

# Heat Source effect on convective heat transfer through wavy channel and nano-fluid

<sup>1</sup>P.Venkataramana, Research scholar, Dept. of Mathematics, Rayalaseema university, Kurnool, AP, India.

<sup>2</sup>G. Srinivas, Dept. of Mathematics, Guru Nanak Institute of Technology, Telangana, India.

<sup>3</sup>Dr.G.V.P.N srikanth, Dept. of Mathematics, Asst.Professor, VNR VJIET, Telangana, India.

**Abstract:** We investigated theoretically the free convective flow of heat transfer through a porous medium in a vertical wavy channel. The constant heat source is also to be considered. The resultant differential equations are solved by RK 6th order method. The numerical computations are presented graphically to show the salient features of the fluid flow and heat transfer characteristics. The Nusselt numbers are also analyzed for several of governing parameters.

**Keywords:** Free convection, Heat source, Nano-fluid, Porous Medium, RK 6th order method, Travelling Thermal Waves, Wavy channel.

## I. INTRODUCTION

Research in the field of Heat transfer challenging the cooling of many systems used in day to day life of kind. The heat transfer enhances enormously when nano-particles are suspended in liquids like water, ethylene glycol etc. GVPN Srikanth etc [4] presented heat transfer with heat source and radiation past a permeable stretching sheet in a nano fluid. The two dimensional free convection flow and heat transfer of a Cu and water nano-fluid in a vertical wavy channel with traveling thermal waves was investigated by P.Venkataramana, S.V. Raganayakulu and G.Srinivas Abdullah Dewar, Zahir Shah, Muhammad Idress, Waris Khan, Saeed Islam and Taza Gul presented the Impact of Thermal Radiation and Heat source /sink on Eyring – powell fluid over an unsteady Oscillatory porous Stretching Surface [1]. The study Heat transfer with Radiation and temperature dependent Heat source in MHD free Convection flow in a porous medium between two vertical wavy walls was presented by M.S Dada and A.B Disu [2] J. Prathap kumar, J.C.Umavathi [3] presented a paper on Free convective flow in an open-ended vertical porous wavy channel with a perfectly conductive Thin Baffle [23]. Combined effects of chemical reaction and temperature dependent Heat source on MHD mixed convective flow of a couple-stress fluid in a vertical wavy porous space with travelling thermal waves, investigated by Muthuraj. R, Srinivas. S and Lourdu immaculate. D [5]. Analytical solution of Thermal radiation and chemical reaction effects on unsteady MHD Convection through porous media with Heat source /sink presented by Abdel-nasser a.osman [6]. The study of viscous fluid flow in two vertical wavy channel was investigated by Tak and Kumar [9] and Kumar [7] presented heat transfer with radiation and temperature dependent heat source in MHD free convection flow confined between two vertical wavy

walls. Kuznestsov and Nield [8] studied the classical problem of free convection boundary layer flow of a viscous and incompressible fluid past a vertical flat plate to the case of nano-fluids. Seetha mahalakshmi et.al. [16] Presented the Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical moving in a porous medium with heat source and suction. Effect of hall currents on unsteady mixed convective heat and mass transfer flow of a chemically reacting viscous fluid in a horizontal channel with traveling thermal waves by S.S. Naga Leela Kumari [11]. Sibanda [12] Magneto hydrodynamic mixed convective flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction. G.Srinivas [13] presented the Finite element of analysis of convective flow and heat transfer through a porous medium with dissipative effects in channel/ducts. Transient convective heat and mass transfer through a porous medium in a vertical channel with solet effects investigated by B. Sreenivasa Reddy, J. Girish kumar [14]. Musfequs Salehin, Mohammad Monjurul Ehsan, and A. K. M. Sadrul Islam [15] presented the Heat transfer enhancement and pumping power optimization using CuO-water nanofluid through rectangular corrugated pipe.

## II. MATHEMATICAL MODELLING

We consider the flow of a nano fluid between the two vertical wavy walls. We choose the x-axis along the direction of flow and y-axis perpendicular to it. The two wavy walls are at  $y = d + a \cos \lambda x$  and  $y = -d + a \cos(\lambda x + \zeta)$ . The buoyancy force is due to the density variation and temperature difference along the flow. The thermal wave is imposed in between the two plates. The velocity  $V$

experiences a resistance  $gV/K$  due to the porous medium where  $K$  is the permeability. We assume that the wave length of the wavy wall is proportional to  $1/\lambda$ .

