International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

A Well-defined Model of Multiplicative Connectivity Invariants of Jahangir Graph $J_G(n,m)$

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Abstract: In this research paper, we make progress to the multiplicative degree based topological indices of Jahangir graph like the Multiplicative invariants of first Zagreb $\prod_{e\in E(G)}(a+b)$, the second Zagreb $\prod_{e\in E(G)}(ab)$, the square of first Zagreb $\prod_{e\in E(G)}(a+b)^2$,the square of second Zagreb $\prod_{e\in E(G)}(ab)^2$,the sum connectivity index $\prod_{e\in E(G)}\left(\frac{1}{\sqrt{a+b}}\right)$,the Randic index $\prod_{e\in E(G)}\left(\frac{1}{\sqrt{ab}}\right)$,the Generalized first Zagreb index $\prod_{e\in E(G)}(a+b)^\gamma$,the Generalized second Zagreb index $\prod_{e\in E(G)}(ab)^\gamma$, the Atom bond connectivity index $\prod_{e\in E(G)}\sqrt{\frac{a+b-2}{ab}}$, the Geometric – Arithmetic index $\prod_{e\in E(G)}\left(\frac{2\sqrt{ab}}{a+b}\right)$ of $J_G(n,m)$ and also find closed forms all the above said indices using M-polynomial of Jahangir graphs $J_G(n,m)$ for all values of $n\geq 2, m\geq 3$.Our results are easier to evaluate and simpler than existing results of this article.

Keywords: M-polynomial, Topological invariants, Jahangir graphs

I. INTRODUCTION

Jahangir Graph is defined by a chemical graph and a cycle graph. In mathematical chemistry, mathematical tools like polynomials and topological-based numbers predict properties of compounds without using quantum mechanics. These tools, in combination, capture information hidden in the symmetry of molecular graphs. A topological index is a function that characterizes the topology of the graph. Most commonly known invariants of such kinds are degree -based topological indices. These are actually the numerical values that correlate the structure with various physical properties, chemical reactivity, and biological activities [1–5]. It is an established fact that many properties such as heat of formation, boiling point, strain energy, rigidity and fracture toughness of a molecule are strongly connected to its graphical structure and this fact plays a synergic role in chemical graph theory. Algebraic polynomials play a significant part in chemistry. Hosoya polynomial is one such well-known example which determines distance-based topological indices. M-polynomial [7-10], introduced in 2015, plays the same role in determining closed forms of many degree-based topological indices [11–18]. The main advantage of M-polynomial [19-40] is the wealth of information that it contains about degree-based graph invariants. The Jahangir graph $J_G(n,m)$ is a graph on nm + 1 vertices and m(n + 1) edges for all n ≥ 2 and m ≥ 3. $J_G(n,m)$ consists of a cycle graph $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance to each other. Figure [1] shows some particular cases of $J_G(n,m)$. The Figure $J_G(2,8)$ is carved on Jahangir's tomb. It is situated 5 km northwest of Lahore, Pakistan. In Laurdusamy et al. computed the pebbling number of Jahangir graph $J_G(2,m)$ for m ≥8.Mojdeh et al. in computed domination number in $J_G(2,m)$ and Ramsey number for $J_G(3,m)$ by the M-Polynomial of $J_G(3,m)$ $J_G(2,m)$, $J_G(3,m)$ and

 DOI : 10.18231/2454-9150.2018.1106

$J_G(4,m)$ are computed all these results are partial and need to be generalized for all values of m and n.

DEFINITION:

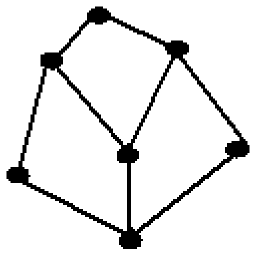
A topological index is a function that characterizes the topology of the Jahangir Graph .Let "V" be the Number of vertices and "E" be the Number of edges for $J_G(n,m)$ graph represents (nm+1) vertices and m(n+1) edges respectively for all $n \geq 2, m \geq 2$ i.e., a graph consisting of a cycle Graph $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$.

