R}}-\Omega\end{vmatrix} = 0$$

On expanding the determinant, we get

$$\Omega^2 + \frac{1}{C\mathcal{R}}\Omega + \frac{1}{ŁC} = 0 \text{ ……. (6)}$$

This equation (6) is quadratic in Ω and its roots are given by

$$\Omega = \frac{-\frac{1}{C\mathcal{R}} \pm \sqrt{\left(\frac{1}{C\mathcal{R}}\right)^2 - \frac{4}{ŁC}}}{2}$$

Or $\Omega = -\frac{1}{2C\mathcal{R}} \pm \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{Ł}}$

Therefore, the roots of the equation (6) are $\Omega_1 = -\frac{1}{2C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{Ł}}$ ……. (7)

And $\Omega_2 = -\frac{1}{2C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{Ł}}$ ……….. (8)

Multiplying equations (7) and (8), we get

$$\Omega_1\Omega_2 = \left(-\frac{1}{2C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{Ł}}\right)\left(-\frac{1}{2C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{Ł}}\right)$$

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On simplifying the right hand side of this equation, we get

$$\Omega_1\Omega_2 = \frac{1}{\text{Ł}C} \;\ldots\ldots\; (9)$$

To find Eigenvectors:

The Eigenvector corresponding to the root $\Omega = \Omega_1 = -\frac{1}{2C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}$ is given by

$$\begin{bmatrix} 0-\Omega_1 & 1 \\ -\frac{1}{\text{Ł}C} & -\frac{1}{C\mathcal{R}} - \Omega_1 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This results

$$-\Omega_1 t_1 + t_2 = 0 \;\ldots\ldots\; (10)$$

And

$$-\frac{1}{\text{Ł}C}t_1 - \left(\frac{1}{C\mathcal{R}} + \Omega_1\right)t_2 = 0 \;\ldots\ldots\; (11)$$

Solving equations (10) and (11), we can write

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \Omega_1 + \frac{1}{C\mathcal{R}} + 1 \\ \Omega_1 - \frac{1}{\text{Ł}C} \end{bmatrix}$$

And the Eigenvector corresponding to the root $\Omega = \Omega_2 = -\frac{1}{2C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}$ is given by

$$\begin{bmatrix} 0-\Omega_2 & 1 \\ -\frac{1}{\text{Ł}C} & -\frac{1}{C\mathcal{R}} - \Omega_2 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This results

$$-\Omega_2 t_1 + t_2 = 0 \;\ldots\ldots\; (12)$$

And

$$-\frac{1}{\text{Ł}C}t_1 - \left(\frac{1}{C\mathcal{R}} + \Omega_2\right)t_2 = 0 \;\ldots\ldots\; (13)$$

Solving equations (12) and (13), we can write

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \Omega_2 + \frac{1}{C\mathcal{R}} + 1 \\ \Omega_2 - \frac{1}{\text{Ł}C} \end{bmatrix}$$

The matrix of Eigenvectors is $\begin{bmatrix} \Omega_1 + \frac{1}{C\mathcal{R}} + 1 & \Omega_2 + \frac{1}{C\mathcal{R}} + 1 \\ \Omega_1 - \frac{1}{\text{Ł}C} & \Omega_2 - \frac{1}{\text{Ł}C} \end{bmatrix}$.

Let $A = \begin{bmatrix} \Omega_1 + \frac{1}{C\mathcal{R}} + 1 & \Omega_2 + \frac{1}{C\mathcal{R}} + 1 \\ \Omega_1 - \frac{1}{\text{Ł}C} & \Omega_2 - \frac{1}{\text{Ł}C} \end{bmatrix}$, then the determinant of A i.e. $|A|$ can be written as

$$|A| = \begin{vmatrix} \Omega_1 + \frac{1}{C\mathcal{R}} + 1 & \Omega_2 + \frac{1}{C\mathcal{R}} + 1 \\ \Omega_1 - \frac{1}{\text{Ł}C} & \Omega_2 - \frac{1}{\text{Ł}C} \end{vmatrix}$$

On expanding the determinant and using equation (9), we obtain

$$|A| = (\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)$$

The inverse of A can be written as

$$A^{-1} = \frac{1}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)}\begin{bmatrix} \Omega_2 - \frac{1}{\text{Ł}C} & -\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) \\ -\left(\Omega_1 - \frac{1}{\text{Ł}C}\right) & \Omega_1 + \frac{1}{C\mathcal{R}} + 1 \end{bmatrix}$$

To find $A\exp(\Omega t)A^{-1}$:

$A\exp(\Omega t)A^{-1} =$

$$\begin{bmatrix} \Omega_1 + \frac{1}{C\mathcal{R}} + 1 & \Omega_2 + \frac{1}{C\mathcal{R}} + 1 \\ \Omega_1 - \frac{1}{\text{Ł}C} & \Omega_2 - \frac{1}{\text{Ł}C} \end{bmatrix}\begin{bmatrix} \exp(\Omega_1 t) & 0 \\ 0 & \exp(\Omega_2 t) \end{bmatrix}\frac{1}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)}\begin{bmatrix} \Omega_2 - \frac{1}{\text{Ł}C} & -\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) \\ -\left(\Omega_1 - \frac{1}{\text{Ł}C}\right) & \Omega_1 + \frac{1}{C\mathcal{R}} + 1 \end{bmatrix}$$

$$= \frac{1}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)} \times \begin{bmatrix} \left(\Omega_1 + \frac{1}{C\mathcal{R}} + 1\right)\exp(\Omega_1 t) & \left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right)\exp(\Omega_2 t) \\ \left(\Omega_1 - \frac{1}{\text{Ł}C}\right)\exp(\Omega_1 t) & \left(\Omega_2 - \frac{1}{\text{Ł}C}\right)\exp(\Omega_2 t) \end{bmatrix}\begin{bmatrix} \Omega_2 - \frac{1}{\text{Ł}C} & -\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) \\ -\left(\Omega_1 - \frac{1}{\text{Ł}C}\right) & \Omega_1 + \frac{1}{C\mathcal{R}} + 1 \end{bmatrix}$$

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$$= \frac{1}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)} \times \begin{bmatrix} (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)(\Omega_2 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) - (\Omega_1 - \frac{1}{\text{Ł}C})(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)\exp(\Omega_2\, t) & (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)[\exp(\Omega_2\, t) - \exp(\Omega_1\, t)] \\ (\Omega_2 - \frac{1}{\text{Ł}C})(\Omega_1 - \frac{1}{\text{Ł}C})\,[\exp(\Omega_1\, t) - \exp(\Omega_2\, t)] & -(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)(\Omega_1 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) + (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)(\Omega_2 - \frac{1}{\text{Ł}C})\exp(\Omega_2\, t \end{bmatrix}$$

The application of initial conditions: $V_1(0) = V(0) = 0$ and $V_2(0) = \text{Đ}_t[V(0)] = \frac{I}{C}$ gives

$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \frac{1}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)} \times$$

$$\begin{bmatrix} (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)(\Omega_2 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) - (\Omega_1 - \frac{1}{\text{Ł}C})(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)\exp(\Omega_2\, t) & (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)[\exp(\Omega_2\, t) - \exp(\Omega_1\, t)] \\ (\Omega_2 - \frac{1}{\text{Ł}C})(\Omega_1 - \frac{1}{\text{Ł}C})\,[\exp(\Omega_1\, t) - \exp(\Omega_2\, t)] & -(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)(\Omega_1 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) + (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)(\Omega_2 - \frac{1}{\text{Ł}C})\exp(\Omega_2\, t \end{bmatrix} \begin{bmatrix} 0 \\ \frac{I}{C} \end{bmatrix}$$

Or

$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \frac{1}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)} \begin{bmatrix} \frac{I}{C}(\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)[\exp(\Omega_2\, t) - \exp(\Omega_1\, t)] \\ \frac{I}{C}[-(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)(\Omega_1 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) + (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)(\Omega_2 - \frac{1}{\text{Ł}C})\exp(\Omega_2\, t)] \end{bmatrix}$$

This results

$$V_1(t) = \frac{\frac{I}{C}(\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)[\exp(\Omega_2\, t) - \exp(\Omega_1\, t)]}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)}$$

Or

$$V(t) = \frac{\frac{I}{C}(\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)[\exp(\Omega_2\, t) - \exp(\Omega_1\, t)]}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)}$$

Or

$$V(t) = \frac{\frac{I}{C}(\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)\,[\exp(\Omega_1\, t) - \exp(\Omega_2\, t)]}{(\Omega_1 - \Omega_2)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)} \ldots\ldots\ldots\ldots (14)$$

And

$$V_2(t) = \frac{\frac{I}{C}[-(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)(\Omega_1 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) + (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)(\Omega_2 - \frac{1}{\text{Ł}C})\exp(\Omega_2\, t)]}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)}$$