In view of the above the governing equations of the flow are as follows: Equation of Continuity,

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} = 0$$

Equation of Momentum,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\mu_{nf}}{\rho_{nf}} u + g \beta_r (T - T_0) u$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \frac{\mu_{nf}}{\rho_{nf}} v$$

Equation of Energy,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \frac{Q}{(\rho c_p)_{nf}} (T - T_0)$$

Where  $\alpha_{nf} = \frac{k_{nf}}{\rho_{nf}}$

The boundary conditions are:

$$u = 0, v = 0, T = T_0(1 + \varepsilon \cos(\lambda x + \omega t)) = T_0' \text{ at } y = d + a \cos \lambda x$$

$$y = d + a \cos \lambda x$$

$$u = 0, v = 0,$$

$$T = T_1[1 + \varepsilon \cos(\lambda x + \omega t)] = T_1' \text{ at } y = -d + a \cos(\lambda x + \omega t)$$

We introduce the following non-dimensional parameters:

$$X = \frac{x}{d}, Y = \frac{y}{d}, t' = \frac{t\gamma}{d^2}, \rho = \frac{\rho d^2}{\rho \gamma^2}, \theta = \frac{T - T_0'}{T_1' - T_0'}$$

$$U = \frac{u d}{y}, V = \frac{v d}{y}, \lambda' = \lambda d, \varepsilon = \frac{a}{d}$$

After introducing non-dimensional parameters and the stream function the governing equations becomes,

$$\frac{\partial^3 \psi}{\partial Y^2 \partial t} + \frac{\partial^3 \psi}{\partial X^2 \partial t} - \frac{\partial \psi}{\partial Y} \left[ \frac{\partial^3 \psi}{\partial Y^2 \partial X} + \frac{\partial^3 \psi}{\partial X^3} \right] + \frac{\partial \psi}{\partial X} \left[ \frac{\partial^3 \psi}{\partial X^2 \partial Y} + \frac{\partial^3 \psi}{\partial Y^3} \right]$$

$$= \frac{\mu_{nf}}{\mu_f} \frac{\rho_f}{\rho_{nf}} \left[ 2 \frac{\partial^4 \psi}{\partial Y^2 \partial X^2} + \frac{\partial^4 \psi}{\partial X^4} + \frac{\partial^4 \psi}{\partial Y^4} \right]$$

$$- d \frac{\mu_{nf}}{\mu_f} \frac{\rho_f}{\rho_{nf}} \left[ \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} \right] - G \theta$$

$$\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial Y} \frac{\partial T}{\partial X} + \frac{\partial \psi}{\partial X} \frac{\partial T}{\partial Y} = \frac{k_{nf}}{k_f} \frac{\rho_f}{\rho_{nf}} \frac{1}{Pr} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \frac{Q}{Pr} \theta$$

The non - dimensional boundary conditions are:

$$-\frac{\partial \psi}{\partial Y} = 0, \frac{\partial \psi}{\partial X} = 0, \theta = 0 \text{ at } Y = 1 + \varepsilon \cos \lambda' X$$

$$-\frac{\partial \psi}{\partial Y} = 0, \frac{\partial \psi}{\partial X} = 0, \theta = 1 \text{ at } Y = -1 + \varepsilon \cos(\lambda' X + \xi)$$

Where  $G = \frac{d^3 g \beta_r (T_1' - T_0')}{\nu^2}$ ,

$Pr = \frac{\nu}{\alpha}$ , Prandtl Number,  $d = \frac{K}{d^2}$ , Darcy Parameter.

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s$$

$$(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_s$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1 - \phi)^{0.25}}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}$$

### III. NUMERICAL METHOD

For computational purpose we use  $f(x, y, t) = f(y)e^{i(ax+st)}$  in all the governing equations as given in Rudraiah (3). The regular Galerkin finite element method is used, the domain is divided into 10 elements. The iterative procedure has been used to attain the desired accuracy and to meet the convergence criteria.

