International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-07, Oct 2018

Best Proximity Point Theorems for Generalized k-Rational Proximal Contraction Mappings in b-Metric Spaces

J. Beny, PG and Research Department of Mathematics, Holy Cross College,
Trichy-2. E.Mail: benykutty@gmail.com
A. Sebastian Selvaraj, PG and Research Department of Mathematics, St. Joseph's College,
Trichy-2. E.Mail: sebawinselva@gmail.com
J. Maria Joseph, PG and Research Department of Mathematics, St. Joseph's College,
Trichy-2.E.Mail: joseph80john@gmail.com

Abstract- **In this paper, we prove existence and convergence of best proximity point theorems for generalized k− rational proximal contraction of first kind and second kind in complete b- metric spaces. Our results generalize and unify some results in the recent literature.**

Keywords : best proximity point, b-metric spaces, fixed point, complete metric space, contraction, convergence .

I. INTRODUCTION AND PRELIMINARIES

The fixed point theorems are broadly contemplated for different reasons. There are number of issues in integral and differential equations, for which arrangements can be proportionately planned as a fixed point of some operator T, on a reasonable space X. So fixed point theorems serve as an effective tool for handling these type of issues. It has additionally discovered various applications in territories like game theory, approximation theory, mathematical economics, theory of differential equations and so on. The most renowned outcome in this field was the Banach's contraction mapping principle [5].

Assume the fixed point condition $Tx_0 = x_0$ does not have an answer then the normal intrigue is to discover an element $x_0 \in X$ such that x_0 is in closeness to Tx_0 in some sense. The idea of best proximity point was discussed [7], fixed point and best proximity point theorems have been investigated by many authors [2,3, 10, 1, 9,8, 11]. For two subsets A and B of X and T is a non-self mapping from A to B, a best proximity pair theorem investigates the conditions affirming the presence of a component x_0 such that d (x_0, Tx_0) = dist (A, B).

Initially, b-metric space was presented by Bakhtin I.A.[4] and examined by Czerwik Stefan [7], Basha S. Sadiq, and Naseer Shahzad [5] proved best proximity point theorems for generalized proximal contractions of the first kind and the second kind in complete metric space, and Raj, A. Antony, J. Maria Joseph, and M. Marudai [13] stretch out that outcome to generalized rational proximal contractions and also some authors[14, 12, 6] developed the b -metric space in different ways. We define a rational proximal b-contraction, which can be applied for proving the existence of best proximity point theorems in complete b-metric spaces.

Definition 1. [10].Let X be a nonempty set and the mapping $d : X \times X \to (0,\infty)$ satisfies:

(bM1). $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;

(bM2). $d(x, y) = d(y, x)$ for all $x, y \in X$.

(bM3).There exists a real number $s \geq 1$ such that $d(x, y) \leq s[d(x, y) + d(z, y)]$ for all $x, y, z \in X$

Then d is called a b-metric on X and (X, d) is called a b-metric space with coefficient s.

Definition 2. [4]. Let (X, d) be a b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$ Then,

a) The sequence $\{x_n\}$ is said to be convergent in (X, d) and converges to x, if for every $\in > 0$ there exists a $n_0 \in N$ such that $d(x_n, x) < \in$ for all $n > n_0$ and this fact is represented by $\lim_{n\to\infty} = x$ or $x_n \to x$ as $n \to \infty$

b) The sequence $\{x_n\}$ is said to be Cauchy sequence in (X, d) if for every $\in > 0$ there exists $n_0 \in$ □ such that $d(x_n, x_{n+p}) < \in$ for all $n < n_0, p > 0$ or equivalently, if $\lim_{n\to\infty}(x_n, x_{n+p}) = 0$ for all p .

c) (X, d) is said to be a complete b-metric space if every Cauchy sequence in X converges to some $x \in X$.

Definition 3. [5] The set B is said to be approximatively compact with respect to A if every sequence $\{y_n\}$ of B

 DOI : 10.18231/2454-9150.2018.1033

satisfying the condition that $d(x,y_n)\to d(x,B)$ for some $x\in A$ has a convergent subsequence.
In this setting, we recall the following notions:

$$dist(A,B)=\inf\{d(a,b)/a\in A,b\in B\}$$

$$A_0=\{a\in A/d(a,b)=dist(A,B) \text{ for some } b\in B\}$$

$$B_0=\{b\in B/d(a,b)=dist(A,B) \text{ for some } a\in A\}$$

II. MAIN RESULTS

Definition 4.

