

# Analysis of Hybrid Lottery Scheduling algorithm using Markov chain Model

**Pradeep kumar Jatav, Research Scholar, Faculty of Computer Science, PAHER University, Udaipur, Rajasthan, India.**

**Rahul Singhai, Sr. Asst.Professor, International Institute of Professional Studies, DAVV, Indore, Madhya Pradesh, India.**

**Saurabh Jain, Professor, Shri Vaishnav Institute of Computer Application Shri Vaishnav Vidyapeeth Vishwavidyalaya, Indore, Madhya Pradesh, India .**

Abstract CPU scheduling algorithms decide which of the available ready queue process should be selected next for the execution so that we can have optimum utilization of CPU. A number of scheduling algorithms have been proposed so far. One of the efficient algorithm among them is lottery scheduling. It is based probability scheduling in which one or more tickets are assigned to each process and when CPU becomes available, a ticket number is generated randomly and the winner process is selected for assignment to CPU. In this paper, we have introduced different schemes for lottery scheduling and analyzed their performance by using Markov chain model. Further on the basis of derived expression for the different proposed schemes, numerical illustrations are identified to plot different graphs and simulation study have been done on the basis of these graphs. In the proposed hybrid model of lottery scheduling under different schemes, It is calculated that if the processes are selected in linear order then the performance is improved and waiting time can be reduced.

**Keyword: Markov chain, CPU- scheduling, lottery scheduling, Transition probability.**

## I. INTRODUCTION

Short-term scheduler select processes from ready queue and dispatch them to the CPU according to different scheduling algorithms so that there can have efficient utilization of CPU and other computer resources. These algorithms involve First Come First Serve, Shortest Job First, Priority scheduling, Round Robin, Lottery Scheduling etc. [1,4, 17].

Lottery scheduling is one of the efficient CPU scheduling algorithms in which at least one ticket is assigned to each process and the scheduler draws random ticket to select the process. In the proposed work, preemption is added in terms of time quantum, and a hybrid lottery scheduling model is drawn that seems more effective than the traditional lottery scheduling algorithm.

## II. LITERATURE REVIEW

Many researchers have analyzed the behaviors of the CPU Scheduling algorithms by using Markov chain model. Sendre and Singhai[1] analyzed and compared distinct schemes of improved round robin using Markov chain model by considering equal and unequal probability matrix. Jain and Jain [2] described and compared the SQMS and MQMS algorithms for Multiprocessing

environment . They also analyzed the behavior of Multiprocessor for load balancing scheme and did simulations studies with different parameters. Jain and Jain [3] proposed a stochastic model along with priority based state scheduling and distributed system. They separated the user factors to improve throughput and response time. Jain and Jain [4] a classified various distributed system based scheduling algorithms model, and implemented the concepts of a Markov chain model. Vyash and Jain [5],[17] developed a hybrid based Markov chain model for lottery-based system. They also did simulation study with a different scheme to analyze the behavior. Vyash and Jain [6] also developed a Markov chain based model on extensive round robin with different schemes. Jain and Jain [7] proposed a multi level feedback queue system and finding states. the effects of wait state on throughput and overall performance of the system. Jain and Jain [8] introduced and compared various CPU scheduling algorithms and applied probability based model. Chavan and Tikekar [9] discussed a comparative study of various CPU scheduling algorithms with a special parameter. Kumar et al. [10] proposed and compared scheduling policies named as FCFS, SJF, RR, and PBS. This comparative study is also analyzed through simulation on different data set in this paper. Goyal and Garg[11]

discussed the various types of CPU scheduling algorithms and compared them according to a different parameter such as throughput and waiting time. Ojha S. et al. [12] designed multilevel queue with lottery schedulers. In this paper they used the ticket generation mechanism for each process. A simulation based analysis is also performed to evaluate the performance of proposed schemes. Shukla. et al. [13] re-evaluated the multilevel queue scheduling on the backdrop of the designed data model with five different cases, which were compared using Markov chain model. Shukla et al. [14] also, developed a K-processing environment in different schemes. and used a random process without any replacement method. Petrou et al. [15] Introduced lottery scheduling algorithms in typical OS schedulers to improve interactive response time and reduce kernel lock collision. They implemented the state forward lottery scheduling techniques, which enabled contention over process execution rates and processor load. they also uses FreeBSD scheduling baseline. Waldspureger. and Williom[16] suggested a novel mechanism that provides efficient and responsive control over the relative executive execution rates of computations using lottery based scheduling. they also examined the use of lotteries for managing memory, virtual circuit bandwidth and multiple resources. This paper proposes lottery based Markov chain model with two type schemes. the performance over lottery scheduling along with various data sets

### III. PROPOSED HYBRID LOTTERY SCHEDULING

Consider a multiprocessing environment, where five process P1, P2, P3, P4, P5 are residing in a ready queue and waiting for their chance to be assign to the CPU. The

processes whose executions were suspended are in waiting queue (W). The selection of process from ready queue is being done according to lottery scheduling. When operating system creates a new process. It assigns a lottery tickets for that process. Each process may have one or more than one ticket thus by giving at least one lottery ticket to each process ensures that each process has non-zero probability of being selected during each scheduling task.

The CPU scheduler generates random ticket numbers and the process having that tickets got the chance of execution thus the winner process is executed next for the assigned time quantum. If the process gets completed within the time quantum then it Scheduling System otherwise it moves to waiting state(W) till it gets the next chance by scheduler so in either case the scheduler picks another ticket and select another process.

The assumptions of the given model are:-

1. The scheduler has random movement over all the processes.
2. The process whose execution is being suspended either due to completion of time quantum or occurrence of any I/O request or any halt conditions are moved to waiting state(W)
3. All processes are either in running state or in waiting state at any time.
4. The scheduler picks any of the process with probability  $P_{ri}$  (where  $i=1, 2, \dots, 6$ )
5. When the execution of any process gets completed than it comes out from the system.

