International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

Construction of control chart for waiting time in (M/M/1): (∞/FCFS) Queuing model using process capability

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Abstract Queue is a very impulsive situation which for all time root pointless setback and decrease the service effectiveness of establishments or service industries. Long queues may make negative effects like wasting of man power, unnecessary blocking which leads to suffocation; even build up complications to customers and also to the establishments. This necessitates the study of waiting time of the customers and the facility. Control chart technique may be applied to analyze the waiting time of the customers in the system to improve the services and the effective performance of concerns. Control chart constructed for random variable W, the time spent in the system, provides the control limits for W. The earlier idea about the expected waiting time, maximum waiting time and minimum waiting time from the parameters of the constructed chart makes effective use of time and guarantees customer's satisfaction. In this article the construction of control chart using process capability for waiting time is proposed and provides suitable tables for M/M/1 queueing model.

Keywords: Waiting time, Poisson arrival, Exponential service time, process capability and average run length.

I. INTRODUCTION

Waiting in line for service is the most unpleasant experiences in this world. Barrer (1957) says, in queuing processes a potential customer is considered "lost" if the system is busy at the time service is demanded. If not served during this time, the customer leaves the system and is considered lost. In queuing system the customer satisfaction can be increased by constructing control charts for average queue length and providing control limits for this so as to make effective utilization of time. Every manufacturing organization is concerned with the quality of its product. Stiff competition in the national and international level and customers' awareness require in the queueing system. Thus the analysis of time spent in the system by the control chart provides improvement of the performance of the system and hence customer satisfaction. In this paper, an attempt is made to construct Shewhart (1931) control chart using process capability for waiting time, W of M/M/1 queueing model. This model finds applications in a number of fields like assembly and repairing of machines, aircrafts, ATM facility of banks etc. where the system is having a single server.

II. CONCEPTS AND TERMINOLOGIES

A. Arrival pattern

Arrival pattern describes the manner in which the units arrive and join the system. The source from which the units come may be finite or infinite. A unit may arrive either singly or in a group. The arrival pattern is often measured in terms of the average number of arrivals per unit time.

B. Service pattern

Service pattern describes the manner in which the service is rendered to the arrivals. Customers may be served either singly or in batches. The time required for serving a unit is called service time and the mean service rate is denoted by μ. The service pattern may be stationary or non-stationary with respect to time and state dependent or independent with respect to number of customers waiting for service.

C. Queue discipline

Queue discipline refers to the manner in which customers are selected for service from the queue. The most common disciplines based on the arrivals of customers into the system are first come first served (FCFS) and last come first served (LCFS). Customers may also be served randomly irrespective of their arrivals to the system called service in random order (SIRO).

D. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

E. Lower specification limit (LSL)

 DOI : 10.18231/2454-9150.2018.1112

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

F. Tolerance level (TL)

It is a statistical interval within which, with some confidence level, a specified proportion of a sampled population falls. It is the difference between USL and LSL, TL = USL-LSL.

G. Process capability (C_P)

Process capability compares the output of an in-control process to the specification limits by using capability indices (Montgomery, 2010). The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by 6 process standard deviation units.i. e. $C_p = \frac{TL}{6\sigma} = \frac{USL-LSL}{6\sigma}$.

H. Average run length (ARL)

The average run length is the number of points that, on average, will be plotted on a control chart before an out of control condition is indicated (*www.micquality.com*).

If the process is in control:

$$ARL = \frac{1}{\alpha}$$

If the process is out of control:

$$ARL = \frac{1}{1-\beta}$$

where α is the probability of a Type I error and β the probability of a Type II error.

III. MODEL DESCRIPTION FOR M/M/1

M/M/1 model has single server, Poisson input, exponential service time and infinite capacity with First Come First Serve (FCFS) queue discipline. Let λ be the mean arrival rate and μ be the average service rate.

A. Steady state equations

The steady state equations of this model are by Kanti Swarup, et al (2011).