Let (X,d) be a complete b-metric spaces with $s\geq 1$. A mapping $T:A\to B$ is said to be generalized k-rational proximal contraction of the first kind if there exist non-negative real numbers l_1,l_2,l_3,l_4,l_5,l_6, with $l_1,l_2,l_3,l_4,l_5+2sl_6<1$ such that the conditions $d(u_1,Tz_1)=d(u_2,Tz_2)=dist(A,B)$ imply that

$$d(u_1,u_2)\leq l_1 d(z_1,z_2)+l_2\frac{d(z_1,u_1)d(z_1,u_2)+d(z_2,u_2)d(z_2,u_1)}{d(z_1,u_2)+d(z_2,u_1)}$$

$$+l_3\frac{1+d(z_1,u_1)d(z_2,u_2)+d(z_2,u_2)d(z_2,u_1)}{1+d(z_1,z_2)}+$$

$$l_4 d(z_1,u_1)+l_5 d(z_2,u_2)+l_6[d(z_1,u_2)+d(z_2,u_1)]$$

for all $u_1,u_2,z_1,z_2\in A.$

Definition 5. Let (X,d) be a complete b-metric spaces with $s\geq 1.A$ mapping $T:A\to B$ is said to be generalized k-rational proximal contraction of the first kind if there exist non-negative real numbers l_1,l_2,l_3,l_4,l_5,l_6 with $l_1+l_2+l_3+l_4+l_5+2sl_6<1$ such that the condition $d(u_1,Tz_1)=d(u_2,Tz_2)=dist(A,B)$ imply that

$$d(Tu_1,Tu_2)\leq l_1 d(Tz_1,Tz_2)+$$

$$l_2\frac{d(Tz_1,Tu_1)d(Tz_1,Tu_2)+d(Tz_2,Tu_2)d(Tz_2,Tu_1)}{d(Tz_1,Tu_2)+d(Tz_2,Tu_1)}+$$

$$l_3\frac{1+d(Tz_1,Tu_1)d(Tz_2,Tu_2)+d(Tz_2,Tu_2)d(Tz_2,Tu_1)}{1+d(Tz_1,Tz_2)}+$$

$$l_4 d(Tz_1,Tu_1)+l_5 d(Tz_2,Tu_2)+$$

$$l_6[d(Tz_1,Tu_2)+d(Tz_2,Tu_1)]$$

for all $u_1,u_2,z_1,z_2\in A.$

Theorem 6. Let (X,d) be a complete b-metric space with $s\geq 1.$ Let A and B be non-empty, closed subsets of X such that B is approximately compact with respect to $A.$ Suppose that A_0 and B_0 be non-empty and $T:A\to B$ be a non-self mapping satisfying the following conditions.

a) T is a generalized k-rational proximal contraction of the first kind.

b) $B_0\supseteq T(A_0)$

Then, there exist a unique element $z\in A$ such that $d(z,Tz)=dist(A,B)$.Then for each $z_0\in A_0$,the sequence $\{z_n\}$ define by $d(z_{n+1},Tz_n)=dist(A,B)$, converges to the best proximity point z.

Proof : Let $z_0\in A_0$. Since B_0 contains $T(A_0)$, so there exist $z_1\in A_0$ such that $d(z_1,Tz_0)=dist(A,B).$

Since $Tz_1\in B_0\supseteq T(A_0)$. Again there exist $z_2\in A_0$ such that $d(z_2,Tz_1)=dist(A,B)$.

Thus we obtain a sequence $\{z_n\}\in A_0$, there exist an element $z_{n+1}\in A_0$ such that $d(z_{n+1},Tz_n)=dist(A,B)$ for every $n\in\mathbb{N}\cup\{0\}$.

By definition (4), we have

$$d(z_n,z_{n+1})\leq l_1 d(z_{n-1},z_n)+$$

$$l_2\frac{d(z_{n-1},z_n)d(z_{n-1},z_{n+1})+d(z_n,z_{n+1})d(z_n,z_n)}{d(z_{n-1},z_{n+1})+d(z_n,z_n)}$$

$$+l_3\frac{1+d(z_{n-1},z_n)d(z_n,z_{n+1})+d(z_n,z_{n+1})d(z_n,z_n)}{1+d(z_{n-1},z_n)}$$