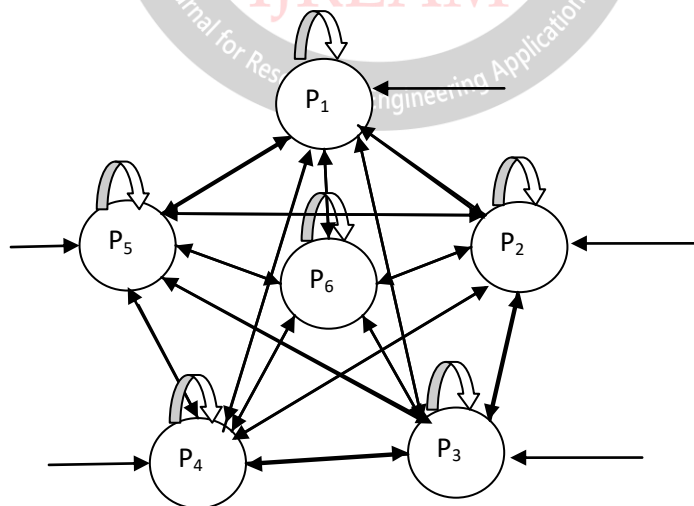


Figure:-3.1: Transition Diagram

### IV. MARKOV CHAIN MODEL

Let  $(X(n), n \geq 1)$  be a Markov chain where  $X(n)$  denotes the state of the lottery based scheduler at different quantum of time. The state space for the random variable  $X(n)$  is  $\{P_1, P_2, P_3, P_4, P_5, P_6\}$  where  $P_6 = W$  (waiting state) and scheduler  $X$  randomly (lottery based) moves stochastically over different processes (state) and waiting states for different quantum of time.

Predefined selections for initial probabilities of states are:

$$\left. \begin{aligned} P[X^{(0)} = p_1] &= pr_1 \\ P[X^{(0)} = p_2] &= pr_2 \\ P[X^{(0)} = p_3] &= pr_3 \\ P[X^{(0)} = p_4] &= pr_4 \\ P[X^{(0)} = p_5] &= pr_5 \\ P[X^{(0)} = p_6] &= pr_6 \end{aligned} \right\} \dots\dots\dots 3.1$$

With

$$pr_1 + pr_2 + pr_3 + pr_4 + pr_5 + pr_6 = \sum_{i=1}^6 pr_i = 1 \text{ where } pr_6 = 0$$

Let  $P_{ij}$  ( $i, j=1,2,3,4,5,6$ ) be the unit step transition probabilities of lottery scheduler over six proposed states then transition probability matrix is as follows:

		$X^{(n)}$					
		$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
$X^{(n-1)}$	$P_1$	$P_{11}$	$P_{12}$	$P_{13}$	$P_{14}$	$P_{15}$	$P_{16}$
	$P_2$	$P_{21}$	$P_{22}$	$P_{23}$	$P_{24}$	$P_{25}$	$P_{26}$
	$P_3$	$P_{31}$	$P_{32}$	$P_{33}$	$P_{34}$	$P_{35}$	$P_{36}$
	$P_4$	$P_{41}$	$P_{42}$	$P_{43}$	$P_{44}$	$P_{45}$	$P_{46}$
	$P_5$	$P_{51}$	$P_{52}$	$P_{53}$	$P_{54}$	$P_{55}$	$P_{56}$
	$P_6$	$P_{61}$	$P_{62}$	$P_{63}$	$P_{64}$	$P_{65}$	$P_{66}$

Figure:-3.2: Transition Probability Matrix

If  $P_{ij}$  ( $i, j= 1,2,3,4,5,6$ ) be Unit step transition probability of scheduler over proposed six states then lottery based transition processing for  $X(n)$  will be

$$P_{ij} = P[X(n)=P_i / X(n-1) = P_j]$$

Unit step transition probability from waiting state W are as follow

$$\left. \begin{aligned} P_{16} &= 1 - \sum_{i=1}^5 p_{1i}, P_{26} = 1 - \sum_{i=1}^5 p_{2i}, \\ P_{36} &= 1 - \sum_{i=1}^5 p_{3i}, \\ P_{46} &= 1 - \sum_{i=1}^5 p_{4i}, P_{56} = 1 - \sum_{i=1}^5 p_{5i} \\ P_{66} &= 1 - \sum_{i=1}^5 p_{6i} \\ 0 &\leq P_{ij} \leq 1 \end{aligned} \right\} \dots\dots\dots 3.2$$

After first quantum the state probability can be determined by the following expressions:-

$$P[X^{(1)}=P_1] = P[X^{(0)}=P_1] .P[X^{(1)}=P_1/ X^{(0)}=P_1] + P[X^{(0)}=P_2].P[X^{(1)}=P_1/ X^{(0)}=P_2] + P[X^{(0)}=P_3].P[X^{(1)}=P_1/ X^{(0)}=P_3] + P[X^{(0)}=P_4].P[X^{(1)}=P_1/ X^{(0)}=P_4] + P[X^{(0)}=P_5].P[X^{(1)}=P_1/ X^{(0)}=P_5] + P[X^{(0)}=P_6].P[X^{(1)}=P_1/ X^{(0)}=P_6]$$

$$P[X^{(1)}=P_1] = \sum_{i=1}^6 pr_i p_{i1}$$

Such That

Hence we obtained the following :

$$\left. \begin{aligned} P[X^{(1)}=P_1] &= \sum_{i=1}^6 pr_i p_{i1} \\ P[X^{(1)}=P_2] &= \sum_{i=1}^6 pr_i p_{i2} \\ \\ P[X^{(1)}=P_3] &= \sum_{i=1}^6 pr_i p_{i3} \\ P[X^{(1)}=P_4] &= \sum_{i=1}^6 pr_i p_{i4} \\ P[X^{(1)}=P_5] &= \sum_{i=1}^6 pr_i p_{i5} \\ \\ P[X^{(1)}=P_6] &= \sum_{i=1}^6 pr_i p_{i6} \end{aligned} \right\} \dots\dots\dots 3.3$$

