Vague infra α −open sets

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Abstract - In this paper we discuss about the fundamental properties of vague infra α −open sets. The relation between vague infra α −open sets and other open sets are studied. Some continuous mappings are defined.

Key words: vague infra open, vague infra α −open, vague infra semi open, vague infra pre open, vague infra semi pre open

I. Introduction

In 1970,Levine[6] started the study of generalised closed sets in topological spaces .In 1965,the concept of fuzzy sets was introduced by Zadeh [8] in 1965.Fuzzy topology was introduced by C. L. Chang[4] in1967 .H.Maki, k.balachandran, R.Devi [7] introduced the concept of alpha generalised closed sets in topological spaces. After that gau and buhrer[5] introduced the concept of vague sets . The basic concepts of vague set theory and its extensions are defined by [3, 5] .Adel .M.AL-Odhari[1] introduced the concept of infra topological spaces.

In this paper we introduce vague infra α −open sets and we discuss the fundamental properties of vague infra α −open sets. The relation between vague infra α −open sets and other open sets are studied. Some continuous mappings are defined.

II. Preliminaries

Definition 2.1:[2] In the universe of discourse U a vague set P is characterized by

i. A true membership function $t_P: U \to [0,1]$
ii. A false membership function $f_P: U \to [0,1]$

where $t_P(u)$ is a lower bound on the grade of membership of u derived from the "evidence for u", $f_P(u)$ is a lower bound on the negation of u derived from the "evidence for u" and $t_P(u) + f_P(u) \le 1$.

Thus the grade of membership u in the vague set A is bounded by subinterval $[t_P(u),\ 1 - f_P(u)]$ of [0, 1]. This indicates that if the actual grade of membership of u is $\mu(u)$, then $t_P(u) \le \mu(u) \le 1 - f_P(u)$. The vague set A is written as A=$\{\langle u, [t_P(u), 1 - f_P(u)]/u \in U\rangle\}$ where the interval $[t_P(u), 1 - f_P(u)]$ is called the vague value of u in P denoted by $V_P(u)$.

Definition 2.2: Let P_1 and P_2 be vague sets of the form P_1 = $\{\langle u, [t_{P_1}(u), 1 - f_{P_1}(u)]/u \in U\rangle\}$ and B = $\{\langle u, [t_{P_2}(u), 1 - f_{P_2}(u)]/u \in U\rangle\}$ then

(i)$P_1 \subseteq P_2$ if and only if $t_{P_1}(u) \le t_{P_2}(u)$ and $1 - f_{P_1}(u) \le 1 - f_{P_2}(u)$ for all u$\in U$.

(ii) P_1=P_2 if and only if $P_1 \subseteq P_2$ and $P_2 \subseteq P_1$.

(iii)${P_1}^c = \{\langle u, [f_{P_1}(u), 1 - t_{P_1}(u)]/u \in U\rangle\}$

(iv)$P_1 \cap P_2 = \{\langle u, min\left(t_{P_1}(u), t_{P_2}(u)\right), min(1 - f_{P_1}(u), 1 - f_{P_2}(u)/u \in U)\rangle\}$

(v)$P_1 \cup P_2$=$\{\langle u, max\left(t_{P_1}(u), t_{P_2}(u)\right), max(1 - f_{P_1}(u), 1 - f_{P_2}(u)/u \in U)\rangle\}$

Definition 2.3: A vague infra topology($VIT\ in\ short$)on X is a family T of vague sets($VS\ in\ short$) in X satisfying the following axioms

(1).0, 1$\in T$

(2).$G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$

In this pair (X, T) is called a vague infra topological space($VITS\ in\ short$) and any vague set in T is known as a vague infra open set($VIOS\ in\ short$) in X. The complement of a VIOS A in a VITS (X, T) is called vague infra closed set ($VICS\ in\ short$)in X.