$$+l_4 d(z_{n-1},z_n)+l_5 d(z_n,z_{n+1})+l_6[d(z_{n-1},z_{n+1})+d(z_n,z_n)]$$

$$\leq l_1 d(z_{n-1},z_n)+l_2 d(z_{n-1},z_n)+l_3 d(z_n,z_{n+1})+l_4 d(z_{n-1},z_n)$$

$$+l_5 d(z_n,z_{n+1})+sl_6 d(z_{n-1},z_n)+sl_6 d(z_n,z_{n+1})$$

$$d(z_n,z_{n+1})\leq\frac{l_1+l_2+l_4+sl_6}{1-(l_3+l_5+sl_6)}d(z_{n-1},z_n)$$

$$d(z_n,z_{n+1})\leq\lambda d(z_{n-1},z_n)\text{,where}$$

$$\lambda=\frac{l_1+l_2+l_4+sl_6}{1-(l_3+l_5+sl_6)}.$$

Repeating this process, we obtain

$$d(z_n,z_{n+1})\leq\lambda^n d(z_0,z_1)$$

For any m,n and $m>n$, we have

$$d(z_n,z_m)\leq s[d(z_n,z_{n+1})+d(z_{n+1},z_m)]$$

$$d(z_n,z_m)\leq sd(z_n,z_{n+1})+sd(z_{n+1},z_m)$$

$$\leq sd(z_n,z_{n+1})+s^2 d(z_{n+1},z_{n+2})+$$

$$s^3 d(z_{n+2},z_m)+\ldots+s^{m-n}d(z_{m-1},z_m)$$

$$=s\lambda^n\left[1+s\lambda+(s\lambda)^2+\ldots+(s\lambda)^{m-n-1}\right]d(z_0,z_1)$$

 DOI : 10.18231/2454-9150.2018.1033

ISSN : 2454-9150 Vol-04, Issue-07, Oct 2018

For $0 \le s\lambda < 1$ we have

$$d(z_n, z_m) = \frac{s\lambda^n}{1 - s\lambda} d(z_0, z_1)$$

Taking $m, n \to \infty$

$$\lim_{m,n\to\infty} d(z_n, z_m) \to 0$$

Hence $\{z_n\}$ is a Cauchy sequence in X. Then there exists $z \in A$ such that the sequence $\{z_n\} \to z$. Thus $d(z,B) \le d(z,Tz_n) = \lim_{n\to\infty} d(z_{n+1},Tz_n) = dist(A,B) \le d(z,B)$.

By Definition (4), we have

$d(z,Tz) = dist(A,B) = d(z_{n+1},Tz_n)$, we get

$$d(z,z_{n+1}) = l_1 d(z,z_n) + l_2 \frac{d(z,x)d(z,z_{n+1}) + d(z_n,z_{n+1})d(z_n,x)}{d(z_n,z) + d(z_n,x)}$$

b)

$$+ l_3 \frac{1 + d(z,x)d(z_n,z_{n+1}) + d(z_n,z_{n+1})d(z_n,x)}{1 + d(z,z_n)} + l_4 d(z,x)$$

$$+ l_5 d(z_n,z_{n+1}) + l_6\left[d(z_n,z_{n+1}) + d(z_n,x)\right]$$

Letting $n \to \infty$

$$d(x,z) \le (l_4 + l_6) d(z,x)$$

This implies, $z = x$, since $l_4 + l_6 < 1$.

Therefore $d(x,\mathrm{Tz}) = d(z,\mathrm{Tz}) = dist(A,B)$

Hence T has a best proximity point $z \in A$.

Next we prove the uniqueness of the best proximity point.

Let y be the another best proximity point of T,

$$d(y,y) = dist(A,B)$$

By Definition (4), we have

$$d(z,y) = l_1 d(z,y) + l_2 \frac{d(z,z)d(z,y) + d(y,y)d(z,y)}{d(z,y) + d(z,y)}$$

$$+ l_3 \frac{1 + d(z,z)d(y,y) + d(y,y)d(y,z)}{1 + d(z,z)}$$

$$+ l_4 d(z,z) + l_5 d(y,y) + l_6\left[d(z,y) + d(z,y)\right]$$

$$d(z,y) \le (l_1 + 2l_6) d(z,y)$$

It follows that $z = y$ since $l_1 + 2l_6 < 1$. Hence T has a unique best proximity point.