In a similar way, the generalized equations for the nth quantum are:-

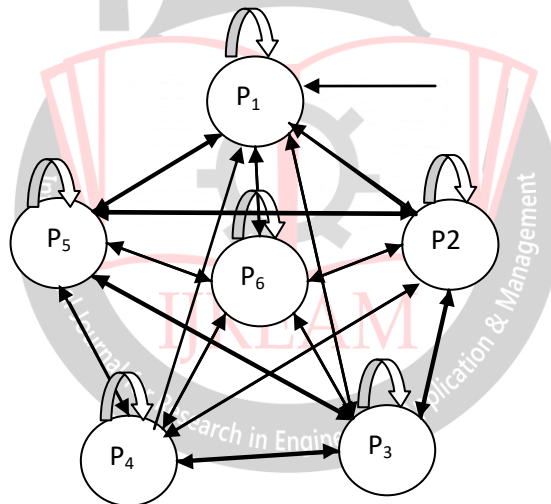
$$\left. \begin{aligned}
 P[X^{(n)}=P_1] &= \sum_{q=1}^6 \dots \sum_{m=1}^6 \{ \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 \{ \sum_{i=1}^6 (pr_i p_{ij}) \} P_{jk} \} P_{kl} \} P_{lm} \} P_{mn} \} P_{n1} \dots P_{q1} \\
 P[X^{(n)}=P_2] &= \sum_{q=1}^6 \dots \sum_{m=1}^6 \{ \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 \{ \sum_{i=1}^6 (pr_i p_{ij}) \} P_{jk} \} P_{kl} \} P_{lm} \} P_{mn} \} P_{n1} \dots P_{q2} \\
 P[X^{(n)}=P_3] &= \sum_{q=1}^6 \dots \sum_{m=1}^6 \{ \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 \{ \sum_{i=1}^6 (pr_i p_{ij}) \} P_{jk} \} P_{kl} \} P_{lm} \} P_{mn} \} P_{n1} \dots P_{q3} \\
 P[X^{(n)}=P_4] &= \sum_{q=1}^6 \dots \sum_{m=1}^6 \{ \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 \{ \sum_{i=1}^6 (pr_i p_{ij}) \} P_{jk} \} P_{kl} \} P_{lm} \} P_{mn} \} P_{n1} \dots P_{q4} \\
 P[X^{(n)}=P_5] &= \sum_{q=1}^6 \dots \sum_{m=1}^6 \{ \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 \{ \sum_{i=1}^6 (pr_i p_{ij}) \} P_{jk} \} P_{kl} \} P_{lm} \} P_{mn} \} P_{n1} \dots P_{q5} \\
 P[X^{(n)}=P_6] &= \sum_{q=1}^6 \dots \sum_{m=1}^6 \{ \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 \{ \sum_{i=1}^6 (pr_i p_{ij}) \} P_{jk} \} P_{kl} \} P_{lm} \} P_{mn} \} P_{n1} \dots P_{q6}
 \end{aligned} \right\} \dots\dots\dots 3.4$$

**V. SOME LOTTERY SCHEDULING SCHEME USING MARKOV CHAIN MODEL**

Following two Schemes are obtained by imposing restrictions and conditions over different states in the given generalized model:-

**5.1 Scheme –I** It is assume that initially process P<sub>1</sub> is going to be executed by earning maximum tickets. After completion of first time quantum. so scheduler can execute

any of the process including P<sub>1</sub> that wins the lottery ticket and after that either the process P<sub>1</sub> or any of the other process make a chance to win the lottery ticket, thus it is possible that the scheduler can select any of the same process again if that process wins the lottery tickets. So in same way scheduler select all the processes till they all gets completed. The transition diagram over various states are given below.



**Figure:-5.1: Transition Diagram Scheme -I**

Initial probability for the scheme-I are:-

$$P[X^{(0)}=P_1] = 1, P[X^{(0)}=P_2] = 0, P[X^{(0)}=P_3] = 0, P[X^{(0)}=P_4] = 0, P[X^{(0)}=P_5] = 0, P[X^{(0)}=P_6] = 0$$

		← X <sup>(n)</sup> →					
		P1	P2	P3	P4	P5	P6
X <sup>(n-1)</sup>	P1	P11	P12	P13	P14	P15	P16
	P2	P21	P22	P23	P24	P25	P26
	P3	P31	P32	P33	P34	P35	P36
	P4	P41	P42	P43	P44	P45	P46
	P5	P51	P52	P53	P54	P55	P56
	P6	P61	P62	P63	P64	P65	P66

**Transition Probability Matrix for Scheme-I**

**Remark 5.1.1:** Using equation 3.3 State probability after the first quantum for scheme –I are as below.

$$P[X^{(1)}=P_1] = P[X^{(0)}=P_1] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_1] + P[X^{(0)}=P_2] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_2] + P[X^{(0)}=P_3] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_3] + P[X^{(0)}=P_4] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_4] + P[X^{(0)}=P_5] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_5] + P[X^{(0)}=P_6] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_6]$$

$$P[X^{(1)}=P_1] = P_{11}$$

Hence for the first quantum, we have

$$P[X^{(1)}=P_1] = P_{11}, P[X^{(1)}=P_2] = P_{12}, P[X^{(1)}=P_3] = P_{13}, P[X^{(1)}=P_4] = P_{14}, P[X^{(1)}=P_5] = P_{15}, P[X^{(1)}=P_6] = P_{16}$$