### IV. DISCUSSION AND RESULTS

Figures 1-15 depicts the variations of  $U, V$  and  $\theta$  for  $Pr = 0.71, \lambda = 0.02, \lambda x = \pi/2, \omega t = \pi/4, \zeta = 0$  in the region  $-1 \leq y \leq 1$  except for the varying parameter. The velocities and temperature profiles are found in wavy form throughout the region. All governing parameters are found maximum in the mid region of the channel. From Fig.1 the velocity  $U$  increases with increase in solid volume fraction particularly in the mid region, the reverse effect is observed for  $V$  from Fig. 6. Both figures stress the importance of solid part in the fluid. 5% or above solid volume fraction accelerates the flow. Magnitude of  $U$  increases gradually with Darcy parameter  $Da$  (Fig.2). Similarly velocity  $v$  shows gradual variations with  $Da$  from Fig. 7. Presence of porous medium affects the flow rapidly; interestingly the reduction of pore diameter is inversely proportional to the velocity. The buoyancy force shows a gradual effect on  $U$  and  $V$  from Figs. 3 & 8 respectively. The buoyancy is directly proportional to the velocity but found more effective near the left plate of the channel. The natural convection is significant even in wavy channel. The variation of  $U$  for  $\varepsilon$  is very clear

from Fig.4, the pattern for V is reversed in Fig. 9. The frequency parameter is more significant on the flow field particularly in the mid region. The frequency parameter has no significance near the hot wall. Heat source parameter enhancing the flow near the cold wall and flow is almost uniform in the remaining part of the channel from figures 5 & 10.

The variation of absolute temperature ( $\theta$ ) is directly proportional to  $\phi$ . The temperature variation is clear near the cold wall due to the nano particle concentration from Fig.11. From Fig. 12 it is observed that presence of porous medium (Da) enhances the absolute temperature significantly. Similar effect of buoyancy (Gr) is observed from Fig.13. The frequency parameter is significant on temperature near the cold wall clearly from Fig 14. Heat source enhances the temperature of the cold plate from Fig. 15.