By taking $l_3 = 0$ in theorem 6, we obtain the following result

Corollary 7. Let (X,d) be a complete b-metric space with $s \ge 1$. Let A and B be non-empty, closed subsets of X such that B is approximately compact with respect to A. Suppose that A_0 and B_0 be non-empty and $T : A \to B$ be a non-self maping satisfying the following conditions.

a) T is a k - rational proximal contraction of the first kind.

b) $B_0 \supseteq T(A_0)$

Then there exists a unique element $z \in A$ such that $d(z,Tz) =$ dist(A,B). Then for each $z_0 \in A_0$, the sequence $\{z_n\}$ definedby $d(z_{n+1},Tz_n) = dist(A,B)$, converges to the best proximity point z.

Corollary 8. Let A and B be two nonempty, closed subsets of a complete b-metric space (X,d), with $s \ge 1$ such that B is approximately compact A. Suppose that A_0 and B_0 are non-empty and $T : A \to B$ satisfying the following conditions

a) There exists a non negative real number $\lambda < 1$ such that for all $u_1, u_2, z_1, z_2 \in A$, the condition

$d(u_1,Tz_1) = dist(A,B)$ and $d(u_2,Tz_2) = dist(A,B)$

imply that $d(u_1,u_2) \le s\lambda d(z_1,z_2)$.

b) $T(A_0) \subseteq B_0$.

Then there exists a unique element $z \in A$ such that $d(x,Tz) = dist(A,B)$. Further, for any fixed $z_0 \in A_0$, there is a sequence z_n, defined by the relation $d(z_{n+1},Tz_n) = dist(A,B)$, converges to the best proximity point x of the mapping T.

Theorem 9. Let (X,d) be a complete b-metric space with $s \ge 1$. Let A and B be non-empty, closed subsets of X such that A is approximately compact with respect to B. Suppose that A_0 and B_0 be non-empty and $T : A \to B$ be a non-self maping satisfying the following conditions.

a)T is continuous generalized k- rational proximal contraction of the second kind.

b) $T(A_0) \subseteq B_0$

Then there exists a unique element $z \in A$ such that $d(z,Tz) = dist(A,B)$. Then for each $z_0 \in A_0$ there is a sequence $\{z_n\}$ defined by $d(z_{n+1},Tz_n) = dist(A,B)$, converges to the best proximity point z of the mapping T.

Proof: As in the proof of Theorem 6, we can find a sequence $\{z_n\}$ in A_0 such that $d(z_{n+1},Tz_n) = dist(A,B)$ for all non-negative integer n, since T is a generalized k-rational proximal contraction of the second kind, we get

$$d(Tz_n,Tz_{n+1}) \le l_1 d(Tz_{n-1},Tz_n) +$$

$$l_2 \frac{d(Tz_{n-1},Tz_n)d(Tz_{n-1},Tz_{n+1}) + d(Tz_n,Tz_{n+1})d(Tz_n,Tz_n)}{d(Tz_{n-1},Tz_{n+1}) + d(Tz_n,Tz_n)}$$

DOI : 10.18231/2454-9150.2018.1033

$$+l_3\frac{1+d(Tz_{n-1},Tz_n)d(Tz_n,Tz_{n+1})+d(Tz_n,Tz_{n+1})d(Tz_n,Tz_n)}{1+d(Tz_{n-1},Tz_n)}$$

$$+l_4d(Tz_{n-1},Tz_n)+l_5d(Tz_n,Tz_{n+1})+l_6\left[d(Tz_{n-1},Tz_{n+1})+d(Tz_n,Tz_n)\right]$$

$$\leq l_1d(Tz_{n-1},Tz_n)+l_2d(Tz_{n-1},Tz_n)+l_3d(Tz_n,Tz_{n+1})+l_4d(Tz_{n-1},Tz_n)$$
$$+l_5d(Tz_n,Tz_{n+1})+sl_6d(Tz_{n-1},Tz_n)+sl_6d(Tz_n,Tz_{n+1})$$
$$d(Tz_n,Tz_{n+1})\leq\frac{l_1+l_2+l_3+sl_6}{1-(l_3+l_5+sl_6)}d(Tz_{n-1},Tz_n)$$

