International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

A Study on Extreme Vertex Steiner Graphs

J. John, Department of Mathematics, Government College of Engineering, Tirunelveli, India. john@gcetly.ac.in

A. Arockiamary, Department of Mathematics, Government College of Engineering, Tirunelveli, India. maryjoseph6468@gmail.com

Abstract - Let S_e be the set of all extreme vertices of G. A x-Steiner set S of G is called an extreme x- Steiner set of G if $S = \begin{cases} S_e \text{ for } x \notin S_e \\ S_e - \{x\} \text{ for } x \in S_e \end{cases}$. A graph G is called an extreme x-Steiner graph if there exists a vertex x in G such that x has an extreme x- Steiner set of G. Some general properties satisfied by these concepts are studied. The extreme vertex Steiner number of some standard graphs are Obtained. For every pair a, b of integers with $2 < a \leq b$, there exists a connected graph G with $Ext(G) = a$ and $s_x(G) = b$ for a vertex x in G.

Key words: Steiner distance, Steiner number, vertex Steiner number, monophonic number, vertex monophonic number, geodetic number, vertex geodetic number,

AMS Subject classification: 05C12

I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. An $u - v$path of length $d(u, v)$ is called an $u - v$ *geodesic*. It is known that the *distance*is a metric on the vertex set of G. For a vertex $v$of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of G is the radius, and denoted by $radG$ and the maximum eccentricity is its diameter, and denoted by $diamG$ of G.

For basic graph theoretic terminology, we refer to Harary [2]. For a nonempty set $W$of vertices in a connected graph G, the *Steiner distance* $d(W)$ of W is the minimum size of a connected subgraph of G containing W. Necessarily, each such subgraph is a tree and is called a *Steiner tree* with respect to W or a Steiner $W - tree$. It is to be noted that $d(W) = d(u, v)$when $W = \{u, v\}$. If v is an end vertex of a Steiner $W - tree$, then $v \in W$. Also if $< W >$ is connected, then any Steiner W-tree contains the elements of W only. The Steiner distance of a graph is introduced in [6]. The set of all vertices of G that lie on some *Steiner* $W - tree$ is denoted by $S(W)$. If $S(W) = V$, then W is called a *Steiner set* of G. A *Steiner set* of minimum cardinality is a minimum *Steiner set* or simply a $s - set$ of G and this cardinality is the *Steiner number* $s(G)$ of G. If W is a *Steiner set* of G and v a cut vertex of G, then v lies in every *Steiner* $W - tree$ of G and so $W \cup \{v\}$ is also a *Steiner set* of G. The Steiner number of a graph was introduced in [7] and further studied in [3,4,8,9,10,12]. A vertex v is a simplicial vertex of a graph G if the subgraph inducedby its neighbors is complete. Let x be a vertex of a connected graph G and $W \subset V(G)$ such that $x \notin W$. Then W is called an x *-Steiner set* of G if every vertex of G lies on some Steiner $W \cup \{x\}$ -tree of G. The minimum cardinality of an x- Steiner set of G is defined as the x - *Steiner number* of G and denoted by $s_x(G)$. Any x- Steiner set of cardinality $s_x(G)$ is called an s_x *-set* of G. Let x be a vertex of a connected graph G and $S \subset V(G)$ such that $x \notin S$. Then S is called an x - *geodetic set* of G if every vertex of G lies on some $x - y$ geodesic, where $y \epsilon S$. The minimum cardinality of an x - geodetic set of G is defined as x - *geodetic number* of G and denoted by $g_x(G)$. Any x - geodetic set of cardinality $g_x(G)$ is called a g_x *-set* of G.The definition of x-geodetic set can also be defined as follows. Let $S \subset V(G)$ and $x \in V$ such that $x \notin S$. Let $I_x[y]$ be the set of all vertices that lies in $x - y$ geodesic including x and y, where $y \in S$ and $I_x[S] = \cup_{y \in S} I_x[y]$. Then S is said to be an x-geodetic set of G, if $I_x[S] = V$. Let x be a vertex of a connected graph G and $M \subset V(G)$ such that $x \notin M$. Then M is called an x - *monophonic set* of G if every vertex of G lies on some $x - y$ monophonic path, where $y \epsilon M$. The minimum cardinality of an x - monophonic set of G is defined as x - *monophonic number* of G and denoted by $m_x(G)$. Any x - monophonic set of cardinality $m_x(G)$ is called a m_x *-set* of G. The definition of x-monophonic set can also be defined as follows. Let $M \subset V(G)$ and $x \in V$ such that $x \notin M$. $J_x[y]$ is the set of all vertices that lies in $x - y$ monophonic path including x and y, where $y \in M$ and $J_x[M] = \cup_{y \in M} J_x[y]$. Then M is said to be an x-monophonic set of G, if $J_x[M] = V$.

 DOI : 10.18231/2454-9150.2018.1096

Throughout the following G denotes a connected graph.
The following theorems are used in the sequel.