Example 2.4: Let U={u,v}and T={0, P_1, P_2, P_3, 1}is a vague infra topology on U where

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P_1= $\{\langle u[0.2,0.6], v[0.3,0.5]\rangle\}$, P_2= $\{\langle u[0.4,0.5], v[0.1,0.6]\rangle\}$, P_3= $\{\langle u[0.2,0.5], v[0.1,0.5]\rangle\}$, 0= $\{\langle u[0,0], v[0,0]\rangle\}$, 1=$\{\langle u[1,1], v[1,1]\rangle\}$.

Here the open sets are 0, P_1, P_2, P_3, 1 and corresponding closed sets are 1, $P_1{}^c, P_2{}^c, P_3{}^c$, 0 respectively.

Example 2.5: Let U= {u, v, w} and P_1, P_2, P_3 are vague sets on U as follows:

P_1= $\{\langle u[0.3,0.6], v[0.2,0.5], w[0.4,0.7]\rangle\}$

P_2= $\{\langle u[0.4,0.5], v[0.3,0.7], w[0.2,0.6]\rangle\}$

P_3= $\{\langle u[0.3,0.5], v[0.2,0.5], w[0.2,0.6]\rangle\}$

Then $P_1 \cap P_2 = P_2 \cap P_3 = P_1 \cap P_3 = P_3$. Then T= {0, P_1, P_2, P_3, 1}is a vague infra topology.

Definition 2.6: Let (U,T) be a vague infra topological space and P_1= $\{\langle u, [t_A(u), 1-f_A(u)]/u \in U\rangle$ be a vague set in U. The vague infra closure of U is defined by

VIcl(P_1)=$\cap\{k$ / k is a vague infra closed set in U and $P_1 \subseteq k\}$

Definition 2.7: Let (U,T) be a vague infra topological space and P_1=$\{\langle u, [t_{P_1}(u), 1-f_{P_1}(u)]/u \in U\rangle$ be a vague set in U. The vague infra interior of P_1 is defined by

VIint(P_1)=$\cup\{k$ / k is a vague infra open set in U and $P_1 \supseteq k\}$

III. VAGUE INFRA α –OPEN SET

Definition 3.1: A set $H \subseteq$U is called vague infra α –open(vague infra α –closed) set if H$\subseteq VIint(VIcl(VIint(H)))(VIcl(VIint(VIcl(H)))\subseteq$H). The class of all vague infra α –open (vague infra α –closed) sets in U will be denoted as $VI\alpha O(U)(VI\alpha C(U))$.

Definition 3.2: For any set H, we have,

(i) $VI\alpha Cl(H) = \cap\{k: k \supseteq H, H\ is\ an\ vague\ infra\ \alpha - closed\ set\ of\ U\}$ is called an vague infra α –closure

(ii) $VI\alpha Int(H) = \cup\{k: k \subseteq H, H\ is\ an\ vague\ infra\ \alpha - open\ set\ in\ U\}$ is called an vague infra $\alpha - interior.$

(iii) $VIsCl(H) = \cap\{k: k \supseteq H, H\ is\ an\ vague\ infra\ semi - closed\ set\ of\ U\}$ is called an vague infra semi closure.

(iv) $VIsInt(H) = \cup\{k: k \subseteq H, H\ is\ an\ vague\ infra\ semi\ open\ set\ in\ U\}$ is called an vague infra semiinterior.

(v) $VIgCl(H) = \cap\{k: k \supseteq H, H\ is\ an\ vague\ infra\ g - closed\ set\ of\ U\}$ is called an vague infra g-closure.

(vi) $VIgInt(H) = \cup\{k: k \subseteq H, H\ is\ an\ vague\ infra\ g\ open\ set\ in\ U\}$ is called an vague infra g- interior.

Theorem 3.3: A set K$\in VI\alpha O(U)$ if and only if there exist an open set H such that H$\subseteq K \subseteq VIint(VIgcl(H))$.

Proof: **Necessity**: If K$\in VI\alpha O(U)$ then K$\subseteq VIint(VIcl(VIint(H)))$. Put H=$VIint\ K$, then H is an vague infra open set and H$\subseteq K \subseteq VIint(VIgcl(H))$

Sufficiency: Let H be an vague infra open set such that H$\subseteq k \subseteq VIint(VIgcl(H))$, this implies that $VIint(VIgcl(H)) \subseteq VIint(VIcl(VIint(H))$ then $k \subseteq VIint(VIcl(VIint(H)))$.