Computational Procedure of M-Polynomial:

M-Polynomial of graph G is defined as If $G=(V,E)$ is a graph and $v \in V$, then $d_v(G)$ (or d_v for short if G is clear from the context) denotes the degree of v. Let G be a graph and let $m_{ab}(G), a,b \geq 1$, be the number of edges e = uv of G such that $\{d_u(G), d_v(G)\} = \{a,b\}$. The M-polynomial of G as $M(G;x,y) = \sum_{a \leq b} m_{ab}(G) x^a y^b$. For a graph $G=(V,E)$, a degree-based topological index is a graph invariant of the form $I(G) = \sum_{e=uv \in E} f(d_u, d_v)$ where $f = f(x,y)$ is a function appropriately selected for possible chemical applications. In this article, we compute closed form of some degree-based topological indices of the Jahangir graph by using the *M*-polynomial. Let "v" be the number of vertices and "e" be the number of edges for $J_G(n,m)$ graph represents (nm+1) vertices and m(n+1) edges respectively for all n≥2 , m≥3. i.e., a graph consisting of a cycle graph $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$. Here we considered the Jahangir graphs for $n=2, m \geq 3$, the results are given in case 1 to case 3 and for $n=3, m \geq 3$, the results are given in case 4 to case 6 and for $n=4, m \geq 3$, the results are given in case 7 to case 9 , for $n=5, m \geq 3$, the results are given in case 10 to case 12, finally, the results are generalized for $n \geq 2, m \geq 3$.

Case 1:

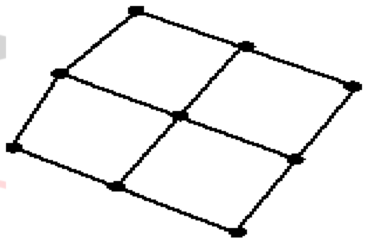
For $J_G(2,3)$, the number of edges with end degrees (2,2) is equal to 0, the number of edges with end degrees (2,3) is equal to 6, the number of edges with end degrees (3,3) is equal to 3, therefore the total number of edges is equal to 9.



Fig(1) $J_G(2,3)$

Case (2):

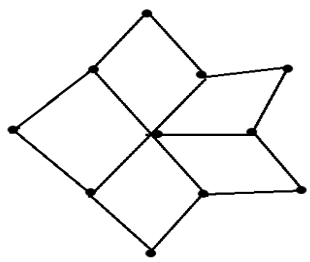
For $J_G(2,4)$, the number of edges with end degrees (2,2) is equal to 0, the number of edges with end degrees (2,3) is equal to 8, the number of edges with end degrees (3,3) is equal to 0,the number of edges with end degrees (3,4) is equal to 4, therefore the total number of edges is equal to 12.



Fig(2) $J_G(2,4)$

Case (3):

For $J_G(2,5)$, the number of edges with end degrees (2,2) is equal to 0, the number of edges with end degrees (2,3) is equal to 10, the number of edges with end degrees (3,3) is equal to 0,the number of edges with end degrees (3,4) is equal to 0,the number of edges with end degrees (3,5) is equal to 5, therefore the total number of edges is equal to 15.

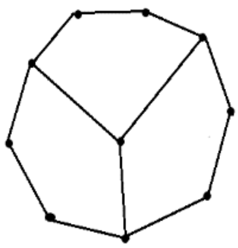


Fig(3) $J_G(2,5)$

Case (4):

For $J_G(3,3)$, the number of edges with end degrees (2,2) is equal to 3, the number of edges with end degrees (2,3) is equal to 6, the number of edges with end degrees (3,3) is equal to 3, the therefore total number of edges is equal to 12.

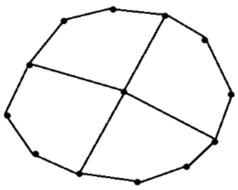
 DOI : 10.18231/2454-9150.2018.1106



Fig(4) $J_G(3,3)$

Case(5):

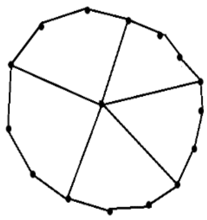
For $J_G(3,4)$, the number of edges with end degrees (2,2) is equal to 4, the number of edges with end degrees (2,3) is equal to 8, the number of edges with end degrees (3,3) is equal to 4, therefore the total number of edges is equal to 16.