Theorem 1.1 [20] (i) Every simplicial vertex of G other than the vertex x (whether x is simplicial or not) belongs to every x - geodetic set for any vertex x in G.

(ii) No cut-vertex of a connected graph G lies in a minimum x- geodetic set of G. (whether x is a cut vertex or not)

Theorem 1.2[22]. (i) Every simplicial vertex of G other than the vertex x (whether x is simplicial or not) belongs to every x - monophonic set for any vertex x in G.

(ii) No cut-vertex of a connected graph G lies in a minimum x- monophonic set of G. (whether x is a cut vertex or not)

Theorem 1.3[22]. For the complete graph K_p $(p \geq 2)$, $m_x(K_p) = p-1$ for every vertex x.

Theorem 1.4[22]. For the nontrivial tree T with k end vertices, $m_x(T) = \begin{cases} k \text{ if } x \text{ is a cut vertex of } G \\ k-1 \text{ if } x \text{ is an end vertex of } G \end{cases}$

Theorem 1.5. [12] Every simplicial vertex of G other than the vertex x (whether x is simplicial or not) belongs to every x- Steiner set for any vertex x in G.

Theorem 1.6. **[12]** No cut-vertex of a connected graph G belongs to any minimum x -Steiner set of G.

Theorem 1.7. **[12]** For the complete graph $K_p(p \geq 2)$, $s_x(K_p) = p-1$ for every vertex x.

Theorem 1.8 [12] For the nontrivial tree T with k end vertices,

$$s_x(T) = \begin{cases} k \text{ if } x \text{ is a cut vertex of } G \\ k-1 \text{ if } x \text{ is an end vertex of } G \end{cases}$$

Theorem 1.9. Every vertex Steiner set of a connected graph $G = (V, E)$ is a vertex monophonic set of x of G.

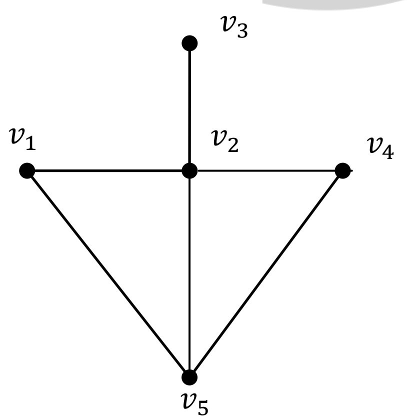
Theorem 1.10. Every vertex geodetic set of a connected graph $G = (V, E)$ is a vertex monophonic set of x of G.

Theorem 1.11. Let x be a vertex of an extreme x- geodesic graph G. Then $g_x(G) = 1$ if and only if there exist only one antipodal extreme vertex y of x such that every vertex of G is on a diametral path joining x and y.

II. EXTREME VERTEX STEINER GRAPHS

Definition 2.1. Let S_e be the set of all extreme vertices of G. A x- Steiner set S of G is called an extreme x- Steiner set of G if $S = \begin{cases} S_e \text{ for } x \notin S_e \\ S_e - \{x\} \text{ for } x \in S_e \end{cases}$. A graph G is called an extreme x- Steiner graph if there exists a vertex x in G such that x has an extreme x- Steiner set of G.

Example 2.2. For the graph G given in Figure 2.1, $S_e = \{v_1, v_3, v_4\}$ is the set of extreme vertices of G so that $Ext(G) = 3$. For the vertex $x = v_5$, $S = S_e$ is a minimum x- Steiner set of G so that $s_x(G) = Ext(G) = 3$. Therefore, G is an extreme x- Steiner graph.



G Figure 2.1

For the complete graph $G = K_p(p \geq 2)$, every vertex is an extreme vertex so that $Ext(G) = p$. By Theorem 1.7, $s_x(G) = p-1$ for every vertex x in G. Thus K_p is an extreme x- Steiner graph. Similarly, for any nontrivial tree with k end vertices, $Ext(G) = k$ and by Theorem 1.8, $s_x(G) = k$ or $k-1$ for every vertex x in G. Thus any non trivial tree is an extreme x- Steiner graph. Since a cycle has no extreme vertices, a cycle is not an extreme x- Steiner graph. Also since a complete bipartite

 DOI : 10.18231/2454-9150.2018.1096

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

graph $G = K_{r,s} (2 \le r \le s)$ has no extreme vertices, a complete bipartite graph is not an extreme x- Steiner graph. We have the following results on an extreme x- Steiner graphs.

Theorem 2.3. Let G be a connected graph. Then $0 \le Ext(G) - 1 \le s_x(G)$ for any vertex x in G.

Proof. This follows from Theorem 1.5

Theorem 2.4. If every vertex of G is either a cut vertex or an extreme vertex, then G is an extreme x- Steiner graph **.**

Proof. This follows from Theorems 1.5 and 1.6 ■

Theorem 2.5. Let x be a vertex of an extreme x- geodesic graph G. Then $s_x(G) = 1$ if and only $g_x(G) = 1$ for some vertex x in G.

