International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

Binomial Labeling of Some Graphs

***S.Chandrakala**

Research scholar, Reg no.9660,Aditanar College,Manonmanium sundaranar university, Tirunelveli, Tamil nadu, India. ckavi2008@gmail.com

Dr. C.Sekar

Associate professor of Mathematics, Aditanar college, Tiruchendur, Tamilnadu, India. sekar.acas@gmail.com

ABSTRACT: In this paper Binomial labeling is defined. Let G be a (p,q) graph. G is said to have a binomial labeling if there exists an injective map $f: V(G) \to \{1, 2, \dots\dots\dots q+1\}$ such that the induced function f^*:E(G)→ N is given by $f^*(uv) = M_{c_m} = \frac{M!}{m!(M-m)!}$ where M = max$\{f(u), f(v)\}$ and $m = \min\{f(u), f(v)\}$, for every $uv \in E$ are all distinct.A graph which has the Binomial labeling is called the Binomial graph.In this paper we investigate binomial labeling on a path graph,cycles,stars,wheel graphs, fan graphs, crown graphs.

Keywords: Binomial labeling, Binomial graph, Path, cycle, wheel graph, crown graph.

I. INTRODUCTION

If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling. Graph labeling is an active area of research in graph theory which has mainly evolved through its many applications in coding theory, communication networks and mobile telecommunication.

A dynamic survey on graph labeling is regularly updated by Gallian [4] and it is published by Electronic Journal of combinatory. The present work is aimed to discuss one such a labeling namely Binomial labeling . The graphs considered in this paper are finite ,undirected and without loops. Let G = (V,E) be a graph with p vertices and q edges . For various graph theoretic notation and terminology we follow Bondy and Murthy [2] and for number theory we follow Burton [3]. The brief summary of definitions and other information which are necessary for the present investigation are given below.

Definition 1.1. If n and r are positive integers with $0 \le r \le n$,then the Binomial coefficient $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ is also an integer.

Definition 1.2. Let G = (p,q) be a graph.G is said to have a Binomial labeling if there exists an injective map $f: V(G) \to \{1,2,\dots,q+1\}$ such that the induced function f^*:E(G)→ N is given by $f^*(uv) = M_{c_m} = \frac{M!}{m!(M-m)!}$, where M= max$\{f(u), f(v)\}$ and $m = \min\{f(u), f(v)\}$, assigns distinct labels for the edges.

Definition 1.3. A graph which has the Binomial Labeling is called the Binomial graph.

Definition 1.4. A fan graph F_n is obtained by joining all vertices of a path P_n to a further vertex called the centre .Thus F_n contains n+1 vertices say $c, v_1, v_2 \dots\dots\dots\dots v_n$ and 2n-1 edges say $cv_i, 1 \le i \le n$ *and* $v_iv_{i+1}, 1 \le i \le n-1$.

Definition 1.5.

A wheel Wn, $n \ge 3$ is a graph obtained by joining all vertices of cycle C_n to a further vertex c ,called the centre.

Definition 1.6. A crown graph R_n is formed by adding to the n points $v_1, v_2, \dots\dots v_n$ of a cycle C_n , n more pendent points $u_1, u_2, \dots\dots.., u_n$ and n more lines u_iv_i ,i=1,2,3,……..n for $n \ge 3$.

II. MAIN RESULTS

Theorem 2.1. The path P_n is a Binomial graph.

Proof:

Let the graph G be a path P_n of length n-1. Let $|V(G)| = n$ *and* $|E(G)| = n-1$.

Define $f: V(G) \to \{1,2,\dots,n\}$ *as follows*:

$f(v_i) = i,\ 1 \le i \le n.$

The induced function $f^*: E(G) \to N$ is given by $f^*(v_iv_{i+1}) = M_{C_m}$

$$= \frac{M!}{m!(M-m)!},\ 1 \le i \le n-1$$

 DOI : 10.18231/2454-9150.2018.1122 © 2018, IJREAM All Rights Reserved.

ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

where M =Max $\{f(v_i), f(v_{i+1})\}$

m=min $\{f(v_i), f(v_{i+1})\}$

Here all the edges have distinct labels $2_{C_1}, 3_{C_2}, \ldots, n_{C_{n-1}}$.

Hence the path P_n admits a binomial labeling . Hence P_n is a Binomial graph.

Example 2.2.

P_6 is a Binomial graph.

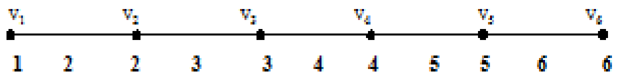


Figure 1 : Binomial labeling of path P_6

Theorem 2.3.

The cycle C_n is a Binomial graph .

Proof:

Let the graph G be a cycle C_n .Let |V(G)| = n and |E(G)| = n .Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n .

Then the edge set $E(G)=\{v_iv_{i+1}/1 \leq i \leq n\} \cup \{v_nv_1\}$.

Define $f: E(G) \to N$ as follows:

$f(v_i) = i, 1 \leq i \leq n-2$; $f(v_{n-1}) = n$; $f(v_n) = n-1$.

Then the induced function $f^*: E(G) \to N$ is defined by

$$f^*(uv) = M_{C_m} = \frac{M!}{Mi(M-m)!} \quad \forall\, uv \in E(G) ,$$

$where\ M = \max\{f(u), f(v)\}\ and\ m = \min\{f(u), f(v)\}$

Here all the edges have distinct labels,

$2c_1, 3c_2, 4c_3, \ldots \ldots \ldots \ldots \ldots \ldots .\, nc_{n-2}, nc_{n-1}, n-1c_1$.

Hence the cycle C_n admits a Binomial Labeling .Hence the graph is Binomial graph.

Example 2.4

C_6 is a Binomial graph

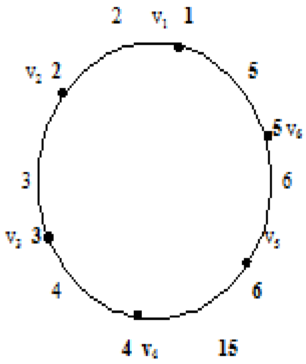


Figure 2 : Binomial labeling of C_6

Theorem 2.5.

Star graph $K_{1,n}$ is a Binomial graph .

Proof :

Let the graph G be a star graph $K_{1,n}$ .Let |V(G)| = n+1 and |E(G) | = n. Let $v_1, v_2, \ldots, v_{n+1}$ be the vertices of $K_{1,n}$ where v_1 is the centre. The edge set $E(G) = \{v_1v_i / 2 \leq i \leq n+1\}$.

Define $f: V(G) \to \{1,2, \ldots, n+1\}\ as\ follows$:

$f(v_i) = i,\ 1 \leq i \leq n+1.$

The induced function $f^*: E(G) \to N$ is given by $f^*(uv) = M_{C_m}$

 DOI : 10.18231/2454-9150.2018.1122 © 2018, IJREAM All Rights Reserved.

ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

$$= \frac{M!}{m!(M-m)!}, \forall\ uv \in E(G)$$

where M =Max $\{f(u), f(v)\}$

m=min $\{f(u), f(v)\}$

Here all the edges have distinct labels 2,3,…,n,n+1.Thus G admits Binomial labeling. Hence the star graph is Binomial graph

Example 2.6.

$K_{1,6}$ is a binomial graph.

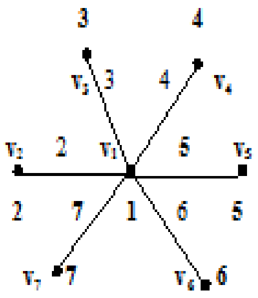


Figure 3: Binomial labeling of $K_{1,6}$

Theorem 2.7

A complete graph K_n is a Binomial graph .

Proof:

Let the graph G be a complete graph K_n.

. Let $|V(G)| = n\ and\ |E(G)| = \frac{n(n-1)}{2}$.