Fig(5) $J_G(3,4)$

Case (6):

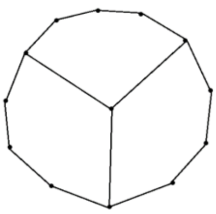
For $J_G(3,5)$, the number of edges with end degrees (2,2) is equal to 5, the number of edges with end degrees (2,3) is equal to 10,the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 20.



Fig(6) $J_G(3,5)$

Case (7):

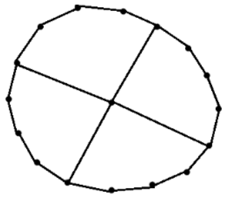
For $J_G(4,3)$, the number of edges with end degrees (2,2) is equal to 6, the number of edges with end degrees (2,3) is equal to 6,the number of edges with end degrees (3,3) is equal to 3, therefore the total number of edges is equal to 15.



Fig(7) $J_G(4,3)$

Case (8):

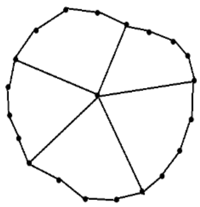
For $J_G(4,4)$, the number of edges with end degrees (2,2) is equal to 8, the number of edges with end degrees (2,3) is equal to 8,the number of edges with end degrees (3,3) is equal to 4, therefore the total number of edges is equal to 20.



Fig(8) $J_G(4,4)$

Case (9):

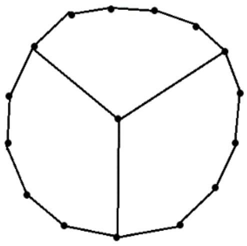
For $J_G(4,5)$, the number of edges with end degrees (2,2) is equal to 10, the number of edges with end degrees (2,3) is equal to 10,the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 25.



Fig(9) $J_G(4,5)$

Case (10):

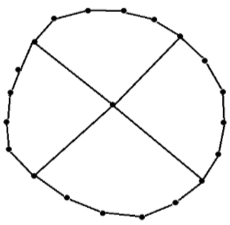
For $J_G(5,3)$, the number of edges with end degrees (2,2) is equal to 10, the number of edges with end degrees (2,3) is equal to 10,the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 25.



Fig(10) $J_G(5,3)$

Case (11):

For $J_G(5,4)$, the number of edges with end degrees (2,2) is equal to 12, the number of edges with end degrees (2,3) is equal to 8,the number of edges with end degrees (3,3) is equal to 4, therefore the total number of edges is equal to 24.

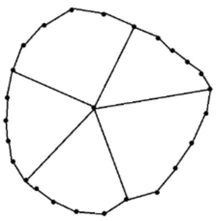


 DOI : 10.18231/2454-9150.2018.1106

Fig(11) $J_G(5,4)$

Case (12):

For $J_G(5,5)$, the number of edges with end degrees (2,2) is equal to 15, the number of edges with end degrees (2,3) is equal to 10, the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 30.



Fig(12) $J_G(5,5)$

M-Polynomial of Jahangir graph is developed and it is given by

$J_G(n,m)$ =

$$\{(n-2)m\}x^2y^2+\{2m\}x^2y^3+\{m\}x^3y^3 \; for\,all\, n\geq 2, m\geq 3$$

Some important Multiplicative degree based Topological invariants of Jahangir graphs are given below:

1. The Multiplicative invariants of first Zagreb $\Rightarrow$

$$P_1[J_G(n,m)]=\prod_e (a+b)$$

2. The Multiplicative invariants of second Zagreb $\Rightarrow$

$$P_2[J_G(n,m)]=\prod_e (ab)$$

3. The Multiplicative invariants of square of first Zagreb $\Rightarrow P_3[J_G(n,m)]=\prod_e (a+b)^2$

4. The Multiplicative invariants of square of second Zagreb $\Rightarrow P_4[J_G(n,m)]=\prod_e (ab)^2$

5. The Multiplicative invariants of sum connectivity index $\Rightarrow P_5[J_G(n,m)]=\prod_e \left(\frac{1}{\sqrt{a+b}}\right)$

