Modified lumped model for Transient heat Conduction in a Hollow sphere

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Abstract - **By using polynomial approximation method a new modified lumped model is developed for a hollow sphere. The boundary condition for the inner side and outer side of the hollow sphere are fixed heat flux and convective heat transfer with the ambient respectively. The non-dimensional equation is solved with the help of second degree polynomial approximation for the non-dimensional temperature. With the application of boundary conditions the constants of the polynomial is determined and the results are plotted in the form of graphs. It is observed that with increase of time the non-dimensional temperature decreases which signify the increase of heat transfer. A new modified Biot number is also developed.**

Keywords **- Biot number, Hollow sphere, Modified lumped model, Polynomial approximation method.**

I. INTRODUCTION

Classical lumped model works well only for lower values of Biot numbers; so there is a need of modifying the existing lumped model. In order to carry out lumped model analysis the Biot number should be less than 0.1 i.e. when the conductive resistance is less than that of the convective. In many engineering application as the analysis of thermo hydraulic nuclear reactor, boiling water reactor involve higher Biot number so classical lumped model is not valid[10]. So the efforts have been done by several authors to improve the lumped model till now by using different analytical methods as perturbation method[3,4], two point Hermite method[1,2,5], polynomial approximation method[6,10].

Clarissa R. Regis *et al.*[1] studied the transient heat conduction in a nuclear fuel rod by improved lumped parameter approach. They used Hermite approximation method for determining the average temperature and heat flux in radial direction.

Jian Su[2] developed an improved lumped model for unsteady cooling of a long slab by two point Hermite integral method which worked well with higher values of Biot numbers.

H. Sadat[3] developed a lumped model of unsteady one-dimensional heat conduction problem by using perturbation method. The center, surface and average temperature for different geometries was obtained for different Biot numbers.

H. Sadat[4] developed a second order model for transient heat conduction in a slab with convective boundary conditions by using perturbation method.

Shijun Liao *et al.*[5] solved a nonlinear model of combined convective and radiative cooling of a spherical body using homotopy analysis method. This series solution agreed well with the numerical solution. It was found that for a nonlinear model of combined convective and radiative cooling of spherical body, the surface temperature of a body decayed more quickly for larger values of Biot number and/or the radiation-conduction parameter.

P Keshavraj and M Taheri[6] developed an improved lumped model by using Polynomial Approximation method. It was found that the improved model was able to calculate average temperature for more higher Biot numbers as compared to that to that of finite difference method.

Devanshu Prasad[7] developed improved lumped model by employing polynomial approximation method with a number of approximate temperature profiles on slab and cylinder under different conditions viz boundary heat flux and heat generation. On the basis of the analysis a modified Biot number was obtained.

Noorul Haque and Amitesh Paul[8] studied the improved lumped parameter in transient heat conduction.

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Zheng Tan *et al.*[9] developed an improved model for combined convective and radiative cooling wall. Two point Hermite approximations for integrals is used to obtain the improved lumped model.

Amit Prakash and Shahid Mahmood[10] developed a modified lumped model for transient heat conduction in a spherical shape with heat generation and with natural convection cooling, for a particular temperature profile using polynomial approximation method. A modified Biot number was obtained.

Jian su *et al.*[11] developed improved lumped models for transient combined convective and radiative cooling of multilayer spherical media. Two point Hermite approximation methods is used to obtain the average temperature and heat flux in each layer. The plain trapezoidal rule was employed in all layers, except for the innermost layer where the second-order two sided corrected trapezoidal rule is used to obtain average temperatures.

II. Contribution

In this paper, a new modified lumped model is developed for a hollow sphere by using polynomial approximation method. The inner side of which is supplied with a specified heat flux and the outer side is exposed to a convective heat transfer. It is found that the newly developed modified lumped model works well for higher values of Biot numbers.

III. Mathematical Formulation

Considering a hollow sphere of inner and outer radii $r_1$and r_2 respectively, former being subjected to heat flux and latter to convective heat transfer as shown in figure 1.