Or

$$\text{Đ}_t[V(t)] = \frac{\frac{I}{C}[-(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_1 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) + (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 - \frac{1}{\text{Ł}C}) \exp(\Omega_2\, t)]}{(\Omega_2 - \Omega_1)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)}$$

Or

$$\text{Đ}_t[V(t)] = \frac{\frac{I}{C}[(\Omega_2 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_1 - \frac{1}{\text{Ł}C}) \exp(\Omega_1\, t) - (\Omega_1 + \frac{1}{C\mathcal{R}} + 1)\,(\Omega_2 - \frac{1}{\text{Ł}C}) \exp(\Omega_2\, t)]}{(\Omega_1 - \Omega_2)\left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right)} \ldots\ldots (15)$$

Using equations (7) and (8), we can find

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$$\left(\Omega_1 + \frac{1}{C\mathcal{R}} + 1\right)\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) = \left(-\frac{1}{2C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}} + \frac{1}{C\mathcal{R}} + 1\right)\left(-\frac{1}{2C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}} + \frac{1}{C\mathcal{R}} + 1\right)$$

Or

$$\left(\Omega_1 + \frac{1}{C\mathcal{R}} + 1\right)\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) = \left(1 + \frac{1}{C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)\left(1 + \frac{1}{C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)$$

Or

$$\left(\Omega_1 + \frac{1}{C\mathcal{R}} + 1\right)\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) = \left(1 + \frac{1}{C\mathcal{R}}\right)^2 - \left(\frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)^2$$

On simplification of the right-hand side of this equation, we obtain

$$\left(\Omega_1 + \frac{1}{C\mathcal{R}} + 1\right)\left(\Omega_2 + \frac{1}{C\mathcal{R}} + 1\right) = \left(\frac{1}{\text{Ł}C} + \frac{1}{C\mathcal{R}} + 1\right) \ldots\ldots\ldots..(16)$$

And

$$(\Omega_1 - \Omega_2) = -\frac{1}{2C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}} - \left(-\frac{1}{2C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)$$

On simplification of the right-hand side of this equation, we obtain

$$(\Omega_1 - \Omega_2) = \frac{1}{C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}} \ldots\ldots\ldots. (17)$$

Using equations (16) and (17) in equation (14), we get

$$V(t) = \frac{\frac{I}{C}[\exp(\Omega_1 t) - \exp(\Omega_2\, t)]}{\frac{1}{C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}}$$

Or $\quad V(t) = \dfrac{I[\exp(\Omega_1 t) - \exp(\Omega_2\, t)]}{\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}} \ldots\ldots\ldots (18)$

Substituting equations (7) and (8) in equation (18), we get

$$V(t) = \frac{I\left\{\exp\left[\left(-\frac{1}{2C\mathcal{R}} + \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)t\right] - \exp\left[\left(-\frac{1}{2C\mathcal{R}} - \frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)t\right]\right\}}{\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}}$$

Or

$$V(t) = \frac{I\exp(-\frac{1}{2C\mathcal{R}}t)\left\{\exp\left[\left(\frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)t\right] - \exp\left[\left(-\frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)t\right]\right\}}{\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}} \ldots\ldots\ldots\ldots. (19)$$

This equation (19) provides an expression for the voltage across a parallel $\text{Ł} - C - \mathcal{R}$ network connected to an excitation source providing a constant current and confirms that the presence of an inductor and a capacitor in the parallel $\text{Ł} - C - \mathcal{R}$ network leads to variation in the voltage across the network even if the excitation source connected to the network provides a constant current, and that voltage across the parallel $\text{Ł} - C - \mathcal{R}$ network depends on the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}$, whether it is real, zero or imaginary. The value of quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}$, in turn, depends on the values of $\frac{1}{\mathcal{R}^2}$ and $\frac{4C}{\text{Ł}}$. We have the following three possibilities:

Possibility I: If the values of network elements Ł, C and $\mathcal{R}$ are so chosen that $\frac{1}{\mathcal{R}} > \left(\frac{4C}{\text{Ł}}\right)^{\frac{1}{2}}$, then the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}$ is real. In such a case, equation (19) can be rewritten as

$$V(t) = \frac{2I\exp(-\frac{1}{2C\mathcal{R}}t)\sinh\left(\frac{1}{2C}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}\right)t}{\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4C}{\text{Ł}}}} \ldots\ldots\ldots\ldots. (20)$$

It is confirmed from the equation (21) that the voltage across the parallel $\text{Ł} - C - \mathcal{R}$ network is non – oscillatory since sinh is non-periodic function, and it decays gradually to zero.