Let $P_n(t)$ = Probability that there are n customers in the system (waiting and in service) at time t.

$$P_0(t+\Delta t) = P_0(t)\,(1-\lambda\Delta t) + P_1(t)\,\mu\,\Delta t + o(\Delta t)$$
$$P_n(t+\Delta t) = P_n(t)\,(1-(\lambda+\mu)\Delta t) + P_{n-1}(t)\lambda\Delta t + P_{n+1}(t)\mu\Delta t + o(\Delta t),\ n \ge 1$$
$$P_0^{|}(t) = -\lambda P_0(t) + \mu P_1(t)$$
$$P_n^{|}(t) = -(\lambda+\mu)P_n + \lambda P_{n-1}(t) + \mu P_{n+1}(t),\ n \ge 1$$

The steady state equations corresponding to the above $P_n^{|}(t)$ are

$$0 = -\lambda P_0 + \mu P_1$$
$$0 = -(\lambda+\mu)P_n + \lambda P_{n-1}(t) + \mu P_{n+1}(t),\ n \ge 1$$

Let ρ=λ/μbe the traffic intensity. The above result yields

$$P_0 = (1-\rho)$$
$$P_n = (1-\rho)\rho^n$$

B. Performance measures

$(i)\ E(L_s) =$ Average number of customers in the system

$$= \sum_{n=0}^{\infty} nP_n = \frac{\rho}{1-\rho}$$

$(ii)\ E(L_q) =$ Average number of customers in the queue

$$= \sum_{n=1}^{\infty} (n-1)P_n$$
$$= \frac{\rho^2}{1-\rho}$$

$(iii)\ E(W_s) =$ Average waiting time of a customer in the system

$$= \frac{1}{\mu(1-\rho)}$$

$(iv)\ E(W_q) =$ Average waiting time of a customer in the queue

$$= \frac{\rho}{\mu(1-\rho)}$$

Let W denote the waiting time of a customer in the system which includes both the waiting time and the service time. The probability density function of the random variable W is given by Gross (1998)

$$f(w) = (\mu-\lambda)e^{-(\mu-\lambda)w},\quad w > 0$$

which is an exponential distribution with parameter (μ -λ) .

Mean $E(w) = \frac{1}{\mu-\lambda}$ and variance $V(w) = \frac{1}{(\mu-\lambda)^2}$

IV. METHODS AND MATERIALS

A. Control chart for waiting time (W) for M/M/1 Model

Shewhart type control charts are constructed by approximating the statistic under consideration by a normal distribution. The parameters of the control chart (Poongodi and Muthulakshmi, 2013) are given by

$$UCL = E(W) + 3\sqrt{V(W)}$$
$$CL = E(W)$$
$$LCL = E(W) - 3\sqrt{V(W)}$$

For M /M /1queueing model the parameters of the control chart for waiting time of the customer in the system are given by

 DOI : 10.18231/2454-9150.2018.1112

ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

$$\text{UCL}=\frac{4}{(\mu-\lambda)}$$

$$\text{CL}=\frac{1}{(\mu-\lambda)}$$

$$\text{LCL}=\frac{-2}{(\mu-\lambda)}$$

B. Waiting time control chart using process capability (Cp) for M/M/1 Model

The capability of a process is a statistical indicator of how well it is functioning, or, in other words, how successful it is at running within its specified limits. In the absence of any special or assignable causes of variation, a process will still have some inherent variability. Process capability is a statistical measure of this inherent variability.

For a specified TL and C_p of the process (Radhakrishnan and Balamurugan, 2012), the value of σ (termed as σ_{cp}) is calculated from $C_p = \left({}^{TL}/_{6\sigma}\right)$ using a JAVA program and presented in Table – A (APPENDIX I) for various combinations of TL and C_p.

$$\text{UCL}_{\bar{X}-q}=\bar{X}_w+\left(\frac{3\sigma_q}{\sqrt{n}}\right)$$

$$\text{CL}_{\bar{X}-q}=E(\hat{W})$$

$$\text{LCL}_{\bar{X}-q}=\bar{X}_w-\left(\frac{3\sigma_q}{\sqrt{n}}\right)$$

V. ILLUSTRATION

As an application of the above theoretical calculations, a real situation, relating to a grocery shop is considered.