**Remark 5.1:** Using equation 3.4 State probability after the first quantum for scheme -I

Generalized expression for nth quantum of scheme-I are

**5.2 Scheme II:** - It is assumed that :

Execution always start from process P<sub>1</sub> On completion of time quantum for Process P<sub>1</sub> the scheduler select next

$$P[X^{(n)}=P_1] = \sum_{q=1}^6 \dots \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (p_{1i}) \} P_{ij} \} P_{j1} \dots P_{q1}$$

$$P[X^{(n)}=P_2] = \sum_{q=1}^6 \dots \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (p_{2i}) \} P_{ij} \} P_{j1} \dots P_{q2}$$

$$P[X^{(n)}=P_3] = \sum_{q=1}^6 \dots \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (p_{3i}) \} P_{ij} \} P_{j1} \dots P_{q3}$$

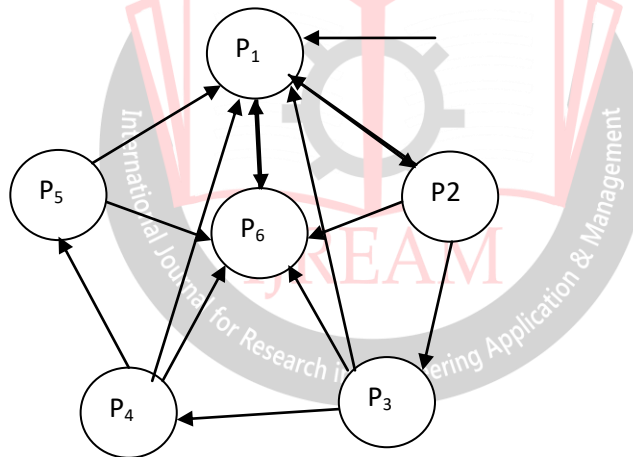
$$P[X^{(n)}=P_4] = \sum_{q=1}^6 \dots \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (p_{4i}) \} P_{ij} \} P_{j1} \dots P_{q4}$$

$$P[X^{(n)}=P_5] = \sum_{q=1}^6 \dots \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (p_{5i}) \} P_{ij} \} P_{j1} \dots P_{q5}$$

$$P[X^{(n)}=P_6] = \sum_{q=1}^6 \dots \sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (p_{6i}) \} P_{ij} \} P_{j1} \dots P_{q6}$$

process in sequentially manner and From any process the scheduler may move to either next process that comes next in sequential order or to waiting state or to process P<sub>1</sub>.

The Transition models according to above assumption are drawn below.



**Figure:-5.2: Transition Diagram Scheme –II**

thus the initial probability are  $P[X^{(0)}=P_1] = 1, P[X^{(0)}=P_2] = 0, P[X^{(0)}=P_3] = 0, P[X^{(0)}=P_4] = 0, P[X^{(0)}=P_5] = 0, P[X^{(0)}=P_6] = 0$

		$X^{(n)}$					
		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
$X^{(n-1)}$	P <sub>1</sub>	0	P <sub>12</sub>	0	0	0	P <sub>16</sub>
	P <sub>2</sub>	P <sub>21</sub>	0	P <sub>23</sub>	0	0	P <sub>26</sub>
	P <sub>3</sub>	P <sub>31</sub>	0	0	P <sub>34</sub>	0	P <sub>36</sub>
	P <sub>4</sub>	P <sub>41</sub>	0	0	0	P <sub>45</sub>	P <sub>46</sub>
	P <sub>5</sub>	P <sub>51</sub>	0	0	0	0	P <sub>56</sub>
	P <sub>6</sub>	P <sub>61</sub>	0	0	0	0	0

**Transition Matrix for Scheme-II**

**Remark 5.2.1** using (3.3), state probabilities after the first quantum for scheme-II are

$$P[X^{(1)}=P_1] = P[X^{(0)}=P_1] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_1] + P[X^{(0)}=P_2] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_2] + P[X^{(0)}=P_3] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_3] + P[X^{(0)}=P_4] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_4] + P[X^{(0)}=P_5] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_5] + P[X^{(0)}=P_6] \cdot P[X^{(1)}=P_1 / X^{(0)}=P_6]$$

$$P[X^{(1)}=P_1] = 0$$

$$P[X^{(1)}=P_2] = P[X^{(0)}=P_1] \cdot P[X^{(1)}=P_2 / X^{(0)}=P_1] + P[X^{(0)}=P_2] \cdot P[X^{(1)}=P_2 / X^{(0)}=P_2] + P[X^{(0)}=P_3] \cdot P[X^{(1)}=P_2 / X^{(0)}=P_3] + \dots$$

$$m_{ij} = \begin{cases} 0 & (i=1, j=1, 3, 4, 5 \text{ for } P_1) \\ & (i=2, j=2, 4, 5 \text{ for } P_2) \\ & (i=3, j=3, 2, 5 \text{ for } P_3) \\ & (i=4, j=2, 3, 4 \text{ for } P_4) \\ & (i=5, j=2, 3, 4, 5 \text{ for } P_5) \\ & (i=6, j=2, 3, 4, 5, 6 \text{ for } P_6) \\ 1 & \text{other wise} \end{cases}$$

$$P[X^{(1)}=P_3] = P[X^{(0)}=P_4] \cdot P[X^{(1)}=P_3 / X^{(0)}=P_4] + P[X^{(0)}=P_5] \cdot P[X^{(1)}=P_3 / X^{(0)}=P_5] + P[X^{(0)}=P_6] \cdot P[X^{(1)}=P_3 / X^{(0)}=P_6]$$