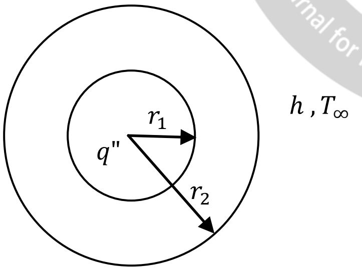

Figure 1: A hollow sphere with inner and outer radius $r_1$and r_2 respectively.

Where $q"$ is the heat flux supplied at the inner side, h is the heat transfer coefficient and T_∞ is ambient temperature at the outside.

A. Analysis of temperature variation with time for a hollow sphere

The governing one dimensional heat conduction equation for a solid sphere of radius r without internal heat generation with temperature $T(x,t)$ and thermal diffusivity α is given by:

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{1}$$

The required boundary conditions for the hollow sphere are:

$$-k\frac{\partial T}{\partial r} = q" \quad at\ r = r_1 \tag{2}$$

$$-k\frac{\partial T}{\partial r} = h(T - T_\infty)\ at\ r = r_2 \tag{3}$$

Dimensionless parameters are:

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, B = \frac{hR}{k}, \tau = \frac{\alpha t}{R^2}, x_i = \frac{r_i}{r_2}, Q = \frac{q"r_2}{k(T_0 - T_\infty)}$$

Where θ is the dimensionless temperature, B the Biot number, τ the dimensionless time, x_i the dimensionless length, Q the dimensionless heat parameter and i indicates for 1 and 2.

The dimensionless governing equation and boundary conditions are:

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial \theta}{\partial x}\right) \tag{5}$$

$$\frac{\partial \theta}{\partial x} = -Q\ at\ x = x_1 = \varepsilon \tag{6}$$

Where $\varepsilon = \frac{r_1}{r_2}$

$$\frac{\partial \theta}{\partial x} = -B\theta\ at\ x = x_2 = 1 \tag{7}$$

For the solution of Eq.(5) by polynomial approximation method, let the assumed temperature profile be:

$$\theta = a_0 + a_1x + a_2x^2 \tag{8}$$

On differentiating the above equation

$$\frac{\partial \theta}{\partial x} = a_1 + 2a_2x \tag{9}$$

On applying first and second boundary conditions, values of a_1 and a_2 are obtained as:

$$a_1 = \frac{-B\theta\varepsilon + Q}{\varepsilon - 1} \tag{10}$$

$$a_2 = \frac{B\theta - Q}{2(\varepsilon - 1)} \tag{11}$$

Utilizing the equation (8), (9) ,(10) and (11) in equation (7), it can be written as:

$$a_0 = \theta\left(1 + \frac{B(2\varepsilon - 1)}{2(\varepsilon - 1)}\right) - \frac{Q}{2(\varepsilon - 1)} \tag{12}$$

The average temperature $(\bar{\theta})$ of the sphere is given by

$$\bar{\theta} = \frac{3}{x_2^3 - x_1^3}\int_{x_1}^{x_2} x^2\,\theta dx \tag{13}$$

$$\bar{\theta} = \frac{3}{1 - \varepsilon^3}\int_{\varepsilon}^{1} x^2(a_0 + a_1x + a_2x^2) \tag{14}$$

$$\bar{\theta} = \theta[1 + B(L_1 - L_4 + L_5) - Q(L_2 - L_3 + L_5)] \tag{15}$$

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Where $L_1 = \frac{2\varepsilon-1}{2(\varepsilon-1)}$, $L_2 = \frac{1}{2(\varepsilon-1)}$, $L_3 = \frac{3(1-\varepsilon^4)}{4(\varepsilon-1)(1-\varepsilon^3)}$, $L_4 = \frac{3\varepsilon(1-\varepsilon^4)}{4(\varepsilon-1)(1-\varepsilon^3)}$, $L_5 = \frac{3(1-\varepsilon^5)}{10(\varepsilon-1)(1-\varepsilon^3)}$

On differentiating Eq. (15),

$$\frac{\partial\bar{\bar{\theta}}}{\partial\tau} = \frac{\partial}{\partial\tau}(\theta[1 + B(L_1 - L_4 + L_5) - Q(L_2 - L_3 + L_5)]) \quad (16)$$