A grocery shop has a single server for billing which starts at 8.00 a.m. An arrival moves immediately into the service facility if it is empty. On the other hand, if the server is busy, the arrival will wait in the queue. Customers are served on first come first served basis. Observed inter arrival time and service time of 200 customers in the system is given in Table-1. In this table customers, arrival time (min.), inter-arrival time, starting time of service, service time (min.), ending time of service and customer's waiting time in system (min.) are given in columns I,II,III,IV,V,VI and VII respectively. Control charts are constructed using the theoretical formula and also estimated values of observed data (Poongodi and Muthulakshmi, 2013).

Table 1: Observed waiting time of customers

I	Arrival time (in Minutes) II	Inter arrival time III	Starting time of service IV	Service time (in minutes) V	Ending time of service VI	Waiting time in system VII
1	8.01	1	8.01	4	8.05	4
2	8.03	2	8.05	3	8.08	5
3	8.04	1	8.08	5	8.13	9
4	8.08	4	8.13	4	8.17	9
5	8.09	1	8.17	2	8.19	10
6	8.10	1	8.19	8	8.27	17
7	8.12	2	8.27	7	8.34	22
8	8.16	4	8.34	10	8.44	28
9	8.19	3	8.44	1	8.45	26
10	8.23	4	8.45	3	8.48	25
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
200	5.11	4	5.11	1	5.12	1

Source: https://research.ijcaonline.org

From Table-1 the average inter arrival time is 2.755 min. and the average service time is 2.59 min. Arrival rate λ = 21.78 customers/hr and service rate μ = 23.17 customers/hr. The parameters of the control limits are given by

$$\text{UCL}=E(W)+3\sqrt{V(W)}=43.25+\left(3\times\sqrt{31.17}\right)=59.99\,\text{min}.$$

$$\text{CL}=E(W) \qquad =43.25\text{min}.$$

$$\text{LCL}=E(W)-3\sqrt{V(W)}=43.25-\left(3\times\sqrt{31.17}\right)=26.50\,\text{min}.$$

The estimated parameters of the control chart (Radhakrishnan and Balamurugan, 2010) for waiting time are calculated based on sample observations.

$$\text{UCL}=\bar{X}_w+3\sigma_w=6.16+(3\times 4.57)=19.88\text{ min}$$

$$\text{CL}=\bar{X}_w \qquad =6.16\text{ min}$$

$$\text{LCL}=\bar{X}_w-3\sigma_w=6.16-(3\times 4.57)=-7.57=0$$

A. Construction of Shewhart control chart for mean waiting time

The 3σ control limits suggested by Shewhart (1931) are

$$\text{UCL}=\bar{X}_w+\left({}^{3}/_{\sqrt{n}}\right)\sigma_w=6.16+\left[\left({}^{3}/_{\sqrt{2}}\right)\times 4.57\right]=15.86\text{ min}$$

$$\text{CL}=E(\hat{W}) \qquad =6.16\text{ min}$$

$$\text{LCL}=\bar{X}_w+\left({}^{3}/_{\sqrt{n}}\right)\sigma_w=6.16-\left[\left({}^{3}/_{\sqrt{2}}\right)\times 4.57\right]=-3.55\text{ min}=0$$