$$P[X^{(1)}=P_2] = P_{12}$$

Hence for the first quantum, we obtained

$$P[X^{(1)}=P_1] = 0 \quad P[X^{(1)}=P_2] = P_{12} \quad P[X^{(1)}=P_3] = 0$$

$$P[X^{(1)}=P_4] = 0 \quad P[X^{(1)}=P_5] = 0 \quad P[X^{(1)}=P_6] = P_{16}$$

Define an indicator functions  $m_{ij}$  ( $i, j=1, 2, 3, 4, 5, 6$ ) such that

Then using (3.3) state probability after second quantum for scheme-II are

$$P[X^{(2)}=P_1] = \sum_{j=1}^6 (m_{1j} P_{1j}) (m_{j1} P_{j1})$$

$$P[X^{(2)}=P_2] = \sum_{j=1}^6 (m_{2j} P_{2j}) (m_{j2} P_{j2})$$

$$P[X^{(2)}=P_3] = \sum_{j=1}^6 (m_{3j} P_{3j}) (m_{j3} P_{j3})$$

$$P[X^{(2)}=P_4] = \sum_{j=1}^6 (m_{4j} P_{4j}) (m_{j4} P_{j4})$$

$$P[X^{(2)}=P_5] = \sum_{j=1}^6 (m_{5j} P_{5j}) (m_{j5} P_{j5})$$

$$P[X^{(2)}=P_6] = \sum_{j=1}^6 (m_{6j} P_{6j}) (m_{j6} P_{j6})$$

$$P[X^{(n)}=P_4] = \sum_{q=1}^6 \dots$$

$$\sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (pr_{li} m_{li}) \} P_{j1} m_{j4} \dots P_{q1} m_{q4} \}$$

$$P[X^{(n)}=P_5] = \sum_{q=1}^6 \dots$$

$$\sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (pr_{li} m_{li}) \} P_{j1} m_{j5} \dots P_{q1} m_{q5} \}$$

$$P[X^{(n)}=P_6] = \sum_{q=1}^6 \dots$$

$$\sum_{l=1}^6 \{ \sum_{k=1}^6 \{ \sum_{j=1}^6 \{ \sum_{i=1}^6 (pr_{li} m_{li}) \} P_{j1} m_{j6} \dots P_{q1} m_{q6} \}$$

**VI. NUMERICAL ANALYSIS USING DATA SETS**

In order to analyze two schemes mentioned in section 4.1 and 4.2 under Markov Chain model with equal and unequal transition elements, the following data sets are considered.

**6.1 Data Set-I**

**Scheme-1:** Let initial probability are

$$Pr_1 = 0.3, Pr_2 = 0.2, Pr_3 = 0.30, Pr_4 = 0.1, Pr_5 = 0.19$$

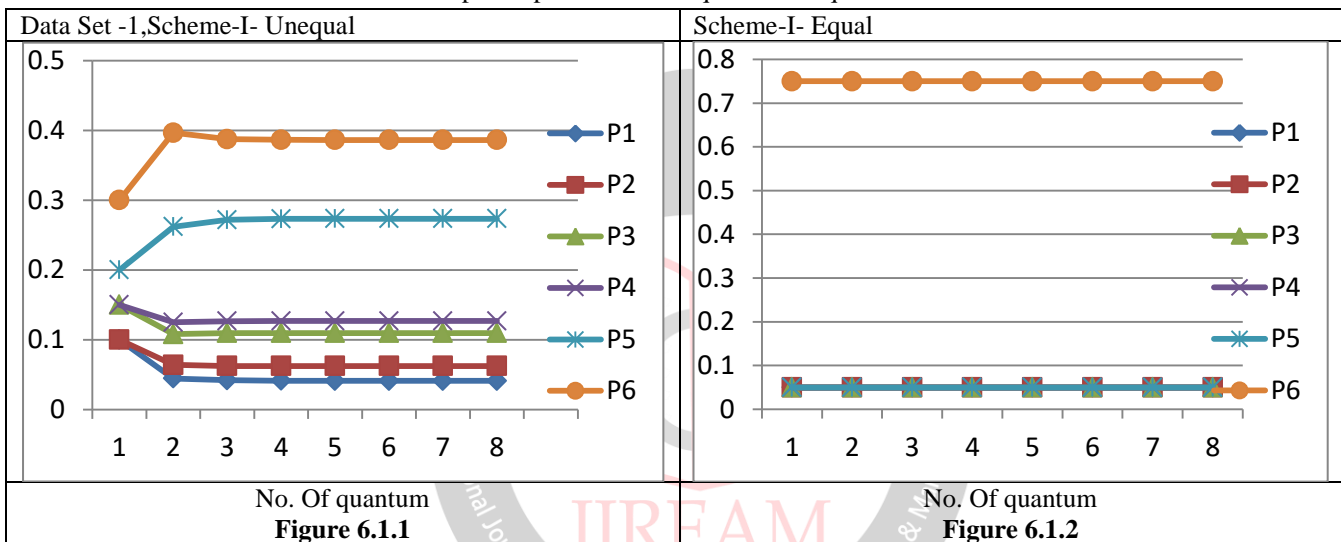
Unequal and equal Probability Matrices are as follows

Unequal								Equal							
← X (n) →								← X (n) →							
		P1	P2	P3	P4	P5	P6			P1	P2	P3	P4	P5	P6
	P1	.10	.10	.15	.15	.20	.30		P1	.05	.05	.05	.05	.05	.75
	P2	.04	.10	.15	.18	.20	.33		P2	.05	.05	.05	.05	.05	.75
	X(n-1)	P3	.02	.03	.04	.06	.25	.6	X(n-1)	P3	.05	.05	.05	.05	.75
		P4	.05	.05	.08	.10	.27	.45		P4	.05	.05	.05	.05	.75
		P5	.01	.07	.15	.16	.36	.25		P5	.05	.05	.05	.05	.75
		P6	.06	.06	.10	.12	.24	.42		P6	.05	.05	.05	.05	.75

Unequal							Equal						
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>		P1	P2	P3	P4	P5	P6
n=1	.10	.10	.15	.15	.20	.30	n=1	.05	.05	.05	.05	.05	0.75
n=2	0.0445	0.064	0.108	0.125	0.262	0.3965	n=2	.05	.05	.05	.05	.05	0.75
n=3	0.0418	0.0624	0.1095	0.1266	0.2719	0.3875	n=3	.05	.05	.05	.05	.05	0.75
n=4	0.0411	0.0623	0.1096	0.1267	0.2732	0.3865	n=4	.05	.05	.05	.05	.05	0.75
n=5	0.04105	0.06227	0.10966	0.12671	0.27340	0.38629	n=5	.05	.05	.05	.05	.05	0.75
n=6	0.04103	0.06227	0.10966	0.12671	0.27342	0.38627	n=6	.05	.05	.05	.05	.05	0.75
n=7	0.04103	0.06227	0.10965	0.12671	0.27342	0.38626	n=7	.05	.05	.05	.05	.05	0.75
n=8	0.04104	0.06227	0.10966	0.12672	0.27342	0.38625	n=8	.05	.05	.05	.05	.05	.075

Table 6.1.1 Transition probability for unequal and equal cases.

