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A Study on Radial graph of Stand Graphs

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Abstract: A graph G for which two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G. In this paper we have determined the radial graph of stand graphs. The geodetic polynomials and detour geodetic polynomials of stand graphs are derived and some important results are proved.

Keywords: Distance, Detour geodetic polynomial, Geodetic polynomial, Radial graph , Stand graph, X -tree.

AMS Classification: 05C12, 05C60 , 05C75

I. INTRODUCTION

In this paper we discuss only finite simple and connected graph. For basic graph theoretical terminology we refer [1]. In [5] the concept of radial graph R(G) is introduced and the characterization for R(G) is proved . The concept of Geodetic polynomials of a graph using Geodetic sets of a graph are introduced in [8]. Geodetic polynomial, detour geodetic polynomial of some graphs are discussed in [7] . Here we have derived some results on radial graph of stand graphs and geodetic polynomial, detour geodetic polynomial of stand graphs .

1.1. Preliminaries

For a graph G, the distance d(u ,v) between a pair of vertices u and v is the length of a shortest path joining them. The eccentricity e(u) of a vertex u is the distance to a vertex farthest from u. The radius r(G) of G is defined as the minimum eccentricity of all the vertices of G and the diameter d(G) of G is defined as the maximum eccentricity of all the vertices of G.

A graph G for which r(G) = d(G) is called a self centred graph. Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G.

II. RADIAL GRAPH OF STAND GRAPHS

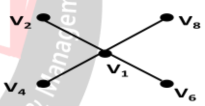
In this section we discuss radial graph of stand graphs and prove some theorems for finding the radial graphs of stand graphs.

A) Definition 2.1

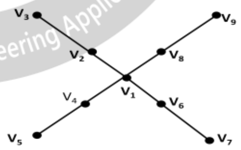
An X- tree is a tree, in which one vertex is of degree 4 and four vertices are pendent vertices, all other vertices if any will be of degree 2. An X –tree with n ≥ 5 vertices is denoted as X_n.

B) Example 2.2

The following are examples for X_5 and X_9



X_5



X_9 :

C) Definition 2.3

Let G be a simple graph with n vertices. In which 4 vertices are pendent vertices and 2 vertices has degree 3, all other vertices has degree 2. Then the graph is called **Stand Graph.** A stand graph with n ≥ 6 vertices is denoted as S_n.

D) Example 2.4

The following are examples for stand graph S_7 and S_8

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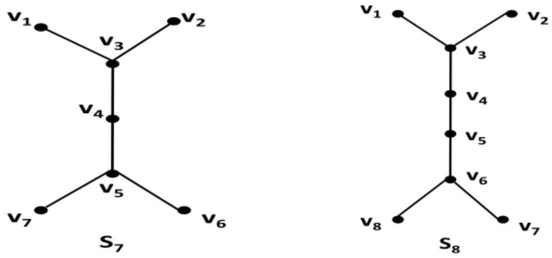


E) Theorem 2.5

Let S_n be a Stand graph with n vertices, then the radial graph of S_n is

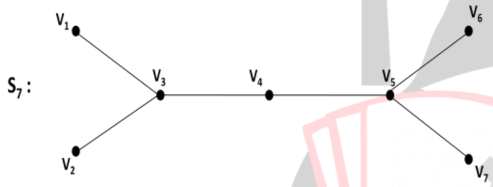
(i) $X_5 \cup \left(\frac{n-5}{2}\right) P_2$ if n is odd, n≥ 7

(ii) $2P_3 \cup \left(\frac{n-6}{2}\right) P_2$ if n is even, n≥ 6

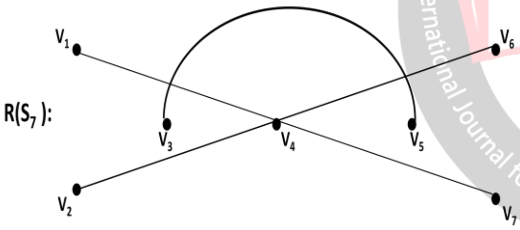
Proof:

Let us prove the theorem by induction on the number of vertices of S_n

Case (i) If n is odd, n ≥ 7. Let n = 7 then S is a stand graph with 7 vertices and it will be of the form,