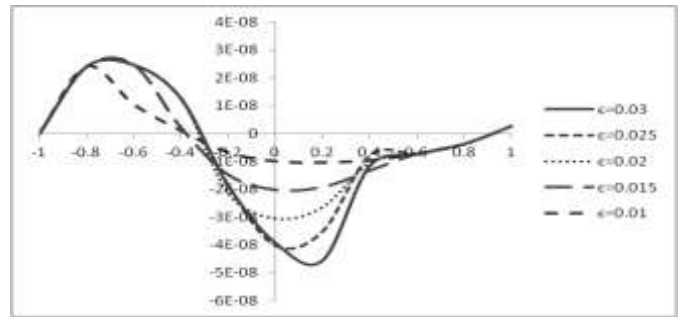


Fig.4 Variation of U with  $\epsilon$

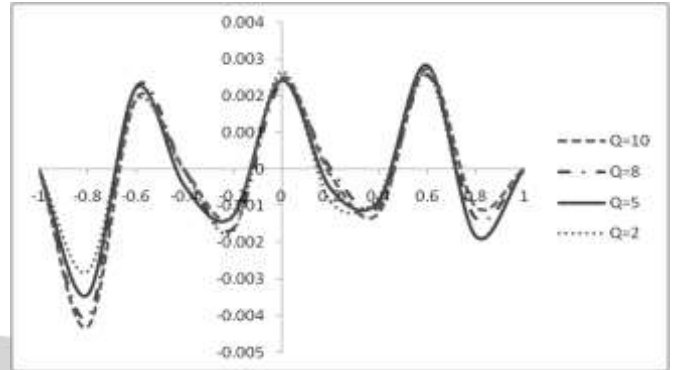


Fig.5 Variation of U with Q

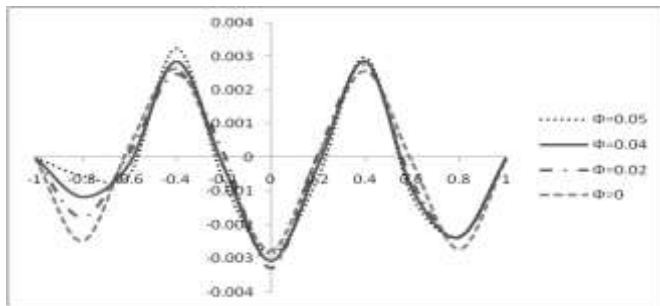


Fig.1 Variation of U with  $\phi$

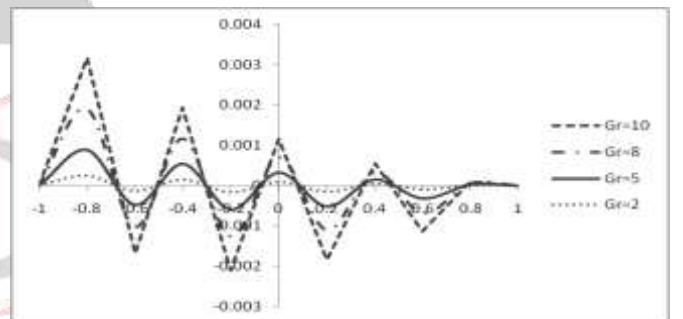


Fig.6. Variation of v with  $\phi$

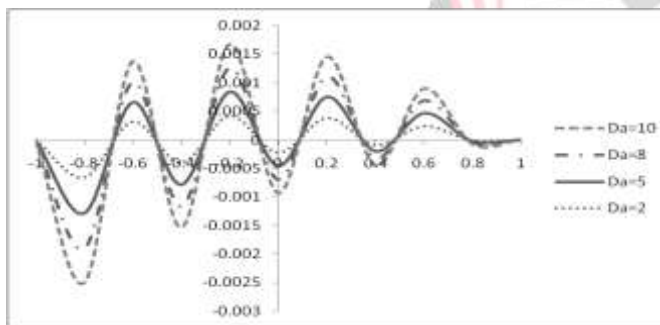


Fig.2 Variation of U with Da

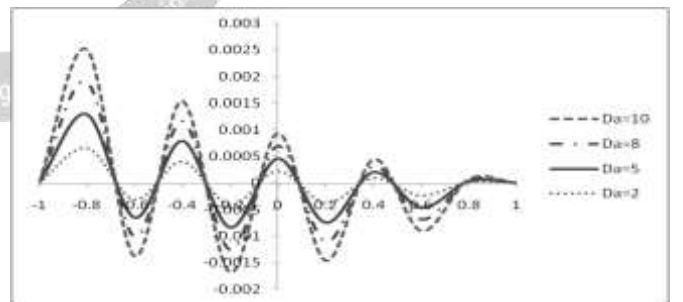


Fig.7 Variation of v with Da

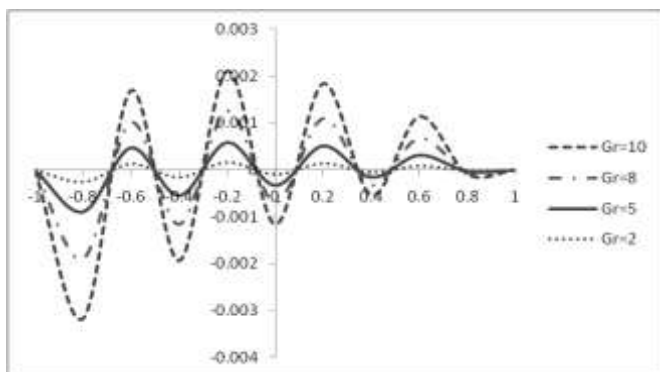


Fig.3 Variation of U with Gr

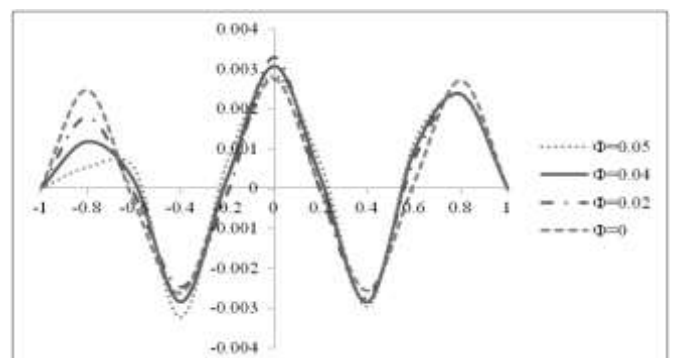


Fig.8 Variation of V with Gr

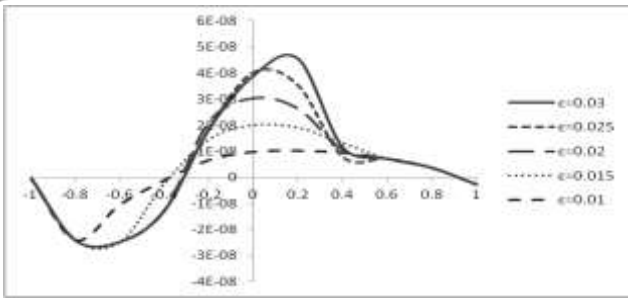


Fig.9 Variation of v with  $\epsilon$

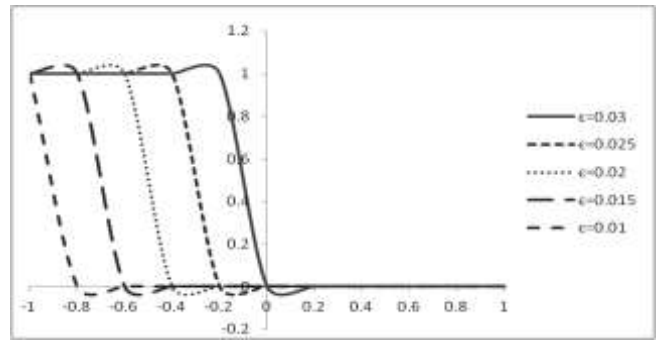