Graphical pattern for Unequal and Equal cases are:



**Unequal:-** the state probability P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub>, P<sub>4</sub>, P<sub>5</sub>, and P<sub>6</sub> of the lottery scheduler makes stable pattern when number of quantum  $n \geq 2$  but for  $n < 2$  it reflects changing in the pattern. The main point is that the probability of wait state P<sub>6</sub> is higher in this data sets than other states probabilities as shown in figure 6.3.1

**Equal:-** The probability for waiting state probability is becoming very high over the all state probabilities showing the inefficiency of Scheme-I with equal transition element.

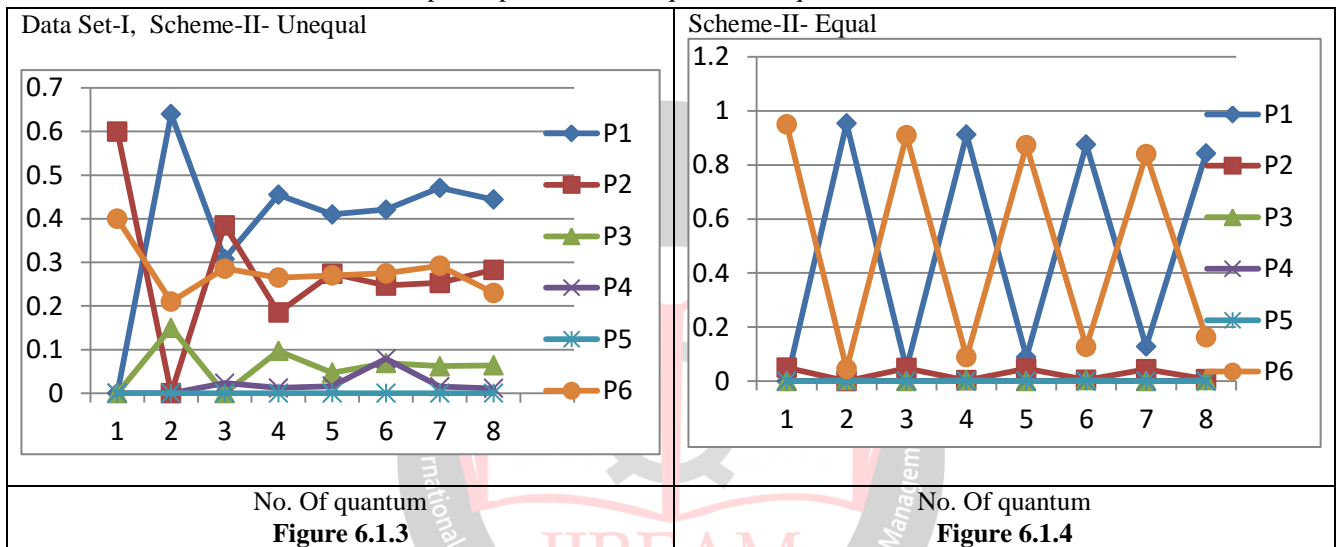
**Scheme-II:** Let initial probabilities are:  $Pr_1 = 1.0, Pr_2 = 0.00, Pr_3 = 0.00, Pr_4 = 0.00, Pr_5 = 0.00$ , Unequal and equal Probability Matrices are as follows:

Unequal $X^{(n)}$								Equal $X^{(n)}$							
		P1	P2	P3	P4	P5	P6			P1	P2	P3	P4	P5	P6
	P1	0	.60	0	0	0	.40		P1	0	.05	0	0	0	.95
	P2	.40	0	.25	0	0	.35		P2	.05	0	.05	0	0	.90
X(n-1)	P3	.65	0	0	.15	0	.20		X(n-1)	P3	.05	0	0	.05	.90
	P4	.65	0	0	0	.05	.30			P4	.05	0	0	0	.90
	P5	.85	0	0	0	0	.15			P5	.05	0	0	0	.95
	P6	1	0	0	0	0	0			P6	1	0	0	0	0

Unequal							Equal						
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
n=1	0	.60	0	0	0	.40	n=1	0	.05	0	0	0	.95
n=2	.64	0	.15	0	0	.21	n=2	.953	0	.003	0	0	.045
n=3	.308	.385	0	.023	0	.286	n=3	.046	.048	0	.0002	0	.909
n=4	.455	.185	.097	.012	0	.265	n=4	.912	.003	.003	0	.0001	.088
n=5	.410	.274	.047	.016	0	.270	n=5	.089	.046	.0002	.0002	0	.872
n=6	.421	.247	.069	.079	0	.275	n=6	.875	.005	.003	.00001	.00001	.127
n=7	.471	.253	.062	.015	0	.292	n=7	.128	.044	.0003	.0002	.000005	.839
n=8	.444	.283	.064	.011	0	.230	n=8	.842	.007	.003	.00002	.00001	.162

Table 6.1.2. Transition probability below for Unequal cases

Graphical pattern for Unequal and Equal cases are:-



**Unequal:** - Graphics reveal a higher and increasing manner of state probabilities at the state P<sub>1</sub>, P<sub>2</sub> and P<sub>6</sub> then the other state probability P<sub>3</sub>, P<sub>4</sub> and P<sub>5</sub> but state P<sub>1</sub> is very high over remaining states so this is good sign for lottery scheduler.

