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On The Ternary Cubic Diophantine Equation $5x^2 - 2y^2 = 3z^2$

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ABSTRACT - The ternary cubic Diophantine equation given by $5x^2 - 2y^2 = 3z^2$ is analyzed for its non-zero discrete integer points on it. Different patterns of integer points for the equation under contemplation are obtained. A few interesting relations between solutions and extraordinary numbers are obtained.

Keywords: Ternary cubic, integer solutions, polygonal numbers, pyramidal numbers. 2010 Mathematics subject classification: 11D25.

I. INTRODUCTION

The ternary cubic diophantine equations propose an infinite field of research due to their selection. For an general of a variety of problems, one may refer. This statement concerns with yet another interesting ternary cubic diophantine equation $5x^2 - 2y^2 = 3z^2$ for formative its infinitely many non-zero integral points. Also, a few interesting relations between the solutions and particular numbers are obtainable.

II. NOTATIONS

$CP_{m,n}$ – Centered pyramidal number of rank n with size m.

$T_{m,n}$ – Polygonal number of rank n with size m.

Pr_n – Pronic number of rank n.

P_n^m - Pyramidal number of rank n with size m.

III. METHOD OF ANALYSIS

The Diophantine equation to be solved for its non-zero distinct integral solution is

$$5x^2 - 2y^2 = 3z^2 \qquad (1)$$

To start with it is seen that (1) is satisfied by the following triples of integers

$$(1,1,1), (-1,2,-1)$$

In what follows, we demonstrate methods of obtaining non-zero distinct integer solutions to (1)

The replacement of linear transformations

$$x = X_1 + 2T, y = X_1 + 5T, z = z_1 \qquad (2)$$

in (1) leads to

$$X_1{}^2 - 10T^2 = z_1{}^3 \qquad (3)$$

Imagine $z_1 = z_1(a,b) = a^2 - 10b^2; a,b > 0$ (4)

Equation (3) is solved in the course of various approaches and different patterns of solutions thus obtained for (1) are illustrated below.

Pattern: I

Mark 1 as

$$1 = \frac{(7+2\sqrt{10})(7-2\sqrt{10})}{9} \qquad (5)$$

By means of (4) and (5) in (3) and employing the process of factorization and equating positive factors, we get

$$\left(X_1 + \sqrt{10}T\right) = \frac{7+2\sqrt{10}}{3} (a + \sqrt{10}b)^3 \qquad (6)$$

Equating the rational and irrational parts of (6), we have

$$X_1 = 63A^3 + 1800B^3 + 1890AB^2 + 540A^2B$$

$$T = 18A^3 + 630B^3 + 540AB^2 + 189A^2B$$

Employing (2), the value of x, y and z satisfying (1) are given by

$$x = x(a,b) = 99A^3 + 3060B^3 + 2970AB^2 + 918A^2B$$

$$y = y(a,b) = 153A^3 + 4950B^3 + 4590AB^2 + 1485A^2B$$

$$z = z(a,b) = 9A^2 - 90B^2$$

A few interesting properties are as follows:

1. $x(A+1,A) - 7047\, CP_A^6 - 3888T_{4,A} - 1215Pr_A - 3j_5 = 6$
2. $y(B,B+1) - 22356P_B^5 - 28674T_{3,B} - Gno_B \equiv 4951\ (mod\ 5101)$
3. $z\big((A+1), A(A+1)\big) + 90Biq_A + 180Cub_A + 7T_{8,A} + 4T_{6,A} + 52T_{4,B} + j_5 + 3J_1 = 3$
4. $[-z(A,A)]$ represents a perfect square.

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5. $x(2A, A+1) - z(2A, A+1) - 5769CP_A^{14} - 3Cub_A - 22992Pr_A - 1794T_{4,B} - j_{11} - 3J_{10} = 78$

Pattern: II

Once more mark (1) as

$$1 = \frac{(13+4\sqrt{10})(13-4\sqrt{10})}{9} \quad (7)$$

Utilize (4) and (7) in (3); employing the system of factorization and equating positive factors, we get

$$(X_1 + \sqrt{10}T) = \frac{13+4\sqrt{10}}{3}(a+\sqrt{10}b)^3 \quad (8)$$

Equating the rational and irrational parts of (8), we have

$$X_1 = \frac{1}{3}\{13a^3 + 400b^3 + 390ab^2 + 120a^2b\}$$

$$T = \frac{1}{3}\{4a^3 + 130b^3 + 120ab^2 + 39a^2b\}$$

As our plan is to discover integer solutions putting $a = 3A, b = 3B$, we get as follows

$$X_1 = 117A^3 + 3600B^3 + 3510AB^2 + 1080A^2B$$

$$T = 36A^3 + 1170B^3 + 1080AB^2 + 351A^2B$$

$$Z_1 = 9A^2 - 90B^2$$

In analysis of (2), the integer solutions of (1) are given by

$$x = x(a,b) = 189A^3 + 5940B^3 + 5670AB^2 + 1782A^2B$$

$$y = y(a,b) = 297A^3 + 9450B^3 + 8910AB^2 + 2835A^2B$$

$$z = z(a,b) = 9A^2 - 90B^2$$

A few interesting properties are as follows:

1. $z(A(A+1), A+2) - 108\,FN_A^4 - 9So_A - 12S_A - 3J_6 \equiv -64 (mod\ 243)$
2. $x(A^2, A+1) - 6791So_A - CP_A^6 - 30942T_{4,A} \equiv 5940\ (mod\ 16699)$
3. $z(A^2, A(A+1)) + 81Biq_A + 180Cub_A + 90T_{4,A} = 0$
4. $x(1,B) - y(1,B) - 3510CP_B^6 + 1620CSq_B^a \equiv -1728 (mod\ 4293)$
5. $y(\mathrm{B}+1, \mathrm{B}) - 9210CP_B^{14} - So_B - 1633S_B - 3Pr_B \equiv -1336 (mod\ 25802)$

IV. Conclusion

In this paper, we have obtainable different sets of non-zero distinct integer solutions to the ternary cubic equation $5x^2 - 2y^2 = 3z^2$.

As the cubic diophantine equations are loaded in range, one may explore for other choices of equations beside with their solutions and dealings involving the solutions.

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