Fig.14 Variation of  $\Theta$  with  $\epsilon$

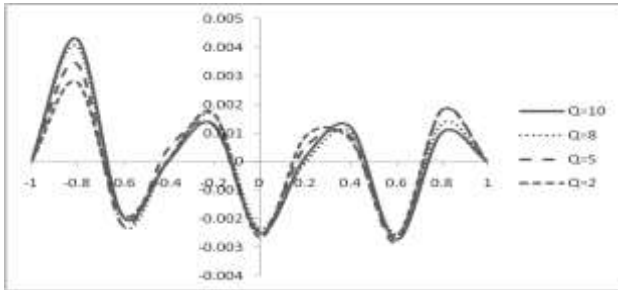


Fig.10. Variation of v with Q

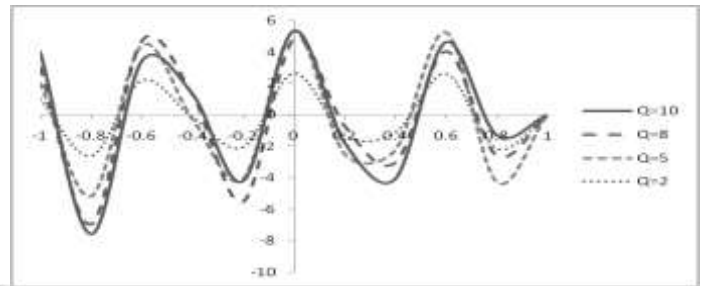


Fig.15. Variation of  $\Theta$  with Q

**Tables for Nusselt numbers:**

**Table-1:**

$\phi$	0	0.02	0.04	0.05
Nu-1	-4.873 $\times 10^{19}$	-3.258 $\times 10^{17}$	-1.867 $\times 10^{20}$	-3.026 $\times 10^{19}$
Nu-2	809.094	-127.886	-81.787	74.545

**Table-2:**

Da	2	5	8	10
Nu-1	-3.026 $\times 10^{19}$	-3.026 $\times 10^{19}$	-3.026 $\times 10^{19}$	-3.0264 $\times 10^{19}$
Nu-2	74.555	74.545	74.538	74.534

**Table-3:**

Gr	2	5	8	10
Nu-1	-3.0258 $\times 10^{19}$	-3.0264 $\times 10^{19}$	-3.0268 $\times 10^{19}$	-3.0272 $\times 10^{19}$
Nu-2	74.495	74.545	74.5962	74.6298