Proof. Let $g_x(G) = 1$. Then by Theorem 1.11 there exist only one antipodal extreme vertex y of x such that every vertex of G is on a diametral path joining x and y. Let $W = \{y\}$. Then every vertex of G lies on a Steiner W_x- tree of G. Since y is an extreme vertex of G, W is an extreme x-Steiner set of G. Therefore $s_x(G) = 1$. Conversely, let $s_x(G) = 1$ and let $W = \{y\}$ be an extreme x-Steiner set of G. Then every vertex of G lies on a Steiner W_x- tree of G. Since every Steiner W_x- tree of G is a $x- y$ geodesic, every vertex of G lies on a $x- y$ geodesic. Hence W is an extreme x-geodetic set of G. Therefore $g_x(G) = 1$. ■

Theorem 2.6. Let G be an extreme x- Steiner graph of order $p \ge 2$. Then x- Steiner number is $p - 1$ for all vertices x in G if and only if $G = K_p$.

Proof. This follows from Theorem 1.7 ■

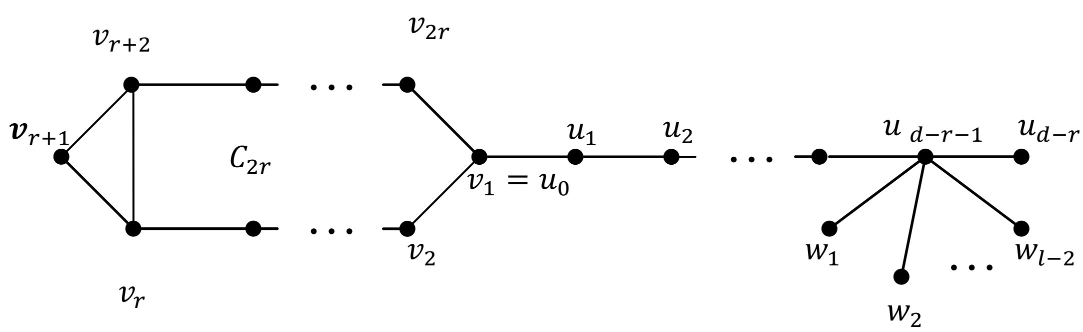
Theorem 2.7. Let x be a vertex of a nontrivial tree T of order p and diameter d, then $s_x(T) = p - d + 1$ or $p - d$ if and only if T is a caterpillar.

Proof. Let T be any nontrivial tree. Let P be a diametral path of length d. Let k be the number of end vertices of T and l the number of internal vertices of T other than the internal vertices of P. Then $d - 1 + l + k = p$. By Theorem 1.8, $s_x(T) = k$ or $k - 1$ for any vertex x in T. Hence $s_x(T) = p - d + 1$ or $p - d$ for any vertex x in T if and only if $l = 0$, if and only if all the internal vertices of T lie on the diametral path P, if and only if T is a caterpillar. ■

For every connected graph, $radG \le diamG \le 2\,radG$. Ostrand[23] showed that every two positive integers a and b with $a \le b \le 2a$ are realizable as the radius and diameter, respectively, of some connected graph. Now, Ostrand's theorem can be extended to an extreme x- Steiner graph so that the x- Steiner number can also be prescribed, when $a < d \le 2a$.

Theorem 2.8. For positive integers r, d and $l \ge 2$ with $r < d \le 2r$, there exists an extreme x- Steiner graph G with $radG = r, diamG = d$ and $s_x(G) = Ext(G) = l$ for some vertex x in G.

Proof. When $r = 1$, let $G = K_{1,l}$. Then $d = 2$ and by Corollary 1.8, $s_x(G) = l = Ext(G)$ for the cut vertex x in G and G is an extreme x- Steiner graph. Now, let $r \ge 2$. Construct a graph G with the desired properties as follows. Let $C_{2r}: v_1, v_2, \dots, v_{2r}, v_1$ be a cycle of order $2r$ and let $P_{d-r+1} : u_0, u_1, u_2, \dots, u_{d-r}$ be a path of order $d - r + 1$. Let H be the graph obtained from C_{2r} and P_{d-r+1} by identifying v_1 in C_{2r} and u_0 in P_{d-r+1}. Now, add $(l - 2)$ new vertices $w_1, w_2, \dots, w_{l-2}$ to H and join each vertex $w_i (1 \le i \le l - 2)$ to the vertex u_{d-r-1} and join the vertices v_r and v_{r+2} and obtain the graph G of Figure 2.2. Then $radG = r$ and $diamG = d$. Let $x = u_0$. Let $S_e = \{v_{r+1}, w_1, w_2, \dots, w_{l-2}, u_{d-r}\}$ be the set of l extreme vertices of G. By Theorem 1.5, S_e is a subset of every x- Steiner set of G. It is clear that S_e is an x- Steiner set and it follows from Theorem 3.4 that $s_x(G) = Ext(G) = l$ and G is an extreme x- Steiner graph. ■