6. The Multiplicative invariants of Randic index $\Rightarrow P_6[J_G(n,m)]=\prod_e \left(\frac{1}{\sqrt{ab}}\right)$

7. The Generalized Multiplicative invariants of first Zagreb $\Rightarrow$

$$P_7[J_G(n,m)]=\prod_e (a+b)^\beta$$

8. The Generalized Multiplicative invariants of second Zagreb $\Rightarrow$

$$P_8[J_G(n,m)]=\prod_e (ab)^\beta$$

9. The Multiplicative invariants of Atom bond connectivity $\Rightarrow$

$$P_9[J_G(n,m)]=\prod_e \sqrt{\frac{a+b-2}{ab}}$$

10. The Multiplicative invariants of Geometric – Arithmetic $\Rightarrow P_{10}[J_G(n,m)]=\prod_e \left(\frac{2\sqrt{ab}}{a+b}\right)$

Theorem:1

Let a graph G consists of a pair (V,E) and cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of first Zagreb is given by $P_1[J_G(n,m)]=m(4n+8)$.

Proof:

The Multiplicative invariants of first Zagreb is denoted by

$$P_1[J_G(n,m)]=\prod_e (a+b)$$

Now, this theorem have to derived by the results of the Multiplicative invariants of first Zagreb of topological invariants is given below :

$$P_1[J_G(n,m)]=\prod_{e\in E(G)} (a+b)$$

$$P_1[J_G(n,m)]=$$
$$(a+b)\{(n-2)m\}+(a+b)\{2m\}+(a+b)\{m\}$$

$$P_1[J_G(n,m)]=$$
$$(2+2)(nm-2m)+(2+3)(2m)+(3+3)(m)$$

$$P_1[J_G(n,m)]=4(nm-2m)+5(2m)+6m$$

$$P_1[J_G(n,m)]=4m(n-2)+10m+6m$$

$$P_1[J_G(n,m)]=4m(n-2)+16m$$

$$P_1[J_G(n,m)]=4nm-8m+10m+6m$$

$$P_1[J_G(n,m)]=4nm+8m$$

Hence, the final result of the Multiplicative degree based topological invariants of first Zagreb of Jahangir graph is equal to

$$P_1[J_G(n,m)]=m(4n+8)$$

Theorem:2

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of second Zagreb is given by $P_2[J_G(n,m)]=4mn+13m$

 DOI : 10.18231/2454-9150.2018.1106

Proof:

The Multiplicative invariants of second Zagreb is denoted by $P_2[J_G(n,m)] = \prod_e (ab)$

Now, this theorem have to derived by the results of the Multiplicative invariants of second Zagreb of topological invariants is given below :

$$P_2[J_G(n,m)] = \prod_{e\in E(G)} (ab)$$

$$P_2[J_G(n,m)]$$

$$(ab)\{(n-2)m\}+(ab)\{2m\}+(ab)\{m\}$$

$$P_2[J_G(n,m)] =$$

$$(2\times2)(nm-2m)+(2\times3)(2m)+(3\times3)(m)$$

$$P_2[J_G(n,m)] = 4nm-8m+12m+9m$$

$$P_2[J_G(n,m)] = 4m(n-2)+21m$$

$$P_2[J_G(n,m)] = 4m(n+13)$$

Hence, the final result of the Multiplicative degree based topological invariants of second Zagreb of Jahangir graph is equal to

$$P_2[J_G(n,m)] = 4mn+13m$$

Theorem:3

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of square of first Zagreb is given by $P_3[J_G(n,m)] = m(16n+54)$

Proof:

Proof:

The Multiplicative invariants of square of first Zagreb is denoted by $P_3[J_G(n,m)] = \prod_e (a+b)^2$

