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Square Sum Labeling of almost Bipartite Graph and Mongolian Tent

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ABSTRACT - Let *G* be a (*p*, *q*)-graph. *G* is said to be a square sum graph if there exist a bijection *f*: $V(G) \to$ {0, 1, ..., *p* - 1} such that the induced function *f* * : $E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$ for every *uv* $\in E(G)$ are all distinct. In this paper, we find the bijection f on the vertex set of *G* to produce square sum labeling on almost bipartite graph and mongolian tent.

KEYWORDS: Almost bipartite graph, Mongolian tent and Square sum labeling.

AMS Subject Classification (2010): 05C78.

I. INTRODUCTION

Labeling of a graph *G* is an assignment of labels to vertices or edges or both following certain rules, A useful survey on graph labeling by J.A.Gallian (2015) can be found in [4]. The perception of labeling to the vertices and edges in graphs has flourished with types of labeling being applied in different areas by the researchers. Prominent among the types of labeling is square sum labeling [1], [2] ,[5], [6], [8], [9]. In this paper we deal only finite, simple, connected and undirected graphs obtained through graph operations .

Definition 1.1 :

An almost-bipartite graph [3] is a non-bipartite graph with the property that the removal of a particular single edge renders the graph bipartite.

Definition 1.2 :

Let *G* be a (*p*, *q*)-graph. *G* is said to be a square sum graph if there exist a bijection *f*: $V(G) \to$ {0, 1, ..., *p* - 1} such that the induced function *f** : $E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$ for every *uv*$\in E(G)$ are all distinct.

Definition 1.3 :

A Mongolian tent [7] M_n is defined as the graph obtained from $P_n \times P_n$ by adding a new vertex above the grid and joining every vertex of the top row to the new vertex.

II. SQUARE SUM LABELING :

The Pairs of positive integers whose sum of squares are distinct, constitute an important part of number theory. For example, in number theory a positive integer *n* has a representation as a sum of two squares if $n = a^2 + b^2$ for some *a*, *b* ∈ *Z*. Square sum graphs are vertex labeled graphs with the labels from the set {0, 1, ..., *p* - 1} such that the induced edge labels as the sum of the squares of the labels of the end vertices are all distinct. Not every graph is square sum, the graph which admits square sum labeling is said to be a square sum graph.

III. MAIN RESULTS

Theorem 3.1 :

The Almost bipartite graph $P_m + e$ is square sum.

Proof:

Consider the graph *G*= $P_m + e$,where P_m is the path $v_1\, v_2 \ldots\ldots v_m$ .Let V_1 and V_2 be the bipartition of the vertex set of G.

Case-1:When *m* is even,*e* = $v_1\, v_{m-1}$

$V_1 = \{ v_1, v_3, \ldots, v_{m-1} \}$ and $V_2 = \{ v_2, v_4, \ldots, v_m \}$.

Case-2:When *m* is odd ,*e* = $v_1\, v_m$

$V_1 = \{ v_1, v_3, \ldots, v_m \}$ and $V_2 = \{ v_2, v_4, \ldots, v_{m-1} \}$.

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For both the cases we define $f: V(G) \to \{0,1, 2,. ., m-1\}$ by $f(v_i)$ =i-1, $1 \le i \le m$.

The function f induces a square sum labeling on G.

For if $v_1 \neq v_2$, then $f(v_1) \neq f(v_2)$.

If $e_i = v_{i-1}v_i$ and $e_j = v_{j-1}v_j$ then

$$f^*(e_i) = [f(v_{i-1})]^2 + [f(v_i)]^2 \neq [f(v_{j-1})]^2 + [f(v_j)]^2 = f^*(e_j).$$

So that f^* is square sum labeling on G.Hence G is a Square sum Graph .

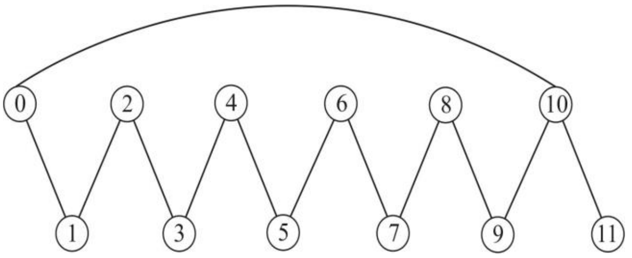

Figure 1: Square sum labeling of almost bipartite graph $P_{12} + e$.

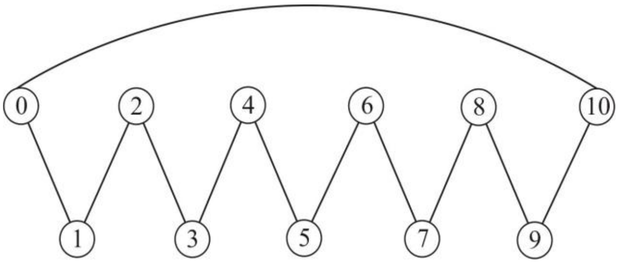

Figure 2: Square sum labeling of almost bipartite graph $P_{11} + e$.