**Table-4:**

$\epsilon$	0.01	0.015	0.02	0.025	0.03
Nu-1	3.6009 $\times 10^{24}$	1.82 $\times 10^{25}$	5.7614 $\times 10^{25}$	1.4066 $\times 10^{26}$	2.916 $\times 10^{26}$
Nu-2	6.586 $\times 10^{-16}$	6.601 $\times 10^{-16}$	6.622 $\times 10^{-16}$	6.648 $\times 10^{-16}$	6.677 $\times 10^{-16}$

**Table-5:**

Q	2	5	8	10
Nu-1	-1.9669 $\times 10^{17}$	-3.026 $\times 10^{19}$	-1.1015 $\times 10^{18}$	-1.286 $\times 10^{18}$
Nu-2	-169.759	74.545	-215.959	65.973

**V. NOMENCLATURE**

d – Half of the distance between the wavy walls  
g – Acceleration right wall of the channel

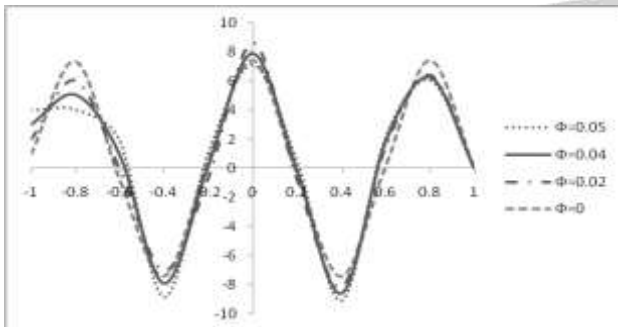


Fig.11 Variation of  $\Theta$  with  $\phi$

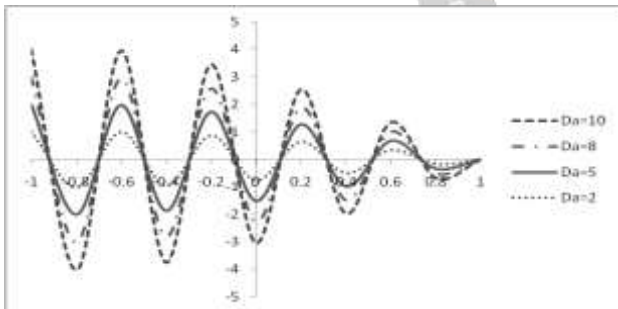


Fig.12 Variation of  $\Theta$  with Da

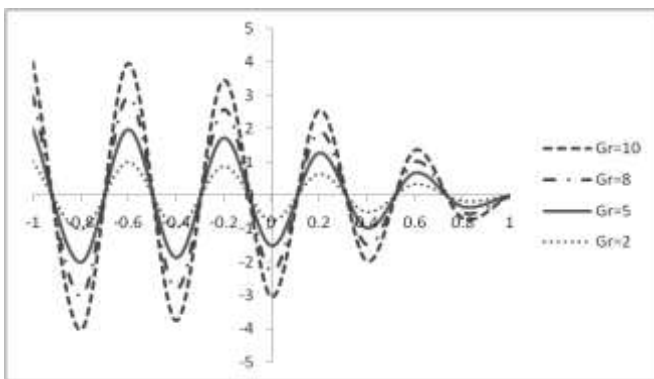


Fig.13 Variation of  $\Theta$  with Gr

$p$  – Pressure  
 $\beta$  – Molecular Diffusivity  
 $\alpha$  – Thermal Diffusivity  
 $\omega$  – Non dimensional due to gravity  
 $a$  – Constant  
 $\lambda$  – Non dimensional wave number  
 $\rho$  – Density  
 $\mu$  – Viscosity  
 $u$  – Velocity in  $x$  - direction  
 $v$  – Velocity in  $y$  - direction  
 $Q$  – Heat transfer rate

## VI. CONCLUSIONS

The cooling effect is found more with metal particle proportion on the left wall and heating is found more on right wall of the channel. This is biggest advantage of nano fluid. The porous medium is showing the behavior as that of solid volume fraction. The rate of heat transfer enhances with buoyancy and frequency parameters. The lesser the frequency of the wave the more the heat transfer rate.