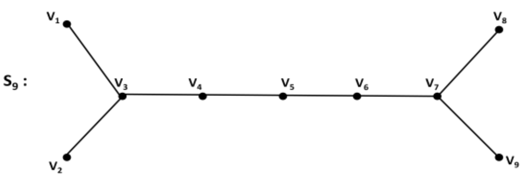
The Radial graph of Stand graph S_7 is



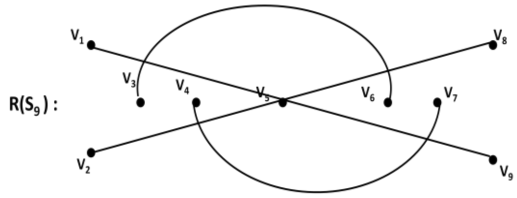
$X_5 \cup P_2$

The vertex v_4 has degree 4 and the vertices v_1,v_2,v_6,v_7 are of degree 1. The path P_2 is existing in the radial graph. Hence the radial graph of S_7 is $X_5 \cup P_2$.

If n = 9 then S is a stand graph with 9 vertices and it is,



The Radial graph of the above stand graph S_9 is



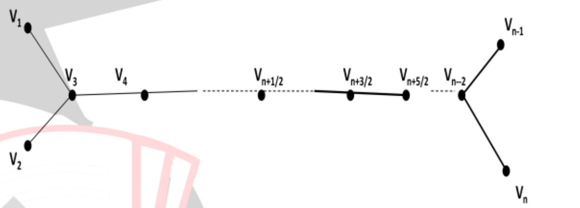
Here the vertex v_5 has degree 4, the vertices v_1,v_2,v_8,v_9 are of degree 1 and two paths are existing. Hence the radial graph of the stand graph S_9 is $X_5 \cup 2P_2$.

The theorem is true for n = 7 and n = 9.

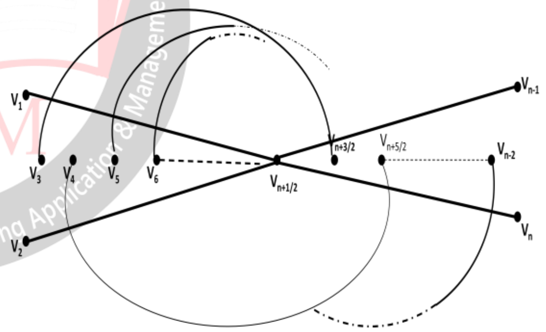
Let us assume that the theorem is true for all stand graph with n-1 vertices and n is odd. i.e, The radial graph of the stand graph S_{n-1} is $X_5 \cup \left(\frac{n-5}{2}\right) P_2$

Now we prove the theorem for stand graph with n vertices

Le S_n is the stand graph with n vertices. n is odd



The radial graph of the above stand graph is

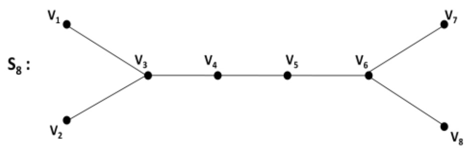


The vertices v_1,v_2,v_{n-1} and v_n has degree 1. The vertices $v_{n+1/2}$ has degree 4. All the other vertices makes a path with distance 1. Here we observe that the $\left(\frac{n-5}{2}\right) P_2$ paths exist. Hence the radial graph of the stand graph S_n is $X_5 \cup \left(\frac{n-5}{2}\right) P_2$.

Case (ii)

Assume that n is even and n = 8.