Now, this theorem have to derived by the results of the Multiplicative invariants of square of first Zagreb invariants is given below :

$$P_3[J_G(n,m)] = \prod_{e\in E(G)} (a+b)^2$$

$$P_3[J_G(n,m)] =$$

$$(a+b)^2\{(n-2)m\}+(a+b)^2\{2m\}+(a+b)^2\{m\}$$

$$P_3[J_G(n,m)] \quad =$$

$$(2+2)^2\,(nm-2m)+(2+3)^2\,(2m)+(3+3)^2\,(m)$$

$$P_3[J_G(n,m)] \quad = 16(nm-2m)+50m+36m$$

$$P_3[J_G(n,m)] = 16nm-32m+50m+36m$$

Hence, the final result of the Multiplicative degree based topological invariants of square of first Zagreb of Jahangir graph is equal to

$$P_3[J_G(n,m)] = m(16n+54)$$

Theorem:4

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of square of second Zagreb is given by $P_4[J_G(n,m)] = m(16n+121)$

Proof:

Proof:

The Multiplicative invariants of square of second Zagreb is denoted by $P_4[J_G(n,m)] = \prod_e (ab)^2$

Now, this theorem have to derived by the results of the Multiplicative invariants of square of second Zagreb invariants is given below :

$$P_4[J_G(n,m)] = \prod_{e\in E(G)} (ab)^2$$

$$P_4[J_G(n,m)] =$$

$$(ab)^2\{(n-2)m\}+(ab)^2\{2m\}+(ab)^2\{m\}$$

$$P_4[J_G(n,m)] =$$

$$(2\times2)^2\,(nm-2m)+(2\times3)^2\,(2m)+(3\times3)^2\,(m)$$

$$P_4[J_G(n,m)] = 16nm-32m+72m+81m$$

$$P_4[J_G(n,m)] \quad 16(nm-2m)+153m$$

Hence, the final result of the Multiplicative degree based topological invariants of square of second Zagreb of Jahangir graph is equal to

$$P_4[J_G(n,m)] = m(16n+121)$$

Theorem:5

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of sum connectivity index is given by

$$P_5[J_G(n,m)] = \frac{m\left(\sqrt{30}n-2\left(\sqrt{5}\left(\sqrt{6}-1\right)-2\sqrt{6}\right)\right)}{2\sqrt{30}}$$

Proof:

The Multiplicative invariants of sum connectivity index is denoted by $P_5[J_G(n,m)] = \prod_e \left(\frac{1}{\sqrt{a+b}}\right)$

 DOI : 10.18231/2454-9150.2018.1106

Now, this theorem have to derived by the results of the Multiplicative invariants of sum connectivity invariants is given below :

$$P_5[J_G(n,m)] = \prod_{e\in E(G)}\left(\frac{1}{\sqrt{a+b}}\right)$$

$$P_5[J_G(n,m)] = \left(\frac{1}{\sqrt{a+b}}\right)\{(n-2)m\}+\left(\frac{1}{\sqrt{a+b}}\right)\{2m\}+\left(\frac{1}{\sqrt{a+b}}\right)\{m\}$$

$$P_5[J_G(n,m)] = \left(\frac{1}{\sqrt{2+2}}\right)(nm-2m)+\left(\frac{1}{\sqrt{2+3}}\right)(2m)+\left(\frac{1}{\sqrt{2+3}}\right)(m)$$

$$P_5[J_G(n,m)] = \left(\frac{nm}{2}\right)-\left(\frac{2m}{2}\right)+\left(\frac{2m}{\sqrt{5}}\right)+\left(\frac{2m}{\sqrt{6}}\right)$$

$$P_5[J_G(n,m)] = \frac{1}{2}(nm-2m)+\frac{m}{\sqrt{6}}+\frac{2m}{\sqrt{5}}$$

$$P_5[J_G(n,m)] = m\left(\frac{n}{2}+\frac{1}{\sqrt{6}}+\frac{2}{\sqrt{5}}-1\right)$$