Theorem 3.4: A set H$\in VI\alpha C(U)$ if and only if there exist a closed set k such that $VIcl(VIgint(K)) \subseteq H \subseteq K$.

Proof: **Necessity**: If H$\in VI\alpha C(U)$ then $VIcl\left(VIgint(VIcl(H))\right) \subseteq H$, put K=VI cl(H), then K is a closed set $VIcl(VIgint(K)) \subseteq H \subseteq K$.

Sufficiency: Let K be a closed set such that $VIcl(VIgint(K)) \subseteq H \subseteq K$, this implies that $VIcl\left(VIgint(VIcl(H))\right) \subseteq VIcl(VIgint(K))$ then $VIcl\left(VIgint(VIcl(H))\right) \subseteq H$.

Theorem 3.5: Let H be a set of U. Then, the following properties are true:

(a) $VIsint(H) = H \cap VIgcl(VIint(H))$
(b) $VIscl(H) = H \cap VIgint(VIcl(H))$

Proof: (a) We know that $VIsint$ is vague infra semi open,

Then $VIsint(H) \subseteq VIgcl\left(VIint(VIsint(H))\right) \subseteq VIgcl(VIint(H))$

So, $VIsint(H) \subseteq H \cap VIgcl(VIint(H))$

We have $VIint(H) \subseteq H \cap VIgcl(VIint(H)) \subseteq VIgcl(VIint(H))$

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Then $H \cap VIgcl(VIint(H))$ is an vague infra semi open set and $H \cap VIgcl(VIint(H)) \subseteq H$, then $H \cap VIgcl(VIint(H)) \subseteq VIsint(H)$

Then $VIsint(H) = H \cap VIgcl(VIint(H))$

Corollory 3.6:. Let H be a set of U. Then, the following properties are true:

(a) If H is a vague infra generalized closed set, then $VIsint(H) = VIgcl(VIint(H))$

(b)If H is a vague infra generalized open set, then $VIscl(H) = VIgint(VIcl(H))$

Proof: We know that $VIgint(H) \subseteq VIgint(VIcl(H))$ but $VIgint(H) = H$, this implies that $H \subseteq VIgint(VIcl(H))$, then $VIscl(H) = VIgint(VIcl(H))$.

Theorem 3.7: For any subset H of a space U, the following implication hold

(i)$\Longrightarrow$ (ii) $\Longrightarrow (iii) \Longrightarrow (iv)$

(i) H$\in VI\alpha C(U)$

(ii) $VIcl(VIgint(K)) \subseteq H \subseteq K$ for closed set K

(iii) $VIsint(k) \subseteq H \subseteq K$ for closed set K

(iv)$VIsint(VIcl(H)) \subseteq H$

Theorem 3.8: For any subset H of a space U, the following statements are hold:

(i) H$\subseteq K \subseteq VIint(VIgcl(H))$ and H$\in VI\alpha O(U)$ then K$\in VI\alpha O(U)$.

(ii) $VIcl(VIgint(K)) \subseteq H \subseteq K$ and H$\in VI\alpha C(U)$ then K$\in VI\alpha C(U)$

Proof:

(i) Let H$\in VI\alpha O(U)$ then there exist M an open set such that M$\subseteq H \subseteq VIint(VIgcl(M))$

This implies that M$\subseteq K$ and $H \subseteq VIint(VIgcl(K))$. Therefore, $VIint(VIgcl(H)) \subseteq VIint(VIgcl(M))$ and M$\subseteq K \subseteq VIint(VIgcl(M))$, then K$\in VI\alpha O(U)$

Proposition3.9:.Let H be the set in U. Then the following statements hold;