Let S_8 is a stand graph with 8 vertices and it is



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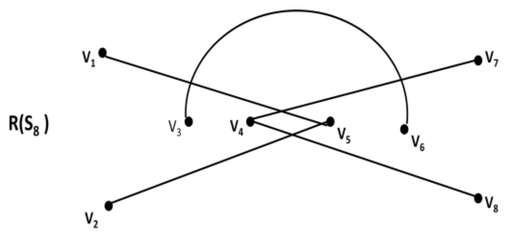
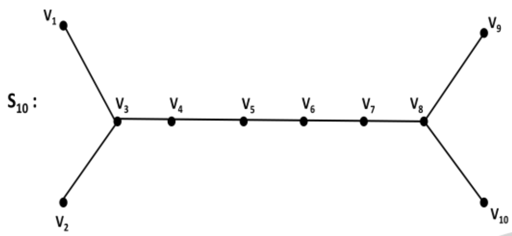
The Radial graph of stand graph S_8 is

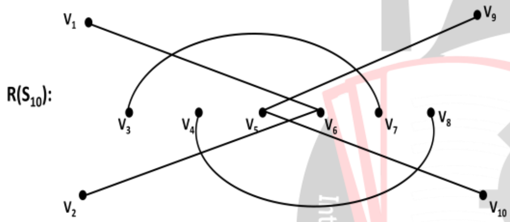


The radial graph of the stand graph contains 4 vertices of degree 1 and 2 vertices of degree 2 and one path. The radial graph of the stand graph S_8 is 2 $P_3 \cup P_2$.

If n = 10. Let S_{10} is a stand graph with 10 vertices.



The radial graph of strand graph S_{10} is



Radial graph of stand graph S_{10} contains 8 vertices has degree 1. 2 vertices has degree 2. Hence the radial graph of stand graph S_{10} is 2 $P_3 \cup$ 2 P_2.

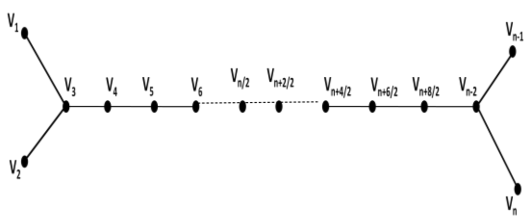
This theorem is true for n = 8, n = 10.

Let us assume that the theorem is true for all the stand graph with n-1 vertices and n is even.

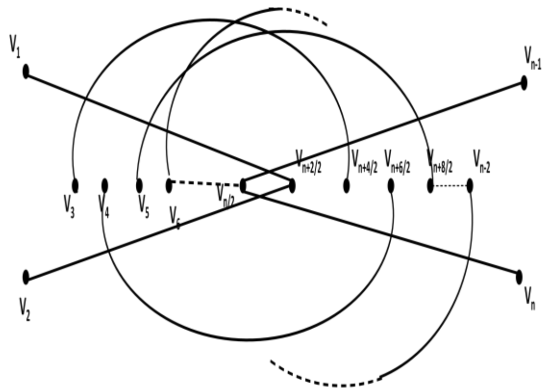
(i.e) the radial graph of the stand graph S_{n-1} is $2P_3 \cup \left(\frac{n-6}{2}\right) P_2$

Now we prove the theorem for stand graph with n vertices

Let S_n is the stand graph with n vertices. n is even



The radial graph of the stand graph S_n is



the vertices v_1, v_2, v_{n-1}, v_n has degree 1, $v_{n/2}, v_{n+2/2}$ is of degree 2. all the other vertices has degree 1. i.e. two path of length 2 is exising in the graph. All other pair of vertices makes a path of length 1.

Hence the radial of the stand graph with n vertices is 2 $P_3 \cup \left(\frac{n-6}{2}\right) P_2$

Hence the proof.

III. Geodetic Polynomial of Stand Graphs

In this section we discuss geodetic polynomial of stand graph

A) Definiton 3.1

Let $\mathcal{G}$ (G,i) be the family of geodetic sets of the graph G with cardinality i and let $g_e(G,i) = |\,\mathcal{G}(G,i)|$. Then the geodetic polynomial $\mathcal{G}(G,x)$ of G is defined as $\mathcal{G}(G,x) = \sum_{i=g(G)}^{|V(G)|} g_e(G,i)\, x^i$ where $g(G)$ is the geodetic number of G.

B) Theorem 3.2

The geodetic polynomial of S_n, if $n \geq 6$ is $\mathcal{G}(S_n, x) = (x+1)^{n-4} x^4$

Proof:

Let S_n be a stand graph with n vertices without loss of generality we choose $n \geq 6$.