Hence, the final result of the Multiplicative degree based topological invariants of sum connectivity of Jahangir graph is equal to

$$P_5[J_G(n,m)] = \frac{m\left(\sqrt{30}n-2\left(\sqrt{5}\left(\sqrt{6}-1\right)-2\sqrt{6}\right)\right)}{2\sqrt{30}}$$

Theorem:6

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of Randic index is given by

$$P_6[J_G(n,m)] = m\left(\frac{n}{2}+\sqrt{\frac{2}{3}}-\frac{2}{3}\right)$$

Proof:

The Multiplicative invariants of Randic index is denoted by

$$P_6[J_G(n,m)] = \prod_e\left(\frac{1}{\sqrt{ab}}\right)$$

Now, this theorem have to derived by the results of the Multiplicative invariants of Randic index is given below :

$$P_6[J_G(n,m)] = \prod_{e\in E(G)}\left(\frac{1}{\sqrt{ab}}\right)$$

$$P_6[J_G(n,m)] = \left(\frac{1}{\sqrt{ab}}\right)\{(n-2)m\}+\left(\frac{1}{\sqrt{ab}}\right)\{2m\}+\left(\frac{1}{\sqrt{ab}}\right)\{m\}$$

$$P_6[J_G(n,m)] = \left(\frac{1}{\sqrt{2\times 2}}\right)(nm-2m)+\left(\frac{1}{\sqrt{2\times 3}}\right)(2m)+\left(\frac{1}{\sqrt{3\times 3}}\right)(m)$$

$$P_6[J_G(n,m)] = \frac{nm}{2}-\frac{2m}{2}+\frac{2m}{\sqrt{6}}+\frac{m}{3}$$

$$P_6[J_G(n,m)] = \frac{1}{2}(nm-2m)+\sqrt{\frac{2}{3}}m+\frac{m}{3}$$

$$P_6[J_G(n,m)] = \frac{1}{6}m\left(3n+2\sqrt{6}-4\right)$$

$$P_6[J_G(n,m)] = \frac{1}{6}\left(3n+2\sqrt{6}-2\right)$$

Hence, the final result of the Multiplicative degree based topological invariants of Randic invariants of Jahangir graph is equal to

$$P_6[J_G(n,m)] = m\left(\frac{n}{2}+\sqrt{\frac{2}{3}}-\frac{2}{3}\right)$$

Theorem:7

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Generalized Multiplicative invariants of first Zagreb is given by

$$P_7[J_G(n,m)] = 2^{2\beta}(nm-2m)+5^{\beta}(2m)+6^{\beta}m$$

Proof:

The Generalized Multiplicative invariants of first Zagreb is denoted by $P_7[J_G(n,m)] = \prod_e (a+b)^{\beta}$

Now, this theorem have to derived by the results of the Generalized Multiplicative invariants of first zagreb index is given below :

$$P_7[J_G(n,m)] = \prod_{e\in E(G)}(i+j)^{\beta}$$

$$P_7[J_G(n,m)] = (a+b)^{\beta}\{(n-2)m\}+(a+b)^{\beta}\{2m\}+(a+b)^{\beta}\{m\}$$

$$P_7[J_G(n,m)] = (2+2)^{\beta}(nm-2m)+(2+3)^{\beta}(2m)+(3+3)^{\beta}(m)$$

DOI : 10.18231/2454-9150.2018.1106

$$P_7[J_G(n,m)] = 4^\beta(nm-2m)+5^\beta(2m)+6^\beta(m)$$
$$P_7[J_G(n,m)] = 4^\beta nm-8^\beta m+5^\beta(2m)+6^\beta m$$
$$P_7[J_G(n,m)] = 4^\beta(nm-2m)+5^\beta(2m)+6^\beta(m)$$

Hence, the final result of the Multiplicative degree based topological invariants of Generalized first Zagreb of Jahangir graph is equal to

$$P_7[J_G(n,m)] = 2^{2\beta}(nm-2m)+5^\beta(2m)+6^\beta m$$

Theorem:8

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Generalized Multiplicative invariants of second Zagreb is given by

$$P_8[J_G(n,m)] = 2^{2\beta}(mn-2m)+3^{2\beta}(m)+6^\beta m$$

Proof:

The Generalized Multiplicative invariants of second Zagreb is denoted by $P_8[J_G(n,m)] = \prod_e (ab)^\beta$

Now, this theorem have to derived by the results of the Generalized Multiplicative invariants of second Zagreb index is given below :

$$P_8[J_G(n,m)] = \prod_{e\in E(G)} (ab)^\beta$$

$$P_8[J_G(n,m)] = (ab)^\beta\{(n-2)m\}+(ab)^\beta\{2m\}+(ab)^\beta\{m\}$$

$$P_8[J_G(n,m)] = (2\times2)^\beta(nm-2m)+(2\times3)^\beta(2m)+(3\times3)^\beta\{m\}$$
$$P_8[J_G(n,m)] = 4^\beta(nm-2m)+6^\beta(2m)+9^\beta m$$
$$P_8[J_G(n,m)] = 4^\beta nm-8^\beta m+12^\beta m+9^\beta m$$
$$P_8[J_G(n,m)] = 4^\beta m(n-2)+6^\beta(2m)+9^\beta m$$

Hence, the final result of the Multiplicative degree based topological invariants of Generalized second Zagreb of Jahangir graph is equal to

$$P_8[J_G(n,m)] = 2^{2\beta}(mn-2m)+3^{2\beta}(m)+6^\beta m$$

Theorem:9

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of Atom bond connectivity is given by $P_9[J_G(n,m)] = m\left(\frac{n}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\sqrt{2}+\frac{2}{3}\right)$

Proof:

The Multiplicative invariants of Atom bond connectivity is denoted by

$$P_9[J_G(n,m)] = \prod_{e\in E(G)}\sqrt{\frac{a+b-2}{ab}}$$

Now, this theorem have to derived by the results of the Multiplicative invariants of Atom – Bond connectivity index is given below :

$$P_9[J_G(n,m)] =$$

$$\left(\sqrt{\frac{a+b-2}{ab}}\right)\{(n-2)m\}+\left(\sqrt{\frac{a+b-2}{ab}}\right)\{2m\}+\left(\sqrt{\frac{a+b-2}{ab}}\right)\{m\}$$

$$P_9[J_G(n,m)] =$$
$$\left(\sqrt{\frac{2+2-2}{2\times2}}\right)(nm-2m)+\left(\sqrt{\frac{2+2-2}{2\times2}}\right)(2m)+\left(\sqrt{\frac{2+2-2}{2\times2}}\right)(m)$$

$$P_9[J_G(n,m)] = \frac{\sqrt{2}}{2}(nm-2m)+\frac{\sqrt{3}}{6}(2m)+\frac{2}{3}m$$

$$P_9[J_G(n,m)] \frac{nm-2m}{\sqrt{2}}+\frac{m}{\sqrt{3}}+\frac{2m}{3}$$

$$P_9[J_G(n,m)] = \frac{nm-2m}{\sqrt{2}}+\frac{m}{\sqrt{3}}+\frac{2m}{3}$$

Hence, the final result of the Multiplicative degree based topological invariants of Atom-Bond Connectivity invariants of Jahangir graph is equal to

$$P_9[J_G(n,m)] = m\left(\frac{n}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\sqrt{2}+\frac{2}{3}\right)$$

Theorem:10

Let a graph G consists of a pair (V,E) and a cycle graph $C_G(n,m)$ if it is a Jahangir Graph defined $J_G(n,m)$ consisting of a cycle $C_G(n,m)$ with one additional vertex which is adjacent to m vertices of $C_G(n,m)$ at distance n to each other on $C_G(n,m)$, then the Multiplicative invariants of Geometric – Arithmetic is given by $P_{10}[J_G(n,m)] = mn+\left(\frac{4\sqrt{6}}{5}-1\right)m$

Proof:

The Multiplicative invariants of Geometric – Arithmetic is denoted by $P_{10}[J_G(n,m)] = \prod_{e\in E(G)}\left(\frac{2\sqrt{ab}}{a+b}\right)$