1.$VI\alpha int(H)$ is the largest vague infra α -open set contained in H

2. $VI\alpha int(H) \subseteq H$

3. $VI\alpha int(H) \subseteq VI\alpha int(k)$

4.$VI\alpha int(VI\alpha int\ (H))= VI\alpha int(H)$

5. H$\in VI\alpha O(U) \Leftrightarrow VI\alpha int(H) = H$

Proposition 3.10: Let H and K be the sets in U and H$\subseteq K$.Then the following statements hold:

1.$VI\alpha icl(H)$ is the smallest vague infra α -open set containing H

2. $H \subseteq VI\alpha cl(H)$

3. $VI\alpha cl(H) \subseteq VI\alpha cl(K)$

4.$VI\alpha cl(VI\alpha cl(H))= VI\alpha cl(H)$

5. H$\in VI\alpha C(U) \Leftrightarrow. VI\alpha cl(H) = H$

Theorem 3.11: Let H be a set of U. Then the following statements are true:

(a) $(VI\alpha int(H))^c = VI\alpha cl(H)$
(b) $(VI\alpha cl(H))^c = VI\alpha int(H)$
(c) $VI\alpha int(H) \subseteq H \cap VIint(VIgcl(VIint(H)))$
(d) $VI\alpha cl(H) \supseteq H \cup VIcl(VIgint(VIcl(H)))$

Corollary 3.12: Let H be a set of U. Then, the following statements are true:

(a) If H is an open set then $VI\alpha int(H) \subseteq VIint(VIgcl(VIint(H)))$
(b) If H is a closed set then $VI\alpha cl(H) \subseteq VIcl(VIgint(VIcl(H)))$

Theorem 3.13:

(a) The arbitrary union of vague infra α-open sets is a vague infra α-open set.
(b) The arbitrary intersection of vague infra α-closed sets is a vague infra α-closed set.

Proof: Let $\{H_i\}$ be the family of vague infra α-open set. Then for each i,$H_i \subseteq VIint(VIgcl(VIint(H_i)))$ and $\cup H_i \subseteq \cup\ (VIint(VIgcl(VIint(H_i))) \subseteq VIint(VIgcl(\cup H_i))$

Hence $\cup H_i$ is a vague infra α-open set.

Theorem 3.14: Let H be a set in U. Then, $VIgint(H) \subseteq VI\alpha int(H) \subseteq H \subseteq VI\alpha cl(H) \subseteq VIgcl(H)$

Proof: We know that $VIgint(H) \subseteq H$,

Then $VI\alpha int(VIgint(H)) \subseteq VI\alpha int(H)$

Then $VI\alpha int(VIgint(H)) = VIgint(H)$ and so VIgint(H)$\subseteq VI\alpha int(H)$

Also we know that H$\subseteq VIgcl(H)$,

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Then $VI\alpha cl(H) \subseteq VI\alpha cl(VIgcl(H))$.

Then $VI\alpha cl(VIgcl(H)) = VIgcl(H)$ and so $VI\alpha cl(H) \subseteq VIgcl(H)$

Then $VIgint(H) \subseteq VI\alpha int(H) \subseteq H \subseteq VI\alpha cl(H) \subseteq VIgcl(H)$

Theorem 3.15:Let H be a set of a VITS U. Then the following statements hold:

a) If H is VI open set then H is VI α-open set.
b) If H is VI α-open set then H is VI $semi$ open set
c) If H is VI open set then H is VI semi open set
d) If H is VI α-open set then H is VI semi pre open set
e) If H is VI semi open set then H is VI semi pre open set
f) If H is VI open set then H is VI semi pre open set
g) If H is VI pre open set then H is VI $semi\ pre$ open set

Remark: The following examples shows that the converses of these relations are not true in general.