Let X= $\{ u_1, u_2, u_3, \ldots \ldots . u_n \}$. The only one geodetic set with minimum cardinality is 4 in X. Therefore $g_e(S_n, 4) = 1$. The geodetic sets with cardinality 5 are $\{ u_1, u_2, u_3, \ldots \ldots . u_{n-4} \}$.

Therefore $g_e(S_n, 5)$ = (n-4)C_1

$g_e(S_n, 6)$ = (n-4) C_2

....

$\mathcal{G}(S_n, x) = \sum_{i=g(G)}^{|V(G)|} g_e(G,i)\, x^i$

$\mathcal{G}(S_n, x) = x^4$ +(n-4)$C_1 x^5$ + (n-4)C_2 x^6 +.............

+ (n-4)$C_{(n-4)}$ x^n

= x^4 [1+(n-4)C_1 x +(n-4)C_2 x^2 +............

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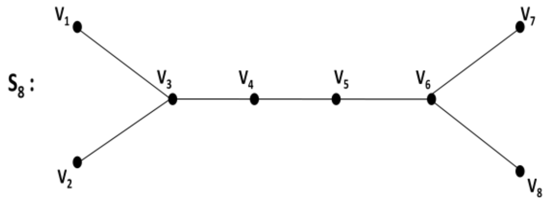
+ (n-4)$C_{(n-4)}$ x^{n-4}]

$\mathcal{G}\ (S_n, x)\ =\ x^4(1+x)^{n-4}$

C) Example 3.3

Let S_8 is a Stand graph with 8 verti



$g_e\ (S_8, 4) =$ {$v_1\ v_7\ v_8\ v_9$}

$|g_e\ (S_8, 4)\ | = 1$

$g_e\ (S_8, 5) =$ {($v_1\ v_2\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_7\ v_8\ v_9$), ($v_1\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_4\ v_7\ v_8\ v_9$), ($v_1\ v_6\ v_7\ v_8\ v_9$)}

$g_e\ (S_8, 6) =$ { ($v_1\ v_2\ v_3\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_4\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_4\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_4\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_4\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_5\ v_6\ v_7\ v_8\ v_9$) }

$|g_e\ (S_8, 6)\ | = 10$

$g_e\ (S_8, 7) =$ {($v_1\ v_2\ v_3\ v_4\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_3\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_3\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_4\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_4\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_5\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_4\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_4\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_5\ v_6\ v_7\ v_8\ v_9$) }

$|g_e\ (S_8, 7)\ | = 10$

$g_e\ (S_8, 8) =$ { ($v_1\ v_2\ v_3\ v_4\ v_5\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_3\ v_4\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_3\ v_5\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_2\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9$), ($v_1\ v_3\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9$) }

$|g_e\ (S_8, 8)\ | = 5$

$g_e\ (S_8, 9) =$ { ($v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9$) }

$|g_e\ (S_8, 9)\ | = 1$

$\mathcal{G}\ (G, x)\ =\ \sum_{i=\ g(G)}^{|V(G)|}\ g_e\ (G, i)\ x^i$

$\mathcal{G}\ (S_8, x)\ =\ \sum_4^9\ g_e\ (S_8, i)\ x^i$

$\mathcal{G}\ (S_8, x\ ) = x^4\ + 5x^5 + 10x^6 + 10x^7 + 5x^8 + x^9$

The Geodetic polynomial of Stand graph S_8 is

$\mathcal{G}\ (S_8, x\ ) =\ x^4 + 5x^5 + 10x^6 + 10x^7 + 5x^8 + x^9$

D) Theorem 3.3

Geodetic polynomial of radial graph of stand graph S_n is

(i) $\mathcal{G}(R(S_n), x)\ =\ x^n$ if n is odd, $n \geq 5$

(ii) $\mathcal{G}\ (R(S_n), x)\ =\ x^{n-2}\ +\ 2x^{n-1}\ +\ x^n$ if n is even $n\ \geq\ 6$.

Proof: (i)

Since $R(S_n)\ =\ X_5 \cup \binom{n-5}{2}\ P_2$ if n is odd , $n \geq 5$.

An X_5 is a tree with 4 end vertices , the only geodetic set with minimum cardinality is 5.