$d(Tz_n,Tz_{n+1})\leq\lambda d(Tz_{n-1},Tz_n)$, where

$$\lambda=\frac{l_1+l_2+l_4+sl_6}{1-(l_3+l_5+sl_6)}$$

Continuing this process, we obtain

$$d(Tz_n,Tz_{n+1})\leq\lambda^n d(Tz_0,Tz_1)$$

For any m,n and $m>n$ we obtain

$$d(Tz_n,Tz_m)\leq s\left[d(Tz_n,Tz_{n+1})+d(Tz_{n+1},Tz_m)\right]$$
$$d(Tz_n,Tz_m)\leq sd(Tz_n,Tz_{n+1})+sd(Tz_{n+1},Tz_m)$$
$$\leq sd(Tz_n,Tz_{n+1})+s^2d(Tz_{n+1},Tz_{n+2})+$$
$$s^3d(Tz_{n+2},Tz_m)+.....+s^{m-n}d(Tz_{m-1},Tz_m)$$
$$=s\lambda^n\left[1+s\lambda+(s\lambda^2)+...+(s\lambda)^{m-n-1}\right]d(Tz_0,Tz_1)$$

For $0\leq s\lambda<1$ we have

$$d(Tz_n,Tz_m)=\frac{s\lambda^n}{1-s\lambda}d(Tz_0,Tz_1)$$

Taking $m,n\to\infty$

$$\lim_{m,n\to\infty}d(Tz_n,Tz_m)\to 0$$

Hence $\{Tz_n\}$ is a Cauchy sequence in X. Since (X,d) is complete and B is closed the sequence $\{Tz_n\}$ converges to some $y\in B.$

$$d(y,A)\leq d(y,z_{n+1})$$
$$\leq d(y,Tz_n)+d(Tz_n,Tz_{n+1})$$
$$\leq d(y,Tz_n)+dist(A,B)$$
$$=d(y,Tz_n)+d(y,A)$$

Letting $n\to\infty$, we obtain

$$d(y,A)\leq\lim_{n\to\infty}d(y,z_{n+1})\leq d(y,A)$$

Thus $d(y,z_n)\to d(y,A)$. Since A is approximately compact with respect to B, the sequence $\{z_n\}$ has a subsequence $\{Tz_{n_k}\}$ converges to some element $z\in A$. By continuity of T, we have

$$d(z,Tz)=\lim_{k\to\infty}d(z_{n_k+1},Tz_{n_k})=dist(A,B)$$

Therefore, z is a proximity point of T.

Next we prove the uniqueness of the best proximity point.

Let q be the another best proximity point of T,

$$d(q,Tq)=dist(A,B)$$
$$d(Tz,Tq)=l_1d(Tz,Tq)+$$

$$l_2\frac{d(Tz,Tz)d(Tz,Tq)+d(Tq,Tq)d(Tz,Tq)}{d(Tz,Tq)+d(Tz,Tq)}$$
$$+l_3\frac{1+d(Tz,Tz)d(Tq,Tq)+d(Tq,Tq)d(Tq,Tz)}{1+d(Tz,Tz)}$$
$$+l_4d(Tz,Tz)+l_5d(Tq,Tq)+l_6[d(Tz,Tq)+d(Tz,Tq)]$$

we get,

$$d(Tz,Tq)\leq(l_1+2l_6)d(Tz,Tq)$$

It follows that $Tz=Tq$ since $l_1+2l_6<1$. Hence T has a unique best proximity point.

This complete proof.

Theorem 10. Let A and B be non-empty, closed subsets of a complete b-metric space (X,d). Suppose that A_0 and B_0 are non empty and non-empty and $T:A\to B$ is a mapping satisfying the following conditions

a) T is the generalized k-rational proximal contraction of the first as well as generalized k - rational proximal contraction of the second kind.

b) $T(A_0)\subseteq B_0$.

Then there exists unique element $z\in A$ such that $d(z,Tz)=dist(A,B)$ and the sequence $\{z_n\}$ Converges to the best proximity point z, where z_0 is any fixed element in A and $d(z_{n+1},Tz_n)=dist(A,B)$ for all $n\geq 0$.

Proof : Proceeding as in the proof of the theorem 6, we find a sequence $\{z_n\}$ in A_0 such that

$$d(z_{n+1},Tz_n)=dist(A,B)$$

For all non-negative integer n.

As in theorem 6, we can show that the sequence $\{z_n\}$ is a Cauchy sequence and hence every converges to some $z\in A$.

As in Theorem 9, it can be prove that the sequence $\{Tz_n\}$ is a Cauchy sequence and hence every converges to some $y\in B$. So, we get

$$d(z,y)=\lim_{n\to\infty}d(z_{n+1},Tz_n)=dist(A,B)$$

Therefore , $z\in A_0$. Since $T(A_0)\subseteq B_0$, we have $d(u,Tz)=dist(A,B)$ for some $u\in A$.