G Figure 2.2

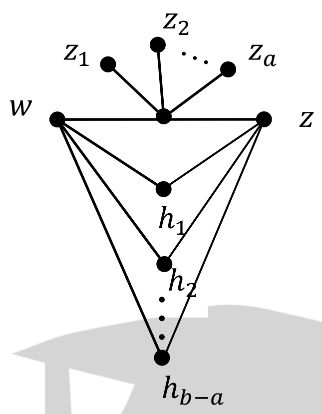
In view of Theorem 2.3 , we have the following realization result.

 DOI : 10.18231/2454-9150.2018.1096

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

Theorem 2.9. For every pair a, b of integers with $2 < a \leq b$, there exists a connected graph G with $Ext(G) = a$ and $s_x(G) = b$ for a vertex x in G.

Proof. Let $P: w, y, z$ be a path on three vertices. Add b new vertices $z_1, z_2, \ldots, z_a$ and $h_1, h_2, \ldots, h_{b-a}$ and join each z_i $(1 \leq i \leq a)$ with y, and join each $h_i (1 \leq i \leq b-a)$ with both w and z in P, there by obtaining the graph G of Figure 2.3. Let $S_e = \{z_1, z_2, \ldots, z_a\}$ be the set of end vertices of G. Then $Ext(G) = a$. Next we show that $s_x(G) = b$. Let S be a s_x-set of G. Let $x = w$. By Theorem 1.5, $S_e \sqsubseteq S$. It is clear that S_e is not a s_x −set of G. We show that each $h_i \in S$ $(1 \leq i \leq b-a.$). Suppose that $h_i \notin S$ for some $i(1 \leq i \leq b-a)$. Then it is clear that h_i does not lie on any Steiner tree joining x and a vertex of S, which is a contradiction. Therefore, each $h_i \in S$ $(1 \leq i \leq b-a)$ and so $s_x(G) \geq a + b - a = b$. Since $S = S_e \cup \{h_1, h_2, \ldots, h_{b-a}\}$ is a s_x-set of G so that $s_x(G) = b$. ■



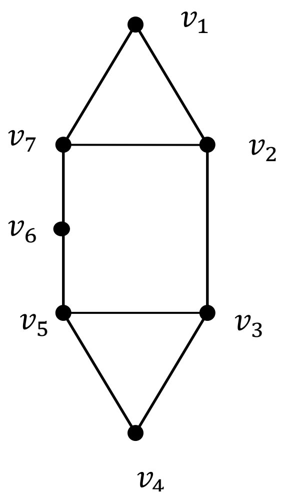
G Figure 2.3

III. EXTREME VERTEX MONOPHONIC GRAPHS

Definition 3.1. Let S_e be the set of all extreme vertices of G. A x- monophonic set S of G is called an extreme x-monophonic set of G if $S = \begin{cases} S_e & \text{for } x \notin S_e \\ S_e - \{x\} & \text{for } x \in S_e \end{cases}$. A graph G is called an extreme x- monophonic graph if there exists a vertex x in G such that x has an extreme x- monophonic set of G.

Example 3.2. For the graph G given in Figure 5.7, $S_e = \{v_1, v_4\}$ is the set of extreme vertices of G so that $Ext(G) = 2$. For the vertex $x = v_6$, $S = S_e$ is a minimum x- monophonic set of G so that $m_x(G) = Ext(G) = 2$. Also for $x = v_1$,

$S = S_e - \{v_1\}$ is a m_x set of G, so that $m_x(G) = Ext(G) - 1$. Therefore, G is an extreme x- monophonic graph.



G Figure 3.1

For the complete graph $G = K_p (p \geq 2)$, every vertex is an extreme vertex so that $Ext(G) = p$. By Theorem 1.3, $m_x(G) = p - 1$ for every vertex x in G. Thus K_p is an extreme x- monophonic graph. Similarly, for any nontrivial tree with

 DOI : 10.18231/2454-9150.2018.1096

k end vertices, $Ext(G) = k$ and by Theorem 1.4, $m_x(G) = k$ or $k-1$ for every vertex x in G. Thus any non trivial tree is an extreme x- monophonic graph. Since a cycle has no extreme vertices, a cycle is not an extreme x- monophonic graph. Also since a complete bipartite graph $G = K_{r,s}(2 \leq r \leq s)$ has no extreme vertices, a complete bipartite graph is not an extreme x- monophonic graph. We have the following results on an extreme x- monophonic graphs.

Theorem 3.3. Let G be a connected graph. Then $0 \leq Ext(G) - 1 \leq m_x(G)$ for any vertex x in G.

Proof. This follows from Theorem 1.2 ∎

Theorem 3.4. If every vertex of G is either a cut vertex or an extreme vertex, then G is an extreme x- monophonic graph.