## REFERENCES

- [1]. Abdullah Dawar, Zahir Shah, Muhammad Idress, Waris Khan, Saeed Islam and Taza Gul; Impact of Thermal Radiation and Heat source /sink on Eyring-powell fluid over an unsteady Oscillatory porous Stretching Surface, Mathematical and Computational Applications (2018) 23, 20.
- [2]. M.S Dada and A.B Disu; Heat transfer with Radiation and temperature dependent Heat source in MHD free Convection flow in a porous medium between two vertical wavy walls, Journal of Nigerian Mathematical Society(2015).
- [3]. J. Prathap kumar, J.C.Umavathi, Free convective flow in an open-ended vertical porous wavy channel with a perfectly conductive Thin Baffle: library.com/journal/htjissue 3, 2015.
- [4]. GVPN Srikanth, Dr.G.Srinivas and Raja Sekar Gorthi: Heat transfer with Heat source and Radiation past a permeable stretching sheet in a nanofluid: International referred research journal, vol-iv issue-iii march-2015.
- [5]. Muthuraj.R, Srinivas. S, and Lourdu immaculate, D.,(2012),“Combined effects of chemical reaction and temperature dependent Heat source on MHD mixed convective flow of a couple-stress fluid in a vertical wavy porous space with travelling thermal waves,” Chemical Industry and Chemical Engineering Quarterly, 18, pp. 305-314.
- [6]. Abdel-nasser,a.osman, S.M.Abo-Dahab and R.A.Mohammed; Analytical solution of Thermal radiation and chemical reaction effects on unsteady MHD Convection through porous media with Heat source /sink; Mathematical problems in Engineering (2011).
- [7]. Kumar H. Heat transfer with radiation and temperature dependent Heat source in MHD free convection flow confined between two vertical wavy walls. Int J Appl Math Mech 2011; 7 (2):77–103.
- [8]. Kunetsov AV, Nield DA (2010) Natural convective boundary layer flow of a nano fluid past a vertical plate.int j term sci 288:243-247.
- [9]. Tak SS, Kumar H. MHD free Convection flow with viscous dissipation in a vertical wavy channel. Ultra Sci Phys Sci 2006; 18 (2M): 221–32.
- [10].S Karunakar Reddy, D Chenna Kesavaiah , M N Raja Shekar: Convective Heat and Mass Transfer Flow from a Vertical Surface with Radiation, Chemical reaction and Heat Source/Absorption, International Journal of Scientific Engineering and Technology (ISSN : 2277-1581) Volume 2 Issue 5, pp : 351-361 1 May 2013 IJSE.
- [11].S.Naga Leela Kumari and G. Sarojamma (2012): Effect of hall currents on unsteady mixed convective heat and mass transfer flow of a chemically reacting viscous fluid in a horizontal channel with traveling thermal waves, International Journal of Mathematical Archive-3(2), 2012, pp. 752-761.
- [12]. O. D. Makinde and P. Sibanda 2008): Magneto hydrodynamic mixed convective flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction. ASME - Journal of Heat Transfer, 130 (11260), pp. 1-8.
- [13]. G.Srinivas, Finite element of analysis of convective flow and heat transfer through a porous medium with dissipative effects in channel/ducts, Ph.D. Thesis, S.K.University, Anantapuramu, 2005.
- [14].B. Sreenivasa Reddy, J. Girish kumar, Transient convective heat and mass transfer through a porous medium in a vertical channel with solet effects, International Journal of Mathematical Archieve, 4(4)(2013)197-207.
- [15]. Musfequs Salehin, Mohammad Monjurul Ehsan, and A. K. M. Sadrul Islam,: Heat transfer enhancement and pumping power optimization using CuO-water nanofluid through rectangular corrugated pipe AIP Conference Proceedings 1851, 020096 (2017);
- [16]. G. V. Seethamahalakshmi, Ramana Reddy and B. D.C. N. Prasad (2012): Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical moving in a porous medium with heat source and suction, IOSR Journal of Engineering, Vol. 1, (1), pp. 28-36.