Hence $\mathcal{G}\ (X_5\,, x)\ =\ x^5$. P_2 is a tree with 2 vertices. The only geodetic set with minimum cardinality is 2. Hence $\mathcal{G}\ (P_2, x)\ =\ x^2$.

Therefore the geodetic polynomial of R (S_n) is $\mathcal{G}\ (R(S_n), x)\ =$

$\mathcal{G}(X_5\,, x). \{\ \mathcal{G}\ (P_2, x)\ .\mathcal{G}\ (P_2, x)\ \ldots\ldots\ldots \binom{n-5}{2}$ times }

$= x^5\ .\ \{\ x^2\ .x^2\ \ldots\ldots\ldots \binom{n-5}{2}$ times }

$=\ x^5 . (x^2)^{\binom{n-5}{2}}$

$=\ x^5\ .x^{n-5}$

$|g_e\ (S_8, 5)\ |\ = 5$

$\mathcal{G}\ (R(S_n),\ x)\ =\ x^n$ if n is odd , $n\ \geq\ 5$.

Proof: (ii)

Since $R(S_n) =\ 2\ P_3\ \cup\ \binom{n-6}{2} P_2$ if n is even ,

$n\ \geq\ 6$.

The geodetic polynomial of $R(S_n)$ is

$\mathcal{G}\ (R(S_n), x)\ =\ \ \mathcal{G}\ (2\ P_3\ ).\ \mathcal{G}\ \binom{n-6}{2} P_2$

$=\ \mathcal{G}\ (\ P_3\ ) . \mathcal{G}\ (\ P_3\ ). \mathcal{G}\ \binom{n-6}{2} P_2$

$= \{\ (x^2 + x^3). (x^2 + x^3). \{\ x^2 . x^2\ \ldots\ldots . \binom{n-6}{2} \}$

$=\ (x^2 + x^3). (x^2 + x^3). (x^2)^{\binom{n-6}{2}}$

$=\ (x^4 + 2x^5 + x^6). x^{n-6}$

$\mathcal{G}\ (R(S_n), x)\ =\ x^{n-2}\ +\ 2x^{n-1}\ +\ x^n$ if n is even , $n \geq 6$.

IV. Detour Geodetic Polynomial of Stand Graphs

In this section we discuss detour geodetic polynomial of stand graph

A) Defintion 4.1

Let D $\mathcal{G}$ (G,i) be the family of detour Geodetic sets of the graph G with cardinality i and let $D g_e\ (G, i) = |D\mathcal{G}\ (G, i)|$. Then the Detour geodetic polynomial $D\mathcal{G}\ (G, x)$ of G is defined as $D\mathcal{G}\ (G, x) = \sum_{i=\ dg(G)}^{dg+(G)}\ Dg_e\ (G, i)\ x^i$ Where $d_g(G)$ is the Detour number of G.

B) Theorem 4.2

The detour geodetic polynomial of the stand graph S_n is $3x^2$+$3x^3$

i.e D $\mathcal{G}$ (S_n , x) = $3x^2 + 3x^3$, n $\geq$ 6

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Proof:

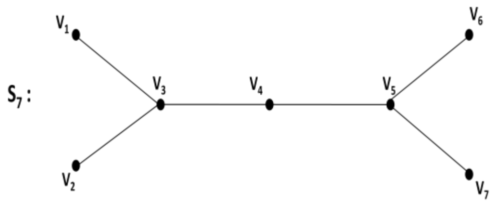
In the Stand graph S_n, n $\geq$ 6, 4 vertices has degree 1 and 2 vertices has degree 3 and all other vertices has degree 2. Hence

$$d_g(G) = 2 \;,\; dg^+(G) = 3 \,.$$

Hence, Three detour geodetic set has cardinality 2 and three detour geodetic set has cardinality 3. Hence the detour geodetic polynomial of the stand graph is $3x^2 + 3x^3$.

Hence the proof.