**Equal:**-all state probabilities are moved independent of quantum variation because the pattern of distribution of

state probabilities is almost similar in this pattern so this equal data set provides chance for job processing with waiting state.

**6.2 Data Set- II**

**Scheme-1:** Let initial probability are Pr<sub>1</sub> =0.20, Pr<sub>2</sub> =0.35, Pr<sub>3</sub> =0.40, Pr<sub>4</sub> =0.01, Pr<sub>5</sub> =0.04, Unequal and equal Probability Matrix are follow:-

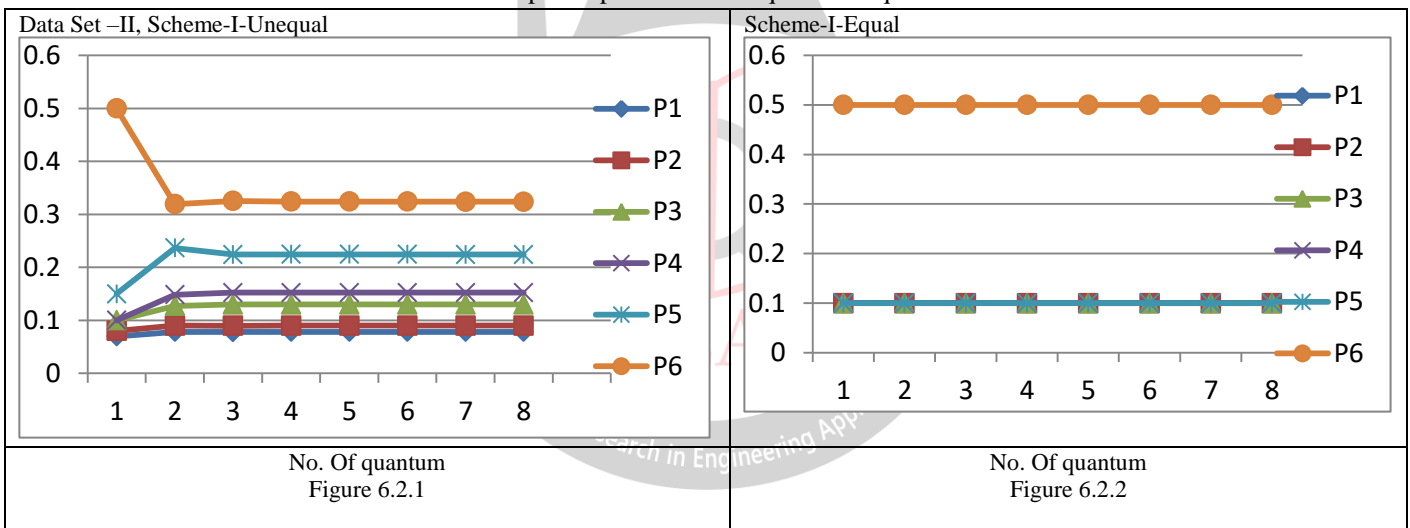
Unequal								Equal							
X <sup>(n)</sup>								X <sup>(n)</sup>							
		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>			P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
	P <sub>1</sub>	.07	.08	.10	.10	.15	.5		P <sub>1</sub>	.10	.10	.10	.10	.10	.50
	P <sub>2</sub>	.05	.08	.13	.15	.25	.34		P <sub>2</sub>	.10	.10	.10	.10	.10	.50
X <sup>(n-1)</sup>	P <sub>3</sub>	.10	.11	.13	.20	.21	.25		X <sup>(n-1)</sup>	P <sub>3</sub>	.10	.10	.10	.10	.50
	P <sub>4</sub>	.09	.10	.15	.17	.20	.29			P <sub>4</sub>	.10	.10	.10	.10	.50
	P <sub>5</sub>	.07	.08	.13	.15	.20	.37			P <sub>5</sub>	.10	.10	.10	.10	.50
	P <sub>6</sub>	.08	.09	.12	.14	.27	.30			P <sub>6</sub>	.10	.10	.10	.10	.50



Unequal							Equal					
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
n=1	.07	.08	.10	.10	.15	.5	.10	.10	.10	.10	.10	.50
n=2	0.0784	0.09	0.1273	0.1485	0.2365	0.3193	.10	.10	.10	.10	.10	.50
n=3	0.0781	0.0899	0.1301	0.1522	0.2242	0.3252	.10	.10	.10	.10	.10	.50
n=4	0.0783	0.0901	0.1301	0.1523	0.2245	0.3240	.10	.10	.10	.10	.10	.50
n=5	0.0784	0.0902	0.1300	0.1522	0.2244	0.3241	.10	.10	.10	.10	.10	.50
n=6	0.0783	0.0901	0.1300	0.1522	0.2244	0.3240	.10	.10	.10	.10	.10	.50
n=7	0.0783	0.0901	0.1300	0.1522	0.2243	0.3239	.10	.10	.10	.10	.10	.50
n=8	0.0783	0.0900	0.1300	0.1522	0.2243	0.3239	.10	.10	.10	.10	.10	.50

Table 6.2.1 Transition probability for Unequal cases and Equal Cases.

Graphical pattern for Unequal and Equal



**Unequal:-**Graphical pattern (figure 6.2.1) reveals initially higher probability at the wait state P<sub>6</sub> than the other states. But after few quantum it decreased and followed a stable pattern and as compared with previous data set state probability P<sub>5</sub> given good sign for this data set. This leads to a better performance over the other states (P<sub>1</sub>, P<sub>5</sub>, P<sub>2</sub>, P<sub>3</sub>, and P<sub>4</sub>) for lottery scheduling specially probability for the states P<sub>1</sub>, P<sub>2</sub> and P<sub>3</sub> is very low as compared to P<sub>4</sub> and P<sub>5</sub> in unequal data set.