Example 3.16: Let U= {u,v}and T={0, P_1, P_2, P_3,1}is a vague infra topology on U where

P_1={⟨$u[0.5,0.6], v[0.4,0.4]$⟩}, P_2={⟨$u[0.3,0.6], v[0.5,0.5]$⟩},P_3={ ⟨$u[0.3,0.6], v[0.4,0.4]$⟩},0={⟨$u[0,0], v[0,0]$⟩},

1={⟨$u[1,1], v[1,1]$⟩}

Then

- A_1 ={⟨$u[0.5,0.8], v[0.5,0.4]$⟩} is VI α-open set but not VI open set.
- A_2 ={⟨$u[0.4,0.7], v[0.5,0.5]$⟩} is VI semi open set but not VI α-open set
- A_3 ={⟨$u[0.5,0.6], v[0.5,0.5]$⟩} is VI semi open set but not VI open set
- A_4 ={⟨$u[0.3,0.5], v[0.5,0.7]$⟩} is VI semi pre open set but not VI α-open set
- A_5 ={⟨$u[0.4,0.7], v[0.6,0.6]$⟩} is VI semi pre open set but not semi open set
- A_6 ={⟨$u[0.4,0.5], v[0.5,0.5]$⟩} is VI semi pre open set but not VI open set
- A_7 ={⟨$u[0.2,0.4], v[0.4,0.5]$⟩} is VI semi pre open set but not VI pre open set

IV. VAGUE INFRA CONTINUOUS MAPPINGS

Definition 4.1: A mapping $f:(U,\tau) \to (V,\sigma)$ is said to be Vague infra continuous if $f^{-1}(K)$ is vague infra open (VI $-closed$) set in U for each vague infra open (closed) set K in V.

Example 4.2 :- Let U = {u_1,u_2} and V={v_1,v_2} and P$_1$={u_1 [0.5,0.6] >,< u_2[0.4,0.5]>},P$_2$={<v_1 [0.5,0.5] >,< v_2 [0.3,0.6]>}, P$_3$={<u_1[0.5,0.5] >,< u_2[0.3,0.5]>}, P$_4$={<v_1[0.4,0.5] >,< v_2[0.5,0.6]>}, P$_5$={<v_1 [0.3,0.6] >,< v_2[0.5,0.5]>}, P$_6$={<v_1 [0.3,0.5] >,< v_2[0.5,0.5]>},.Then τ={0,P$_1$,P$_2$,P$_3$,1} and σ={0, P$_4$, P$_5$, P$_6$,1} are vague infra topological spaces on P and Q respectively. Define a mapping f: (U,τ) → (V,σ) by f(a)= v_1 ,f(u_2) = v_2. If A= { <v_1 [0.4,0.5] >,< v_2 ,[0.5,0.6]>} is a vague infra open set in Q. So f^{-1}(A) = {<,u_1 [0.5,0.6] >,< u_2,[0.4,0.5]>} is a vague infra open set in P. So f is a vague infra continuous mapping.

Definition 4.3: A mapping $f:(U,\tau) \to (V,\sigma)$ is said to be Vague infra $semi$ continuous if $f^{-1}(K)$ is vague infra $semi$-open (VI $semi-closed$) set in U for each VI open (VI closed) set k in V.

Example 4.4 :- Let U = {u_1, u_2} and Y={v_1, v_2} and P$_1$={<u_1,[0.4,0.6] >,< u_2,[0.4,0.5]>},P$_2$={<u_1 [0.5,0.5] >,< u_2, [0.3,0.6]>}, P$_3$={<u_1,[0.4,0.5] >,< u_2,[0.3,0.5]>}, P$_4$={<v_1,[0.4,0.5] >,< v_2,[0.4,0.6]>}, P$_5$={<v_1,[0.3,0.6] >,< v_2,[0.5,0.5]>}, P$_6$={<v_1,[0.3,0.5] >,< v_2,[0.4,0.5]>},.Then τ={0,P$_1$, P$_2$, P$_3$,1} and σ={0, P$_4$, P$_5$, P$_6$,1} are vague infra topological spaces on P and Q respectively. Define a mapping f: (U,τ) → (V,σ) by f(u_1)= v_1 ,f(u_2) = v_2. If A= { <v_1,[0.4,0.5] >,< v_2 ,[0.4,0.6]>} is a vague infra open set in Q. So f^{-1}(A) = {<u_1, [0.4,0.6] >,< u_2,[0.4,0.5]>} is a vague infra semi open set in P. So f is a vague infra semi continuous mapping.