Define f: V(G) $\rightarrow \left\{1,2,\ldots,\frac{n(n-1)}{2}+1\right\}$

by $f(v_i) = (n-2)+i\,,\ 1 \le i \le n$

Clearly f is an injection.

The induced function $f^*: E(G) \rightarrow N$ is given by $f^*(uv) = M_{C_m}$

$$= \frac{M!}{m!(M-m)!}, \forall\ uv \in E(G)$$

where M =Max $\{f(u), f(v)\}$

m=min $\{f(u), f(v)\}$

Clearly all the edges have distinct labels .Hence K_n is a Binomial graph .

Example 2.8.

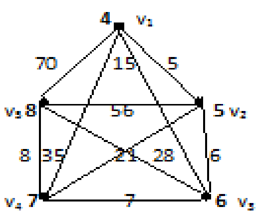


Figure 4: Binomial labeling of K_5

Theorem 2.9.

Wheel graph W_n, $n \ge 3$ is a Binomial graph.

Proof:

Let W_n,$n \ge 3$ be the wheel graph .

 DOI : 10.18231/2454-9150.2018.1122 © 2018, IJREAM All Rights Reserved.

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

Let V(W_n) = $\{c, v_1, v_2, \dots, v_n\}$ and

E(W_n) = $\{cv_i/1 \le i \le n\} \cup \{v_i v_{i+1}/1 < i < n-1\} \cup \{v_n v_1\}$.

Then $|V(W_n)| = n+1$ and $|E(W_n)| = 2n$.

Define f :V(G) $\rightarrow \{1,2, \dots ,2n+1\}$ as follows:

$f(c) = n$;

$f(v_i) = n + i$, 1$\le i \le n$

Clearly f is an injection.

The induced function $f^*: E(G) \rightarrow N$ is given by $f^*(uv) = M_{C_m}$

$= \frac{M!}{m!(M-m)!}$,$\forall\ uv \in E(G)$

where M =Max $\{f(u), f(v)\}$

m=min $\{f(u), f(v)\}$

Clearly all the edges have distinct labels.

Hence the wheel W_n admits a Binomial Labeling .Hence the wheel graph W_n, n$\ge$ 3 is a Binomial graph.

Example 2.10

W_6 is a Binomial graph.

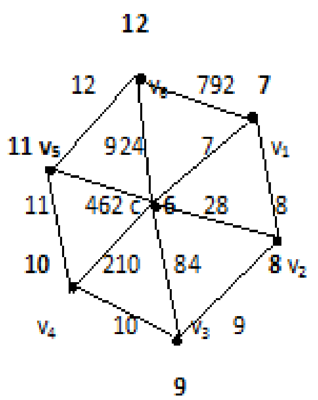


Figure 5: Binomial labeling of W_6

Theorem 2.11.

The fan graph F_n, (n$\ge$ 3) is a Binomial graph.

Proof :

Let G be the fan graph $F_n(n \ge 3)$. Let $V(G) = \{c, v_1, v_2 \dots \dots \dots . v_n\}$.

Let $E(G) = \{cv_i/1 \le i \le n\} \cup \{v_i v_{i+1}/1 \le i \le n-1\}$.

Then $|V(G)| = n+1$ and $|E(G)| = 2n-1$.

Define $f: V(G) \rightarrow \{1,2, \dots ,2n\}$ as follows:

$f(c) = n$; $f(v_i) = n + i, 1 \le i \le n$.

The induced function $f^*: E(G) \rightarrow N$ *is defined by* $f^*(uv) = M_{C_m} = \frac{M!}{m!(M-m)!}$

$\forall e = uv\ \in E(G)$

where $M = \max\{f(u), f(v)\}$

$m = \min\{f(u), f(v)\}$

Here the label of the edges are $(n+1)_{C_n}, (n+2)_{C_n}, \dots, (2n-1)_{C_n}, (2n-1)_{C_{2n-2}} \dots \dots \dots \dots . . (n+2)_{C_{n+1}}$.