C) Example 4.3



DS_1= { v_1 v_5}, DS_2 = { v_1 v_6}, DS_3 = { v_1 v_7} , DS_4 = { v_1 v_4 v_5} ,

DS_5 = { v_1 v_4 v_6} , DS_6= { v_1 v_2 v_7}

$d_g(S_7) = 2$,$dg^+(S_7) = 3$

Detour geodetic polynomial of Stand Graph S_7 is

$$D\,\mathcal{G}\,(S_7\,,x) = \sum_{i=2}^{3} D g_e\,(S_7, i)\, x^i$$

The detour geodetic polynomial of stand graph S_7 is

$$D\,\mathcal{G}\,(S_7\,,x) = 3x^2 + 3x^3.$$

D) Theorem 4.4

Detour geodetic polynomial of radial graph of stand graph S_n is

$$D\,\mathcal{G}\,(R(S_n\,), x) = x^n \text{ if n is odd , n} \geq 5\,.$$

Proof:

Since $R(S_n) = X_5 \cup \binom{n-5}{2} P_2$ if n is odd , n $\geq$ 5. An X_5 is a tree with 5 vertices, $d_g(X_5) = 5$, $dg^+(X_5) = 5$. There is only one detour set with cardinality 5. Hence D $(X_5\,) = x^5$. P_2 is a tree with two end vertices, $d_g(P_2) = 2$,$dg^+(P_2)$ = 2. There is only one detour set with cardinality 2. Hence $D\,\mathcal{G}\,(P_2\,) = x^2$.

$$D\,\mathcal{G}\,(R(S_n\,), x) = D\,\mathcal{G}\,(\,X_5 \cup \binom{n-5}{2}\, P_2\,)$$

$$= D\,\mathcal{G}\,(X_5\,).\{\,D\,\mathcal{G}\,(P_2). D\mathcal{G}\,(P_2). D\mathcal{G}\,(P_2) \dots\dots$$

$$\dots . \binom{n-5}{2} \text{ times } \}$$

$$= x^5 .(\,x^2 . x^2 \dots\dots\dots \binom{n-5}{2} \text{ times })$$

$$= x^5 . (x^2)^{\binom{n-5}{2}}$$

$$= x^5 . x^{n-5} = x^n$$

$$D\,\mathcal{G}\,(R(S_n\,), x\,) = x^n.$$

E) Theorem 4.5

Detour geodetic polynomial of radial graph of stand graph S_n is

$$D\,\mathcal{G}\,(R(S_n\,), x) = x^n \text{ if n is even ,n} \geq 6.$$

Proof: Since $R(S_n) = 2P_3 \cup \binom{n-6}{2} P_2$ if n is odd , n $\geq$ 5.An P_3 is a tree with 3 vertices, $d_g(P_3) = 3$, $dg^+(P_3) = 3$. There is only one detour set with cardinality 3. Hence D $(P_3\,) = x^3$. P_2 is a tree with two end vertices, $d_g(P_2) = 2$,$dg^+(P_2) = 2$. There is only one detour set with cardinality 2. Hence $D\,\mathcal{G}\,(P_2\,) = x^2$.

$$D\,\mathcal{G}\,(R(S_n\,), x) = D\,\mathcal{G}\,(\,2P_3 \cup \binom{n-6}{2}\, P_2\,)$$

$$= D\mathcal{G}(P_3\,).D\;\mathcal{G}(P_3\,).\{\,D\,\mathcal{G}(P_2\,).D\,\mathcal{G}(P_2\,) \dots\dots \binom{n-6}{2} \text{ times } \}$$

$$= (\,x^3\,).(\,x^3\,).\{\,x^2 . x^2 . x^2 \dots\dots\dots\dots \binom{n-6}{2} \text{ times } \}$$

$$= (\,x^3\,).(\,x^3\,).\,(x^2)^{\binom{n-6}{2}}$$

$$= x^6 . x^{n-6} = x^n$$

$$D\,\mathcal{G}\,(\,R(S_n\,), x\,) = x^n \text{ if n is even , n} \geq 6.$$

V. CONCLUSION

This paper is concerned about radial graph, geodetic polynomial and detour geodetic polynomial of stand graphs. The detour geodetic polynomial of radial graph of stand graphs is also determined. Further study can be made in finding the radial graph, geodetic polynomial and detour geodetic polynomial of other types of graphs such as ladder graph ,helm graph etc.

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