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Possibility II: If the values of network elements $\L, \mathrm{C}$ and $\mathcal{R}$ are so chosen that $\frac{1}{\mathcal{R}} = \left(\frac{4\mathrm{C}}{\L}\right)^{\frac{1}{2}}$, then the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}$ is zero. In such a case, equation (19) reveals that the current in the series $\L - \mathrm{C} - \mathcal{R}$ network is indeterminate, which is impossible. If the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}$ is so small that it approaches to zero, then on expanding the exponential terms containing the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}$ and taking only the first two terms, we can rewrite equation (19) as

$$\mathrm{V(t)} = \frac{\mathrm{I}\exp\left(-\frac{1}{2\mathrm{C}\mathcal{R}}\mathrm{t}\right)\left\{1 + \left(\frac{1}{2\mathrm{C}}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}\right)\mathrm{t} - [1 - \left(\frac{1}{2\mathrm{C}}\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}\right)\mathrm{t}]\right\}}{\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}}$$

Or

$$\mathrm{V(t)} = \frac{\mathrm{I}}{\mathrm{C}}\mathrm{t}\exp\left(-\frac{1}{2\mathrm{C}\mathcal{R}}\mathrm{t}\right) \text{……………………..} (21)$$

It is confirmed from the equation (21) that the voltage across the parallel $\L - \mathrm{C} - \mathcal{R}$ network is non – oscillatory and it decays to zero in the minimum time.

Possibility III: If the values of network elements $\L, \mathrm{C}$ and $\mathcal{R}$ are so chosen that $\frac{1}{\mathcal{R}} < \left(\frac{4\mathrm{C}}{\L}\right)^{\frac{1}{2}}$, then the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}$ is imaginary. We can write the quantity $\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}}$ as

$$\sqrt{\frac{1}{\mathcal{R}^2} - \frac{4\mathrm{C}}{\L}} = \mathrm{i}\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}} \text{…………..} (22)$$

Using equation (22), we can rewrite equation (19) as

$$\mathrm{V(t)} = \frac{\mathrm{I}\exp\left(-\frac{1}{2\mathrm{C}\mathcal{R}}\mathrm{t}\right)\left\{\exp\left[\left(\frac{1}{2\mathrm{C}}\mathrm{i}\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}\right)\mathrm{t}\right] - \exp\left[\left(-\frac{1}{2\mathrm{C}}\mathrm{i}\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}\right)\mathrm{t}\right]\right\}}{\mathrm{i}\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}}$$

Or

$$\mathrm{V(t)} = \frac{2\mathrm{I}\exp\left(-\frac{1}{2\mathrm{C}\mathcal{R}}\mathrm{t}\right)\sin\left[\left(\frac{1}{2\mathrm{C}}\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}\right)\mathrm{t}\right]}{\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}} \text{…………..} (23)$$

It is confirmed from the equation (23) that the voltage across the parallel $\L - \mathrm{C} - \mathcal{R}$ network is oscillatory and its amplitude $\frac{2\mathrm{I}\exp\left(-\frac{1}{2\mathrm{C}\mathcal{R}}\mathrm{t}\right)}{\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}}$ is decreasing exponentially with time, and its oscillating frequency is $\frac{1}{4\pi\mathrm{C}}\sqrt{\frac{4\mathrm{C}}{\L} - \frac{1}{\mathcal{R}^2}}$.

IV. Conclusions

In this paper, we have obtained the response of a parallel $\L - \mathrm{C} - \mathcal{R}$ network connected to an excitation source providing a constant current by matrix method. The discussion concludes that the nature of response (i.e. voltage) can be oscillatory or non-oscillatory depending on the values of elements $\L, \mathrm{C}$ and $\mathcal{R}$ of the network. The nature of voltage can be made oscillatory if values of network elements $\L, \mathrm{C}$ and $\mathcal{R}$ are so chosen that $\frac{1}{\mathcal{R}} < \left(\frac{4\mathrm{C}}{\L}\right)^{\frac{1}{2}}$, and in this situation, the amplitude decreases exponentially with time and the frequency of oscillation is independent of excitation source providing a constant current. The nature of voltage is non-oscillatory if values of network elements $\L, \mathrm{C}$ and $\mathcal{R}$ are so chosen that $\frac{1}{\mathcal{R}} > \left(\frac{4\mathrm{C}}{\L}\right)^{\frac{1}{2}}$ or $\frac{1}{\mathcal{R}}$ is slightly greater than $\left(\frac{4\mathrm{C}}{\L}\right)^{\frac{1}{2}}$ and in these situation, the amplitude decays gradually to zero.

V. References

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