 DOI : 10.18231/2454-9150.2018.1122 © 2018, IJREAM All Rights Reserved.

International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

Therefore all the edges have distinct labels. Hence the fan graph admits a Binomial labeling. Hence the fan graph is the Binomial graph.

Example 2.12

Fan graph F_5 is a Binomial graph .

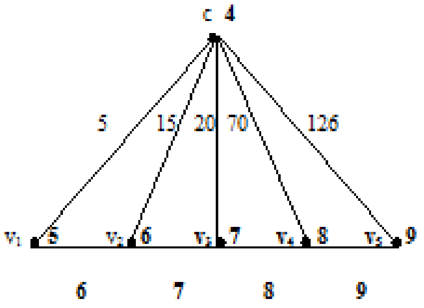


Figure 6: Binomial Labeling of F_5

Theorem 2.13.

A crown graph R_n , $n \geq 3$ is a binomial graph.

Proof :

Let G be the crown graph $R_n, n \geq 3. Let\ V(G) = \{v_1, v_2 \dots \dots . v_n, u_1, u_2 \dots \dots u_n\} and$

$$E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_i u_i, 1 \leq i \leq n\}$$

Then |V(G)| = 2n and |E(G)| = 2n.

Define $f: V(G) \rightarrow \{1,2, \dots, 2n+1\}$ $as\ follows$:

$$f(v_i) = n + i\ ,\ 1 \leq i \leq n$$

$$f(u_i) = i\ ,\ 1 \leq i \leq n$$

The induced function f^*: E(G) → $N\ is\ defined\ by$

$$f^*(uv) = M_{cm} = \frac{M!}{m!(M-m)!}\ ,\ \forall\ uv \in E(G)$$

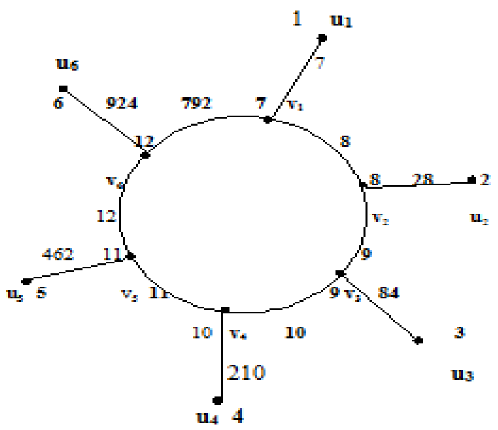
where $M = \max\{f(u), f(v)\}$

$m = \min\{f(u\}, f(v\}$

Clearly all the edges have distinct labels .Hence G is a Binomial graph.

Example 2.14.

R_5 *is a Binomial graph.*



Binomial labeling of R_6

 DOI : 10.18231/2454-9150.2018.1122 © 2018, IJREAM All Rights Reserved.

ISSN : 2454-9150 Vol-04, Issue-08, Nov 2018

III. CONCLUSION

We have introduced a new labeling namely Binomial labeling of graphs .We prove that path, cycle,star,fan graph ,wheel graph, crown graph are all Binomial graphs .Extending the study to other families of graph is an open area of research .

IV. REFERENCES

[1] Balakrishnan and Regannathan.N ,A Text Book on Graph theory,springer verlag,2000.

[2] Bondy .J.A and Murthy U.S.R, “Graph Theory and Application” (North Holland). New York (1976).

[3] Burton,D.M.(2007) Elementry Number Theory, sixth Edition,Tata McGraw-Hill publishing company Limited, New Delhi.

[4] Gallian J.A, “A Dynamic survey of Graph labeling”, The Electronic Journal of Combinatorics, 18(2011), 147, #DS6.

[5] Tout.A, Dabboucy .A.N and Howalla. K, “Prime Labeling of Graphs”, Nat.Acad .Sci letters 11 (1982) 365-368 .

 DOI : 10.18231/2454-9150.2018.1122 © 2018, IJREAM All Rights Reserved.