 DOI : 10.18231/2454-9150.2018.1106

Now, this theorem have to derived by the results of the Multiplicative invariants of Geometric - Arithmetic index is given below :

$$P_{10}[J_G(n,m)] = \prod_{e \in E(G)} \left(\frac{2\sqrt{ab}}{a+b} \right)$$

$$P_{10}[J_G(n,m)] = \left(\frac{2\sqrt{ab}}{a+b} \right)\{(n-2)m\} + \left(\frac{2\sqrt{ab}}{a+b} \right)(2m) + \left(\frac{2\sqrt{ab}}{a+b} \right)\{m\}$$

$$P_{10}[J_G(n,m)] = \left(\frac{2\sqrt{2\times 2}}{2+2} \right)(nm-2m) + \left(\frac{2\sqrt{2\times 3}}{2+3} \right)(2m) + \left(\frac{2\sqrt{3\times 3}}{3+3} \right)(m)$$

$$P_{10}[J_G(n,m)] = \frac{4}{4}\left(nm-2m+\frac{2\sqrt{6}}{5}(2m)+\frac{6}{6}m \right)$$

$$P_{10}[J_G(n,m)] = nm-2m+\frac{2\sqrt{6}}{5}(2m)+m$$

$$P_{10}[J_G(n,m)] = nm+\frac{4\sqrt{6}}{5}m-m$$

$$P_{10}[J_G(n,m)] = nm+\frac{4\sqrt{6}}{5}m-m$$

$$P_{10}[J_G(n,m)] = \frac{1}{5}m\left(5n+4\sqrt{6}-5\right)$$

$$P_{10}[J_G(n,m)] = m\left(n+\frac{4\sqrt{6}}{5}-1 \right)$$

Hence, the final result of the Multiplicative degree based topological invariants of Geometric-Arithmetic invariants of Jahangir graph is equal to

$$P_{10}[J_G(n,m)] = mn+\left(\frac{4\sqrt{6}}{5}-1 \right)m$$

THE RESULTS OF MULTIPLICATIVE INVARIANTS OF DEGREE BASED TOPOLOGICAL INDICES OF JAHANGIR GRAPH $J_G(n,m)$.

Topological Index	Formulas obtained
The product of first Zagreb	$P_1[J_G(n,m)] = m(4n+8)$
The Product of second Zagreb	$P_2[J_G(n,m)] = 4mn+13m$
The Product of square of first Zagreb	$P_3[J_G(n,m)] = m(16n+54)$
The Product of square of second Zagreb	$P_4[J_G(n,m)] = m(16n+121)$
The Product of sum connectivity index	$P_5[J_G(n,m)] = \frac{m\left(\sqrt{30}n-2\left(\sqrt{5}\left(\sqrt{6}-1\right)-2\sqrt{6}\right)\right)}{2\sqrt{30}}$
The Product of Randic index	$P_6[J_G(n,m)] = m\left(\frac{n}{2}+\sqrt{\frac{2}{3}}-\frac{2}{3}\right)$
The Generalized product of first Zagreb	$P_7[J_G(n,m)] = 2^{2\beta}(nm-2m)+5^{\beta}(2m)+6^{\beta}m$
The Generalized product of second Zagreb	$P_8[J_G(n,m)] = 2^{2\beta}(mn-2m)+3^{2\beta}(m)+6^{\beta}m$
The Product of Atom bond connectivity	$P_9[J_G(n,m)] = m\left(\frac{n}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\sqrt{2}+\frac{2}{3}\right)$
The Product of Geometric – Arithmetic index	$P_{10}[J_G(n,m)] = mn+\left(\frac{4\sqrt{6}}{5}-1\right)m$

Conclusion:

In this article, we computed closed forms of Multiplicative invariants of degree based topological indices of $J_G(n,m)$ for all $n \geq 2$, $m \geq 3$. These results will also play a vital role in industries and pharmacy in the realm of that molecular graph which contains $J_G(n,m)$.

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