Proof. This follows from Theorems 1.2 (i) and (ii) ∎

Definition 3.5. Let x be any vertex in G. A vertex y in G is said to be an x-monophonic superior vertex if for any vertex z with $d_m(x,y) < d_m(x,z)$, z lies on an x - y monophonic path.

Theorem 3.6. For a vertex x in a graph G, $m_x(G) = 1$ if and only if there exists an x-monophonic superior extreme vertex y in G such that every vertex of G is on an x - y monophonic path.

Proof. Let $m_x(G) = 1$ and let $S_x = \{y\}$ be a m_x-set of G. If y is not an x-monophonic superior extreme vertex, then there is a vertex z in G with $d_m(x,y) < d_m(x,z)$ and z does not lie on any x - y monophonic path. Thus S_x is not a m_x-set of G, which is a contradiction. The converse is clear from the definition. ∎

Theorem 3.7. Let G be an extreme x- monophonic graph G of order $p \geq 2$. Then x- monophonic number is $p-1$ for all vertices x in G if and only if $G = K_p$

Proof. This follows from Theorems 1.2 and 1.3 ∎

Theorem 3.8. Let x be a vertex of a nontrivial tree T of order p and monophonic diametral path d, then $m_x(T) = p - d_m + 1$ or $p - d_m$ if and only if T is a caterpillar.

Proof. Let T be any nontrivial tree. Let P be a monophonic diametral path of length d_m. Let k be the number of end vertices of T and l the number of internal vertices of T other than the internal vertices of P. Then $d_m - 1 + l + k = p$. By Theorem 1.4, $m_x(T) = k$ or $k-1$ for any vertex x in T. Hence $m_x(T) = p - d_m + 1$ or $p - d_m$ for any vertex x in T if and only if $l = 0$, if and only if all the internal vertices of T lie on the monophonic diametral path P, if and only if T is a caterpillar.

Theorem 3.9. Let G be an extreme x- geodesic graph G of order $p \geq 2$. Then G is an extreme x- monophonic graph.

Proof. Let G be an extreme x- geodesic graph G and x be an vertex of G. Then there exists an extreme x- geodetic set Z such that $I_x(Z) = V$. By Theorem 1.10, Z is an extreme x-monophonic set of G such that $J_x(Z) = V$. Therefore G is an extreme x-monophonic graph. ∎

Theorem 3.10. Let G be an extreme x- Steiner graph G of order $p \geq 2$. Then G is an extreme x- monophonic graph.

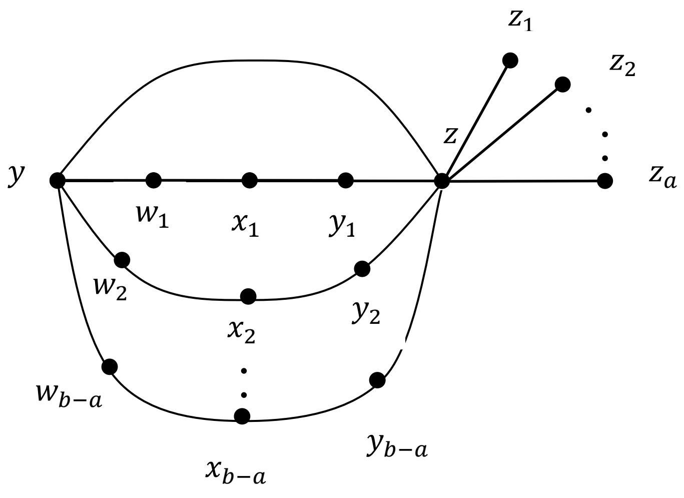
Proof. Let G be an extreme x- Steiner graph and x a vertex of G. Then there exists an extreme x-Steiner set Z such that $S_x(Z) = V$. By Theorem 1.9, Z is an extreme x-monophonic set of G such that $J_x(Z) = V$. Therefore G is an extreme x-monophonic graph. ∎

In view of Theorem 3.3, we have the following realization result.

Theorem 3.11. For every pair a, b of integers with $2 < a \leq b$, there exists a connected graph G with $Ext(G) = a$ and $m_x(G) = b$ for a vertex x in G.