 DOI : 10.18231/2454-9150.2018.1112

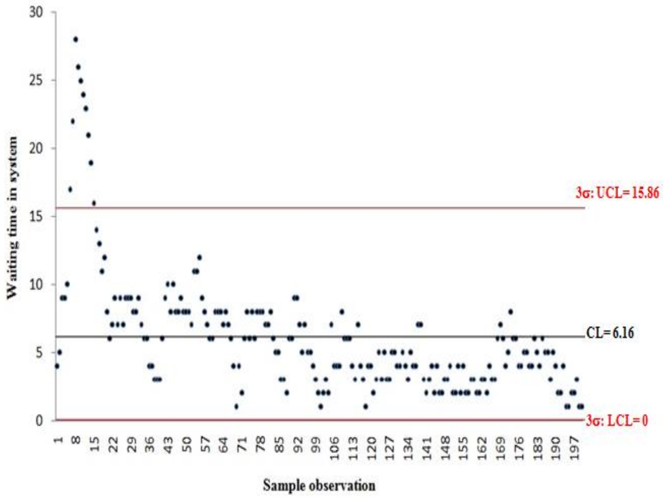

Figure 1: Shewhart control chart for mean waiting time

However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, the control limit interval will be determined using the expression for the Shewhart control chart for mean waiting time:

$$\mathrm{CLI}_{\bar{X}} = \left(\frac{6\sigma}{\sqrt{n}}\right) = \left(\frac{6 \times 4.57}{\sqrt{2}}\right) = 19.39$$

From the resulting Figure-1, it is clear that the process is out of control, since the customer numbers 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 lie outside the control limits and the control limit interval is 19.39.

B. Construction of Shewhart proposed control chart using process capability for mean waiting time

Difference between upper specification and lower specification limits is 4.45 (USL - LSL = 28- 1), which termed as tolerance level (TL) and choose the process capability (C_p) is 2.0, the value of σ_q is 2.25. The control limits using process capability for mean for a specified tolerance level with the control limits $\bar{X} \pm \left(\frac{3\sigma_q}{\sqrt{n}}\right)$

$$\mathrm{UCL}_{\bar{X}-q} = \bar{X}_w + \left(\frac{3\sigma_q}{\sqrt{n}}\right) = 6.16 + \left(\frac{3 \times 2.25}{\sqrt{2}}\right) = 10.93 \text{ min}$$

$$\mathrm{CL}_{\bar{X}-q} = E(\hat{W}) = 6.16 \text{ min}$$

$$\mathrm{LCL}_{\bar{X}-q} = \bar{X}_w - \left(\frac{3\sigma_q}{\sqrt{n}}\right) = 6.16 - \left(\frac{3 \times 2.25}{\sqrt{2}}\right) = 1.38 \text{ min}$$

However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, the control limit interval will be determined using the expression for the mean waiting time control chart using process capability:

$$\mathrm{CLI}_{\bar{X}-q} = \left(\frac{6\sigma_{RPC}}{\sqrt{n}}\right) = \left(\frac{6 \times 2.25}{\sqrt{2}}\right) = 9.55$$

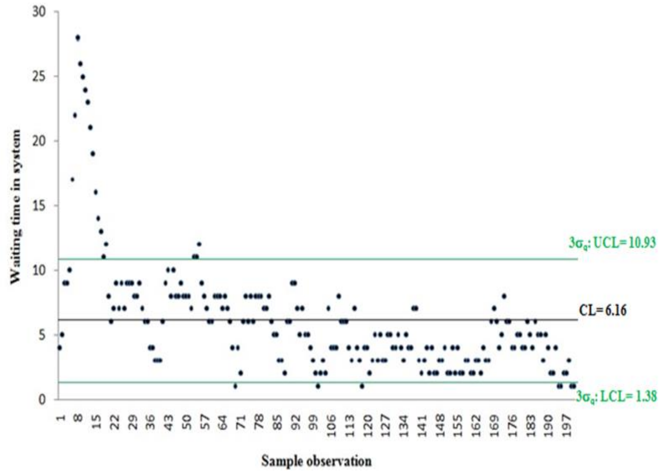

Figure 2: Shewhart control chart using process capability for mean waiting time

From the resulting Figure-2, it is clear that the process is out of control, since the customer numbers 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 53, 54 and 55 lie above the upper control limit and the customer numbers 69, 101, 118, 194, 195, 199 and 200 lie below the lower control limit and the control limit interval is 9.55.