Since T is generalized k-rational proximal contraction of the first kind we obtain.

 DOI : 10.18231/2454-9150.2018.1033

$$d\left(u,z_{n+1}\right)=l_1 d\left(z,z_n\right)+$$

$$l_2\frac{d\left(z,u\right)d\left(z,z_{n+1}\right)+d(z_n,z_{n+1})d(z_{n,}u)}{d\left(z,z_{n+1}\right)+d(z_n,u)}$$

$$+l_3\frac{1+d(z,u)d(z_n,z_{n+1})+d(z_n,z_{n+1})d(z_n,u)}{1+d(z,z_n)}$$

$$+l_4 d(z,u)+l_5 d(z_n,z_{n+1})+l_6\left[d(z,z_{n+1})+d(z_n,u)\right]$$

Letting $n\to\infty$

$$d(u,z)\le(l_4+l_6)d(u,z)$$

Since $\left(l_4+l_6\right)<1\Rightarrow d\left(u,z\right)=0$ therefore $u=z$.

Thus it follows, $d\left(z,Tz\right)=d\left(u,Tz\right)=dist\left(A,B\right)$.

Hence z is a best proximity point of T. We can prove that the uniqueness of the best proximity point of the mapping T as in Theorem 6. This completes the proof.

III. Conclusion

In this research article, we proved best proximity point theorems for new class of generalized k-rational proximal contraction in the setting of b-metric spaces. And also its a interesting paper to find the proximal contraction of first as well as second kind. We presented many interesting results in this paper and these new results would attract many researchers in the recent trends.

References

[1] Al-Thagafi, M. A., and Naseer Shahzad. “Convergence and existence results for best proximity points.” *Nonlinear Analysis: Theory, Methods & Applications* 70.10 (2009): 3665-3671.

[2] Ajay Kumar Sharma and Balwant Singh Thakur. “ Best Proximity points for k-rational proximal contraction of first and second kind”Annals of the University of Bucharest(mathematical series) 6(LXIV) (2015),105-118

[3] Anuradha, J., and P. Veeramani. “Proximal pointwise contraction.” Topology and its Applications 156.18 (2009): 2942-2948.

[4] Bakhtin, I. A. “The contraction mapping principle in quasimetric spaces.” *Functional Analysis*, Gos.Ped. Inst. Unianowsk 30 (1989): 26-37.

[5] Basha, S. Sadiq, and Naseer Shahzad. “Best proximity point theorems for generalized proximal contractions.” *Fixed Point Theory and Applications* 2012.1 (2012): 42.

[6] Cobza, S. “B-metric spaces, fixed points and Lipschitz functions.” arXiv preprint arXiv:1802.02722 (2018).

[7] Czerwik, Stefan. “Contraction mappings in b-metric spaces.” *Acta Mathematica et Informatica Universitatis Ostraviensis* 1.1 (1993): 5-11.

[8] Eldred, A. Anthony, and P. Veeramani. “Existence and convergence of best proximity points.” *Journal of Mathematical Analysis and Applications* 323.2 (2006): 1001-1006.

[9] George, Reny, Cihangir Alaca, and K. P. Reshma. “On best proximity points in b- metric space.” Journal of Nonlinear Analysis and Application 2015.1 (2015): 45-56.

[10] Joseph, J. Maria, D. Dayana Roselin, and M. Marudai. “Fixed point theorems on multivalued mappings in b-metric spaces.” SpringerPlus 5.1 (2016): 217.

[11] Kirk, W. A., P. S. Srinivasan, and P. Veeramani. “Fixed points for mappings satisfying cyclical contractive conditions.” *Fixed point theory* 4.1 (2003): 79-89.

[12] Miculescu, Radu, and Alexandru Mihail. “New fixed point theorems for set-valued contractions in b-metric spaces.” *Journal of Fixed Point Theory and Applications* 19.3 (2017): 2153-2163.

[13] Raj, A. Antony, J. Maria Joseph, and M. Marudai. “Theorems On Best Proximity Points For Generalized Rational Proximal Contractions.” *Theoretical Mathematics & Applications* 4.2 (2014): 135-147. 8

[14] Suzuki, Tomonari. “Basic inequality on a b-metric space and its applications.” *Journal of Inequalities and Applications* 2017.1 (2017): 256.9

 DOI : 10.18231/2454-9150.2018.1033