Proof. Let $P_i: w_i, x_i, y_i (1 \leq i \leq b-a)$ be a copy of path on three vertices. Let H be the graph obtained from $P_i (1 \leq i \leq b-a)$ by adding new vertices y and z and joining each w_i $(1 \leq i \leq b-a)$ with y and each $y_i (1 \leq i \leq b-a)$ with z and joining y with z. Let G be the graph obtained from H by adding new vertices $z_1, z_2, \ldots, z_a$ and joining each z_i $(1 \leq i \leq a)$ with z. The graph G is given in Figure 5.8. Let $x = y$ and let $Z = \{z_1, z_2, \ldots, z_a\}$ be the set of extreme vertices of G such that $Ext(G) = a$. Since $J_x(Z) \neq V$, Z is not a vertex monophonic set of G. Let $H_i = \{x_i, y_i\}(1 \leq i \leq b-a)$. It is easily observed that every m_x -set of G must contain at least one vertex from each H_i $(1 \leq i \leq b-a)$ and so $m_x(G) \geq b - a + a = b$. Let $M = Z \cup \{x_1, x_2, \ldots, x_{b-a}\}$. Then $J_x[M] = V$, so that $m_x(G) = b$. ∎

 DOI : 10.18231/2454-9150.2018.1096

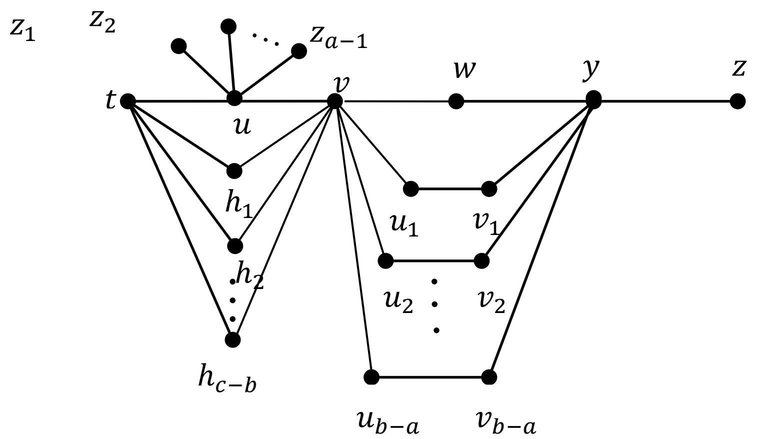


G Figure 3.2

Theroem 3.12. For every three integers a, b, c with $2 \leq a \leq b \leq c$, there exists an extreme x- monophonic graph G which is neither extreme x- geodesic graph nor extreme x- Steiner graph such that $m_x(G) = a$, $g_x(G) = b$ and $s_x(G) = c$ for a vertex x in G.

Proof. Let $P: t, u, v, w, y, z$ be a path on six vertices. Let $P_i: u_i, v_i (1 \leq i \leq b-a)$ be a copy of path on two vertices. Let H be the graph obtained from P_i by joining each u_i $(1 \leq i \leq b-a)$ with v and each $v_i (1 \leq i \leq b-a)$ with y. Let G be the graph obtained from P and H by adding new vertices $z_1, z_2, \dots, z_{a-1}$ and $h_1, h_2, \dots, h_{c-b}$ by joining each h_i $(1 \leq i \leq c-b)$ with t and v and each z_i $(1 \leq i \leq a-1)$ with u. The graph G is given in Figure 3.3. Let $S_e = \{z_1, z_2, \dots, z_{a-1}, z\}$ be the set of extreme vertices of G. Then $Ext(G) = a$. Let $x = t$. First we show that $m_x(G) = a$. Let M be m_x - set of G. Then by Theorem 1.72, $S_e \subseteq M$. It is clear that S_e is a m_x – set of G so that $m_x(G) = a$. Let Z be a x- geodetic set of G. Then by Theorem 1.1, $S_e \subseteq Z$ and so $g_x(G) \geq a$. It is clear that S_e is not an x- geodetic set. We observe that every g_x-set must contain v_i $(1 \leq i \leq b-a)$ and so $g_x(G) \geq a + b - a = b$. Now $Z = S_e \cup \{v_1, v_2, \dots, v_{b-a}\}$ is an x- geodetic set of G so that $g_x(G) = b$. Next we show that $s_x(G) = c$. Let W be a vertex Steiner set of x of G. Then by Theorem 1.5, $S_e \subseteq W$. It is clear that S_e is not an x- Steiner set of G. For $1 \leq i \leq b-a$, let $H_i = \{u_i, v_i\}$. We observe that s_x -set of G must contain atleast one vertex from H_i $(1 \leq i \leq b-a)$ and so $s_x(G) \geq a + b - a = b$. It is easily observed that if the vertex h_i $(1 \leq i \leq c-b)$ does not belong to W, then vertex h_i $(1 \leq i \leq c-b)$ does not lie on

any Steiner W_x -tree of G and so $s_x(G) \geq c - b + b = c$. Let $Z = S_e \cup \{v_1, v_2, \dots, v_{b-a}\} \cup \{h_1, h_2, \dots, h_{c-b}\}$. Since $S_x(W) = V$, W is the unique minimum x- Steiner set of G and so $s_x(G) = c$. ■