Integrating equation (5) with respect to x

$$\int_\varepsilon^1 x^2\frac{\partial\theta}{\partial\tau}dx = \int_\varepsilon^1 \frac{\partial}{\partial x}\left(x^2\frac{\partial\theta}{\partial x}\right)dx \quad (17)$$

$$\int_\varepsilon^1 x^2\frac{\partial\theta}{\partial\tau}dx = \frac{(Q - B\theta\varepsilon)(1-\varepsilon^2)}{\varepsilon - 1} + \frac{(B\theta - Q)(1-\varepsilon^3)}{\varepsilon - 1}$$

On solving further, it is obtained as

$$\frac{\partial\bar{\theta}}{\partial\tau} = -B\theta(-M_1 - M_2) + Q(-M_3 - M_4) \quad (18)$$

Where $M_1 = \frac{3\varepsilon(1+\varepsilon)}{1-\varepsilon^3}$, $M_2 = \frac{3}{\varepsilon-1}$, $M_3 = \frac{3(1+\varepsilon)}{1-\varepsilon^3}$ and

$$M_4 = \frac{3}{\varepsilon - 1}$$

From Eq. (16), it can be written as

$$\frac{\partial\theta}{\partial\tau} = -\frac{B\theta(-M_1 - M_2)}{1 + B(L_1 - L_4 - L_5)} + \frac{Q(-M_3 - M_4)}{1 + B(L_1 - L_4 - L_5)} \quad (19)$$

Integrating the above equation, it is obtained:

$$\theta = \frac{e^{-X\tau} + Y}{X} \quad (20)$$

$$\text{Where } X = \frac{B(-M_1 - M_2)}{1 + B(L_1 - L_4 - L_5)},$$

$$Y = \frac{Q(-M_3 - M_4)}{1 + B(L_1 - L_4 - L_5)}$$

Equation (20) represents the non-dimensional temperature distribution along the radius of the hollow sphere with variation of non-dimensional time τ.

IV. RESULTS AND DISCUSSION

The non-dimensional temperature θ is calculated with respect to τ using equation (20) at various values of B at fixed value of Q for internal and external radius 0.5 and 1 respectively, which is shown in figure 2. It is observed that upto a fixed interval of time τ, θ decreases at different fixed Biot numbers. The interval of non-dimensional time increases with increase of Biot number.

Figure 3 shows the variation of temperature drop with effect of Biot number. From this figure, at a particular instant of time, the temperature drop increases with increase of Biot number forconstant Q (here Q=1).

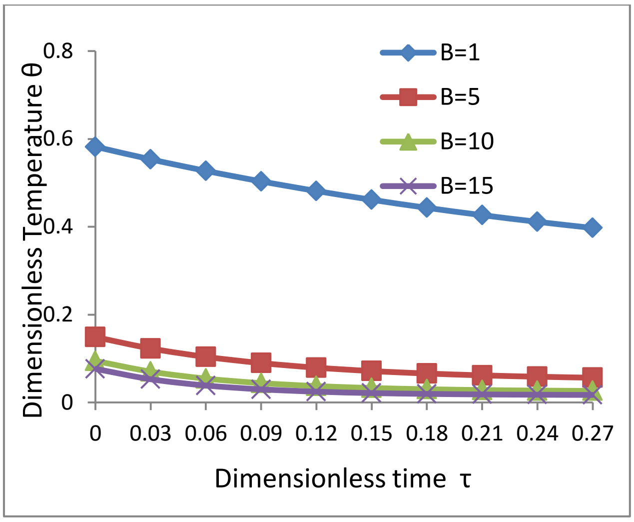

Figure 2: Dimensionless temperature Vs Dimensionless time for a hollow sphere at different Biot numbers and constant heat flux ie Q=1.