Definition 4.5: A mapping $f:(U,\tau) \to (V,\sigma)$ is said to be Vague infra $semi\ pre$ continuous if $f^{-1}(K)$ is vague infra $semi\ pre$open (VI $semi\ pre-closed$) set in U for each open (closed) set K in V.

Example 4.6 :- Let U = {u_1, u_2} and V={v_1, v_2} and P$_1$={<u_1,[0.5,0.6] >,< u_2,[0.4,0.5]>},P$_2$={<u_1 [0.5,0.5] >,< u_2, [0.3,0.6]>}, P$_3$={<u_1,[0.5,0.5] >,< , u_2 [0.3,0.5]>}, P$_4$={<v_1,[0.4,0.5] >,< v_2,[0.5,0.6]>}, P$_5$={<v_1,[0.3,0.6] >,<v_2,[0.5,0.5]>}, P$_6$={<v_1,[0.3,0.5] >,< v_2,[0.5,0.5]>},.Then τ={0,P$_1$, P$_2$, P$_3$,1} and σ={0, P$_4$, P$_5$, P$_6$,1} are vague infra topological spaces on P and Q respectively. Define a mapping f: (U,τ) → (V,σ) by f(u_1)= v_1 ,f(u_2) = v_2. If A= { <v_1,[0.3,0.6] >,< v_2 ,[0.5,0.5]>} is a vague infra open set in Q. So f^{-1}(A) = {<u_1, [0.5,0.5] >,< u_2,[0.3,0.6]>} is a vague infra semi open set in P. So f is a vague infra semi pre continuous mapping.

Definition 4.7: A mapping $f:(U,\tau) \to (V,\sigma)$ is said to be Vague infra pre continuous if $f^{-1}(K)$ is vague infra pre-open (VI $pre-closed$) set in U for each open (closed) set k in V.

Example 4.8:- Let U = {u_1, u_2} and Y={v_1, v_2} and G$_1$={<u_1,[0.5,0.6] >,< u_2,[0.4,0.5]>},G$_2$={<v_1 [0.5,0.5]

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>,< v_2, [0.3,0.6]>}, G_3={<u_1,[0.5,0.5] >,< u_2,[0.3,0.5]>}, G_4={<v_1,[0.4,0.5] >,< u_2,[0.5,0.6]>}, G_5={<v_1,[0.3,0.6] >,<v_2,[0.5,0.5]>}, G_6={<v_1,[0.3,0.5] >,< v_2,[0.5,0.5]>},.Then τ={0,P_1, P_2, P_3,1} and σ={0, P_4, P_5, P_6,1} are vague infra topological spaces on P and Q respectively. Define a mapping f: (U,τ) ⟶ (V,σ) by f(u_1)= v_1 ,f(u_2) = v_2. If A= { <v_1,[0.3,0.5] >,< v_2 ,[0.5,0.5]>} is a vague infra open set in Q. So f^{-1}(A) = {<u_1, [0.5,0.5] >,< u_2,[0.3,0.5]>} is a vague infra pre open set in P. So f is a vague infra pre continuous mapping.

Definition 4.9: A mapping $f:(U,\tau)\to(V,\sigma)$ is said to be an vague infra α – open (vague infra α – closed) if $f(H)$ is a vague infra α – open (vague infra α – closed) set in V for each open (closed) set H in U.

Theorem 4.10: If $f:(U,\tau)\to(V,\sigma)$ is a vague infra α – open then $f\big(VIint(H)\big)\subseteq VI\,\alpha int\big(f(H)\big)$, for each set H$\epsilon$U.

Proof: Let f be an infra α – open mapping and H be a set in U, $f\big(VIint(H)\big)\subseteq f(H)$. We have $VI\,\alpha int(f\big(VIintf(H)\big)$. Then $f\big(VIint(H)\big)\subseteq VI\,\alpha int\big(f(H)\big)$.

Corollary 4.11: If $f:(U,\tau)\to(V,\sigma)$ is a vague infra α – closed then $f\big(VI\alpha cl(H)\big)\subseteq VI\,cl\big(f(H)\big)$, for each set H$\epsilon$V

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