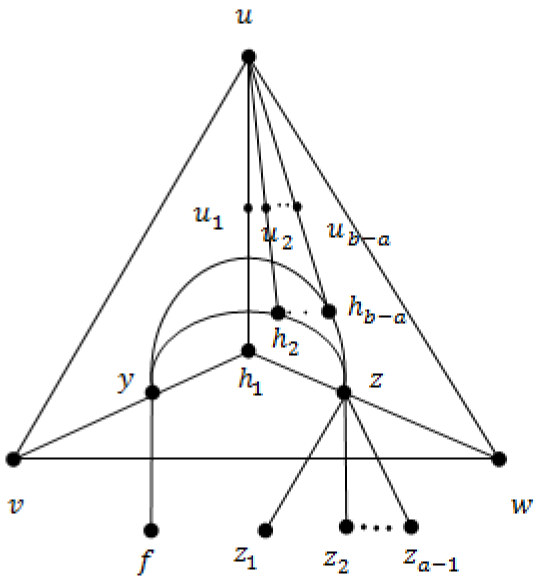
G Figure 3.3

 DOI : 10.18231/2454-9150.2018.1096

Theroem 3.13. For every pair of integers a, b with $2 \leq a \leq b$, there exists an extreme x- monophonic graph G which is also an extreme x- Steiner graph but not an extreme x- geodesic graph such that $m_x(G) = s_x(G) = a$ and $g_x(G) = b$ for a vertex x in G.

Proof. Let C_3 be u, v, w, u. Let H be the graph obtained from C_3 by adding new vertices $y, z, u_1, u_2, \dots, u_{b-a}, h_1, h_2, \dots, h_{b-a}$ by joining each h_i $(1 \leq i \leq b-a)$ with y and z, y with v and z, z with w. Also join u_i with $h_i(1 \leq i \leq b-a)$ and u_i with u. Let G be the graph obtained from H by adding new vertices $z_1, z_2, \dots, z_{a-1}, f$ by joining f with y and each $z_i(1 \leq i \leq a-1)$ with

z. The graph G is given in Figure 3.4. Let $x = u$. First we show that G is an extreme x- Steiner graph. Let $Z = \{f, z_1, z_2, \dots, z_{a-1}\}$ be the set of extreme vertices of G so that $Ext(G) = a$. Then by Theorem 1.5, Z is a subset of every x- Steiner set of G. Since $S_x(Z) = V$, $s_x(G) = a = Ext(G)$. Therefore G is an extreme x- Steiner graph. By Theorem 3.10, G is an extreme x- monophonic graph so that $m_x(G) = a = Ext(G)$. Next we show that G is not an extreme x- geodesic graph. By Theorem 1.1, Z is a subset of every x- geodetic set of G. Since $I_x(Z) \neq V$, Z is not an x- geodetic set of G so that G is not an extreme x- geodesic graph. It is easily observed that every x- geodetic set contains each h_i $(1 \leq i \leq b-a)$ and so $g_x(G) \geq a + b - a = b$. Let $S = Z \cup \{h_1, h_2, \dots, h_{b-a}\}$. Then $I_x(S) = V$ so that $g_x(G) = b$.



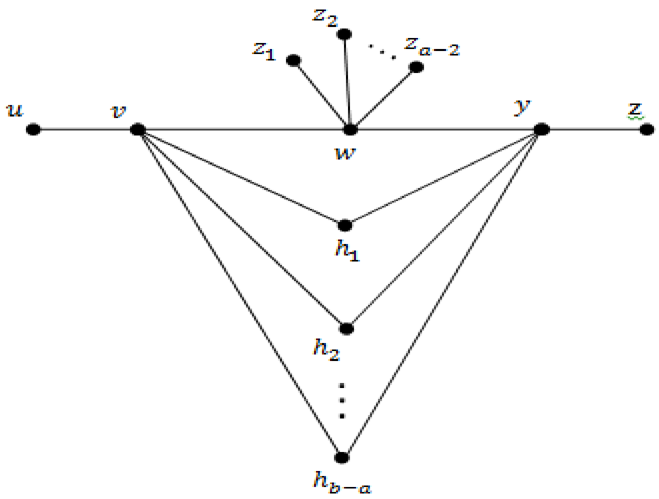
G Figure 3.4

Theroem 3.14. For every pair of integers a, b with $2 \leq a \leq b$, there exists an extreme x- monophonic graph G which is also an extreme x- geodesic graph but not an extreme x- Steiner graph such that $m_x(G) = g_x(G) = a$ and $s_x(G) = b$ for a vertex x in G.

Proof. Let $P: u, v, w, y, z$ be a path on five vertices. Add new vertices $z_1, z_1, \dots, z_{a-2}$, $h_1, h_2, \dots, h_{b-a}$ and join each $z_i(1 \leq i \leq a-2)$ to w, and join each $h_i(1 \leq i \leq b-a)$ with v and y there by obtaining the graph G of Figure 3.5. Let $x = w$. First we show that G is an extreme x- geodesic graph and extreme x- monophonic graph. Let $Z = \{u, z, z_1, z_1, \dots, z_{a-2}\}$ be the set of all extreme vertices of G so that $Ext(G) = a$. Then by Theorem 1.67, Z is a subset of every vertex geodetic set of x of G. Since $I_x[Z] = V$, Z is an extreme x- geodetic set of G and so that $g_x(G) = a = Ext(G)$.