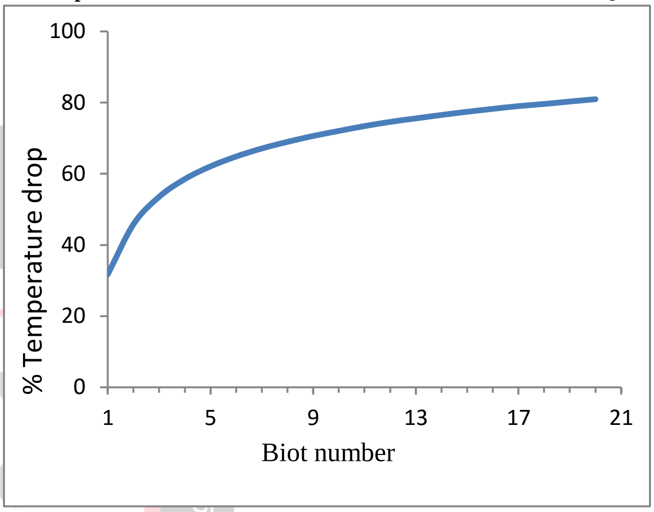

Figure 3: Percent temperature drop Vs Biot number

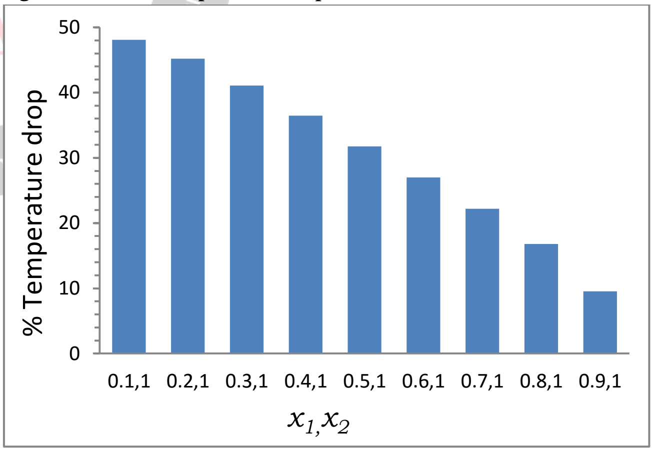

Figure 4: Percent temperature drop Vs thickness of hollow sphere.

It can be seen from figure 4 that on reducing the thickness of the sphere, the percentage of temperature drop also decreases for a particular value of dimensionless heat source parameter and Biot number (Here B=1 and Q=1).

Figure 5 shows the variation of τ and θ by varying heat source parameter Q. It is found that for higher values of Q, there is a very negligible variation (remains almost constant) of non-dimensional temperature with time, whereas for the lower values of Q the nondimensional

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temperature changes gradually with time.

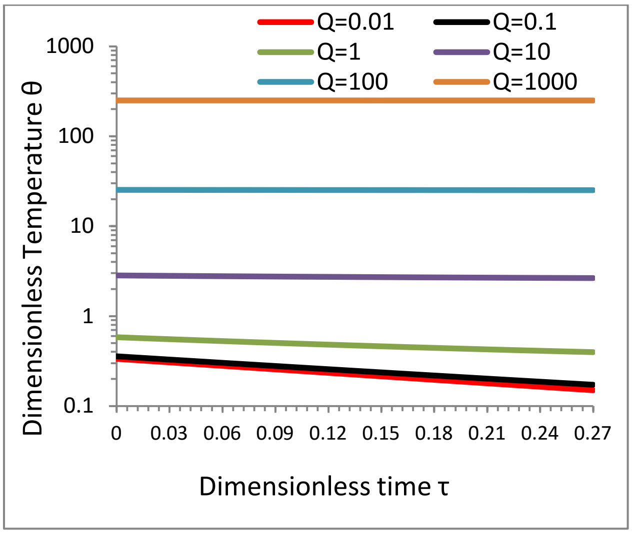

Figure 5: Dimensionless temperature Vs Dimensionless time for a hollow sphere with different values of Q at B=1.

V. Conclusion

A Modified lumped model is developed which works well for higher values of Biot number. A temperature versus time relationship for a hollow sphere with heat flux supplied at the inner side and external side been exposed to a convective heat transfer mode is obtained by polynomial approximation method. It is used to find the temperature distribution with effect of time, Biot number and heat source parameter. The temperature drop decreases with respect to time upto certain region. The percentage of temperature drop decreases for reduction of thickness of the hollow sphere at a fixed B and Q. The present approach is also further utilized to determine the temperature variation with respect to time for a hollow sphere with different values of Q at B=1.

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