Theorem 3.2 : The Mongolian tent M_n is square sum .

Proof:

Let M_n be a Mongolian tent graph with vertex set $\{u, x_{1,1}, x_{1,2}, \ldots, x_{1,n};\ x_{2,1}, x_{2,2}, \ldots, x_{2,n}; \ldots, x_{n,1}, x_{n,2}, \ldots, x_{n,n}\}$ where u is the vertex above the grid and $x_{i,j}$ be the j^{th} vertex in the i^{th} row.

We note that $|V(M_n)| = n^2 + 1$ and $|E(M_n)| = n(2n-1)$.

Define $f: V(M_n) \to \{0,1, 2, ..., n^2\}$ as given below:

let $q_1, q_2, \ldots, q_n$ be prime numbers such that $n^2 > q_1 > q_2 > \ldots > q_n$ and there exists no prime number q such that $n^2 > q > q_1$,or $q_i > q > q_{i+1}$, $1 \le i \le n-1$.

Let $\{m_1, m_2, \ldots, m_{n^2-n}\} = \{0,1,2,3,..., n^2-1\} - \{q_1, q_2, \ldots, q_n\}$, where $m_1 \le m_2 \le \ldots \le m_{n^2-n}$

Now we define $f(u) = n^2$;

$f(x_{1,j}) = q_j$, $1 \le j \le n$;

$f(x_{2,(n-j)+1}) = m_j$, $1 \le j \le n$;

$f(x_{i,(n-j)+1}) = m_{(i-2)n+j}$; if i is odd, $3 \le i \le n$, $1 \le j \le n$;

$f(x_{i,j}) = m_{(i-2)n+j}$; if i is even, $4 \le i \le n$, $1 \le j \le n$.

The function f induces a square sum labeling on M_n .

For, if $e_1 = u_1v_1$ and $e_2 = u_2v_2$ then

(i)$u_1 = u_2 \Rightarrow f(v_1) < f(v_2)$ or $f(v_1) > f(v_2)$,

then $f^*(e_1) = [f(u_1)]^2 + [f(v_1)]^2 \neq [f(u_2)]^2 + [f(v_2)]^2 = f^*(e_2)$.

(ii)If $u_1 \neq u_2$, then $f(u_1) < f(u_2) \Rightarrow f(v_1) < f(v_2)$.

Hence $f^*(e_1) = [f(u_1)]^2 + [f(v_1)]^2 \neq [f(u_2)]^2 + [f(v_2)]^2 = f^*(e_2)$.

so that f^* is a square sum labeling of M_n .Hence M_n is a square sum graph.

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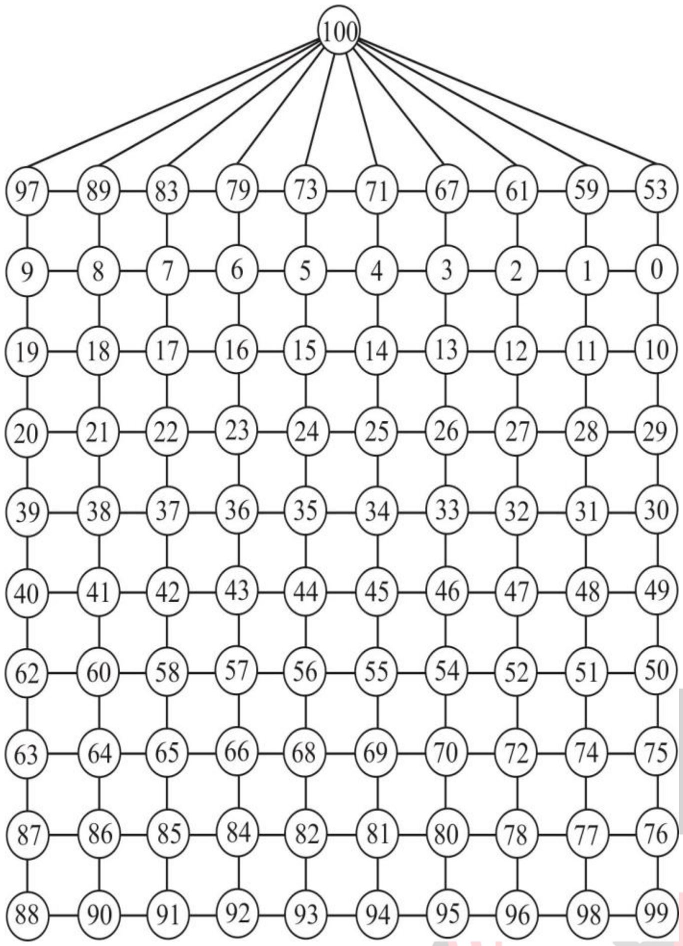

Figure 3: square sum labeling of the Mongolian tent M_{10} .

IV. CONCLUSION

It is very fascinating to study graphs which admit square sum labeling.To examine equivalent results for different types of graphs is an open area of research.

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