Therefore G is an extreme x- geodesic graph. By Theorem 1.8, G is an extreme x- monophonic graph so that $m_x(G) = a = Ext(G)$. Next we show that G is not an extreme x- Steiner graph such that $s_x(G) = b$. By Theorem 1.5, Z is a subset of every x- Steiner set of G. Since $S_x(Z) \neq V$, Z is not an extreme x- Steiner set of G, so that G is not an extreme x- Steiner graph. It is easily observed that every x- Steiner set of G contains each h_i $(1 \leq i \leq b-a)$. Let $W = Z \cup \{h_1, h_2, \dots, h_{b-a}\}$. Then $S_x(W) = V$, so that W is not an extreme x- Steiner set of G such that $s_x(G) = b$.

 DOI : 10.18231/2454-9150.2018.1096



G Figure 3.5

IV. CONCLUSION

Theory of Extreme Vertex Steiner Graphs is one of the potential areas of research. Many domination models in Extreme Vertex Steiner Graphs are available in the literature. The Steiner graphs are one such model which depends upon steiner distance in graphs.

REFERENCES

[1] F. Buckley, F. Harary and L.V. Quintas, Extremal results on the Geodetic Number of a graph, *Scientia A*2 (1988) 17–26
[2] F. Buckley and F. Harary, Distance *in Graphs*, Addition-Wesley, Redwood City, CA, 1990.
[3] B.Bresar M Changat Steiner intervals geodesic intervals, betweenness, *Discrete Mathematics* 309 (20) 2009 6114-6125.
[4] M. Changat, A.K.Lakshmikuttyamma, J.Mathews, A note on 3- Steiner intervals and betweenness, *Discrete Mathematics* 311(22) 2011, 2601-2609.
[5] G. Chartrand, Oellermann, Ortred, Tian Song, Song Ling ,Zou and Hung Kin,Steiner distance in graphs, *Caspopis pro pestovaniMathematiky* (114) (1989) 399-410
[6] G. Chartrand, F. Harary, P. Zhang, Extremal problems in geodetic graph theory, *Congressus Numerantium* 130 (1998) 157–168.
[7] G. Chartrand, P. Zhang, The Forcing Geodetic Number of a Graph, *Discussiones Mathematicae* Vol. 19, (1999) 45 – 48.
[8] G. Chartrand, F. Harary and P. Zhang, On the Geodetic Number of a Graph, *Networks* Vol. 39 (1), (2002) 1 – 6.
[9] G. Chartrand, P. Zhang, Extreme geodesic graphs, *Czech.Math. Journal*, 52 (2002) (127 771 – 780).
[10] G. Chartrand and P. Zhang, The Steiner number of a graph, *Discrete Mathematics* 242 (2002) 41 – 54.
[11] G. Chartrand, and P. Zhang, Geodetic Sets in Graphs, *Networks* Vol. 39 (1), 197 – 218 (2011).
[12] J. John and A. Arockiamary, The vertex Steiner number of a Graph.
[13] Carmen Hernando, Tao Jiang, Merce Mora, Ignacio. M. Pelayo and Carlos Seara, On the Steiner, geodetic and hull number of graphs, *Discrete Mathematics* 293 (2005) 139 −154.
[14] R. Eballe, S. Canoy, Jr., Steiner sets in the join and composition of graphs, *Congressus Numerantium*, 170 (2004) 65 – 73.
[15] Esamel M. paluga, Sergio R. Canoy, Jr., Monophonic numbers of the join and Composition of connected graphs, *Discrete Mathematics* 307 (2007)1146 – 1154.
[16] F. Harary, Graph Theory, *Addision-Wesley* (1969).
[17] F. Harary, E. Loukakis and C. Tsouros. The Geodetic Number of a Graph, *Mathl.Comput.Modeling*17, No.11, (1993) 89 – 95.
[18] P.A. Ostrand Graphs with specified radius and diameter, *Discrete Mathematics*, 4(1973) 71 – 75
[19] Raines, M., Zhang, P. The Steiner distance dimension of graphs. Australasian *J. Combin*. 20, 133–143 (1999)
[20] A. P. Santhakumaran and P. Titus, Vertex Geodomination in Graphs, *Bulletin of Kerala Mathematics Association* 2 (2), (2005) 45 – 57
[21] A. P. Santhakumaran and P. Titus, Monophonic Distance in Graphs, *Discrete Mathematics, Algorithms and Applications* 03, 159 (2011).
[22] A. P. Santhakumaran and P. Titus, The Vertex Monophonic Number of a Graph, *Discussiones Mathematicae Graph Theory* 32 (2), (2012), 191 – 204.

 DOI : 10.18231/2454-9150.2018.1096