Table 2: Assessment of Shewhart control chart and control chart using process capability for mean waiting time

Control limits	Shewhart control chart	Control chart using process capability
LCL	-3.55≈0	1.38
CL	6.16	6.16
UCL	15.86	10.93
CLIs	19.39	9.55

The average run length (ARL) and the false alarm rate are obtained as follows:

Table 3: Average run length (ARL) for control charts

multiple of σ	Shewhart control chart	Control chart using process capability
0.5	333.69	244.44
1	249.51	110.57
1.5	171.15	50.09
2	114.47	24.48
2.12	103.72	20.88
2.5	76.83	13.02
3.0	52.32	7.52

 DOI : 10.18231/2454-9150.2018.1112

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

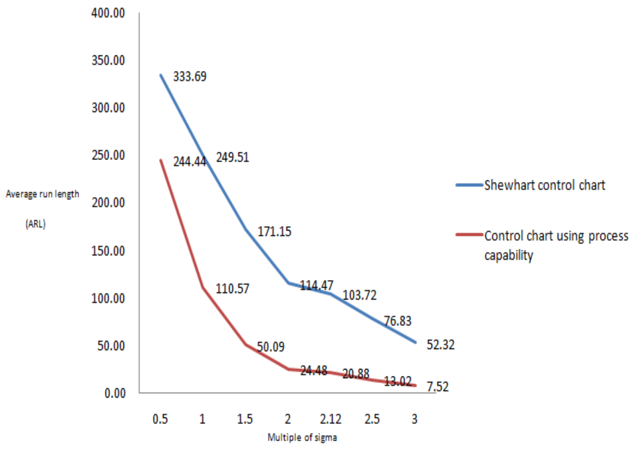

Figure 3: Average run length (ARL) for Shewhart control chart and control chart using process capability

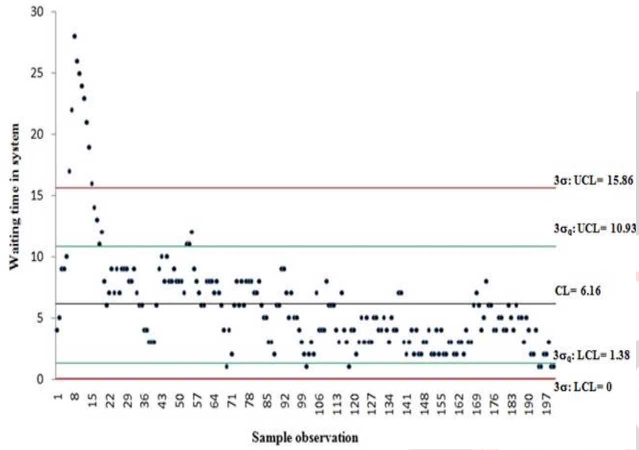

Figure 4: Comparison between Shewhart control chart and control chart using process capability for waiting time

From the Figure-3, it can be observed that the proposed mean waiting control chart using process capability is efficiently detects the shift in the process than the existing Shewhart 3σ control chart with multiple of sigma.

It is found from the Figure-4 which presents the control chart based on theoretical formula using observed data in that the process is out of control when the control limits of Shewhart control chart and control chart using process capability for mean waiting time are adopted and also the variation of process is very undersized when compared to the control limits of Shewhart 3σ. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system.

VI. CONCLUSION

Shewhart (1931) control chart for monitoring the process variability is based on some assumptions with standard deviation (σ), we offered the control chart using process capability for waiting time in the system. The outcome of numerical example shows that the proposed method leads better as many points fall outside the control limits than the existing control charts and the control limits interval of control chart using process capability is smaller than the control limits interval of Shewhart. It is clear that the performance of the system is in shortage than the requirement based on the control chart using process capability. The proposed control chart using process capability for average waiting time will not only assist the producer in providing better quality but also increase the fulfilment and self-assurance of the consumers. In this research article, it also reveals that the mean waiting control chart using process capability is compatible, better performance and efficient than the Shewhart 3σ control chart through the average run length (ARL) for approximately symmetric distributions.

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 DOI : 10.18231/2454-9150.2018.1112