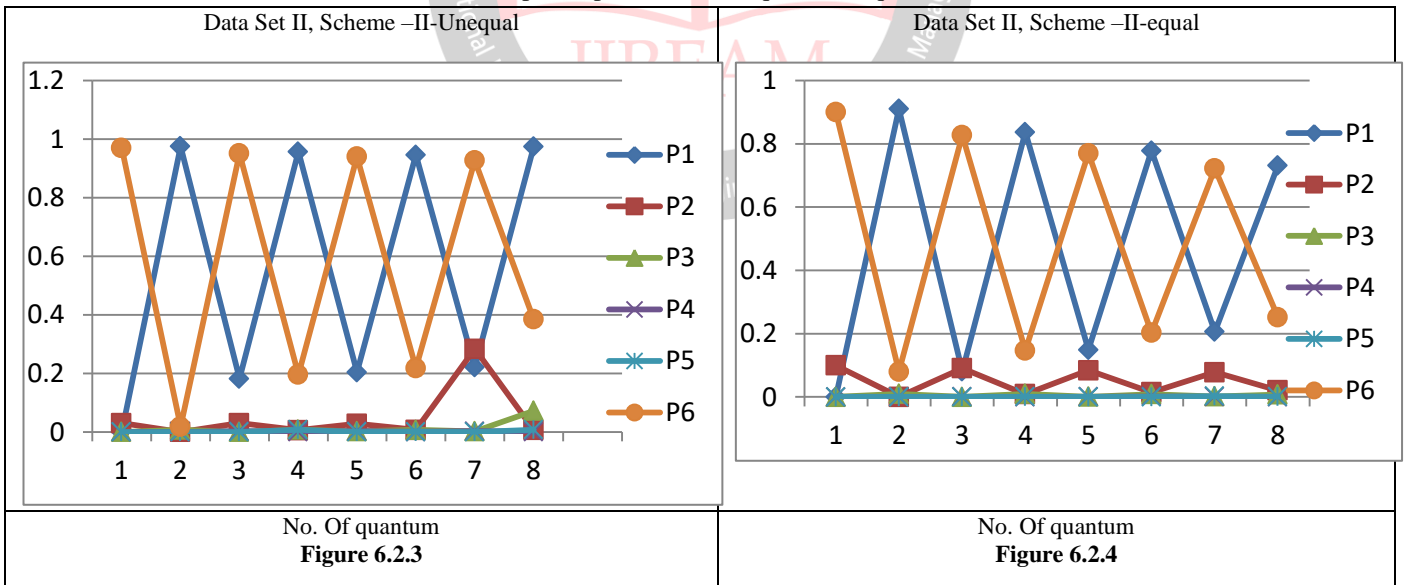
**Equal:-** The probability for waiting state is becoming very high over the all state showing the inefficiency of Scheme-I with equal transition element.

**Scheme-II:** Let initial probability are Pr<sub>1</sub>=1.0, Pr<sub>2</sub>=0.00, Pr<sub>3</sub>=0.00, Pr<sub>4</sub>=0.00, Pr<sub>5</sub>=0.00, Unequal and equal Probability Matrix are follow:

Unequal $X^{(n)}$								Equal $X^{(n)}$							
↑		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	↑		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
	P <sub>1</sub>	0	.03	0	0	0	.97		P <sub>1</sub>	0	.10	0	0	0	.90
	P <sub>2</sub>	.16	0	.25	0	0	.59		P <sub>2</sub>	.10	0	.10	0	0	.80
X <sup>(n-1)</sup>	P <sub>3</sub>	.15	0	0	.30	0	.55	X <sup>(n-1)</sup>	P <sub>3</sub>	.10	0	0	.10	0	.80
	P <sub>4</sub>	.21	0	0	0	.25	.54		P <sub>4</sub>	.10	0	0	0	.10	.80
	P <sub>5</sub>	.63	0	0	0	0	.37		P <sub>5</sub>	.10	0	0	0	0	.90
↓	P <sub>6</sub>	1	0	0	0	0	0	↓	P <sub>6</sub>	1	0	0	0	.0	0

Unequal							Equal						
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
n=1	0	.03	0	0	0	.97	n=1	0	.10	0	0	0	.90
n=2	.975	0	.008	0	0	.018	n=2	.91	0	.01	0	0	.08
n=3	.182	.030	0	.003	0	.951	n=3	.081	.091	0	.001	0	.827
n=4	.957	.006	.008	0	.008	.196	n=4	.837	.009	.010	0	.001	.147
n=5	.204	.028	.002	.003	0	.940	n=5	.149	.084	.0009	.001	0	.770
n=6	.946	.007	.007	.0006	.0008	.218	n=6	.778	.015	.009	.002	.0002	.203
n=7	.221	.284	.002	.003	.0002	.927	n=7	.206	.078	.002	.0009	.003	.722
n=8	.974	.007	.072	.0007	.008	.385	n=8	.731	.021	.008	.0003	.0009	.252

**Table 6.2.1** The Transition probabilities for Unequal and Equal probabilities  
Graphical pattern for Unequal and Equal



**Unequal:-** In this graphical pattern (figure 6.3.3), we observed that state probability P<sub>1</sub> is showing the better performance as compared to other states. The special remark is that state probability P<sub>2</sub> also perform little bit high as compare to other processes (P<sub>3</sub>, P<sub>4</sub> and P<sub>5</sub>). Although the scheduler execute more jobs as compared to previous one. But it still shows average performance efficiency due

to this equal data set provides chance for job processing with waiting state.

**Equal:-** In this graphical pattern (figure 6.3.2.3), we observed that state probability P<sub>1</sub> is showing the better performance as compared to other states. The special remark is that state probability P<sub>2</sub> also perform little bit high as compare to other processes (P<sub>3</sub>, P<sub>4</sub> and P<sub>5</sub>). Although

the scheduler execute more jobs as compared to previous one. But it still shows average performance efficiency and in this equal data set provides chance for job processing with waiting state.

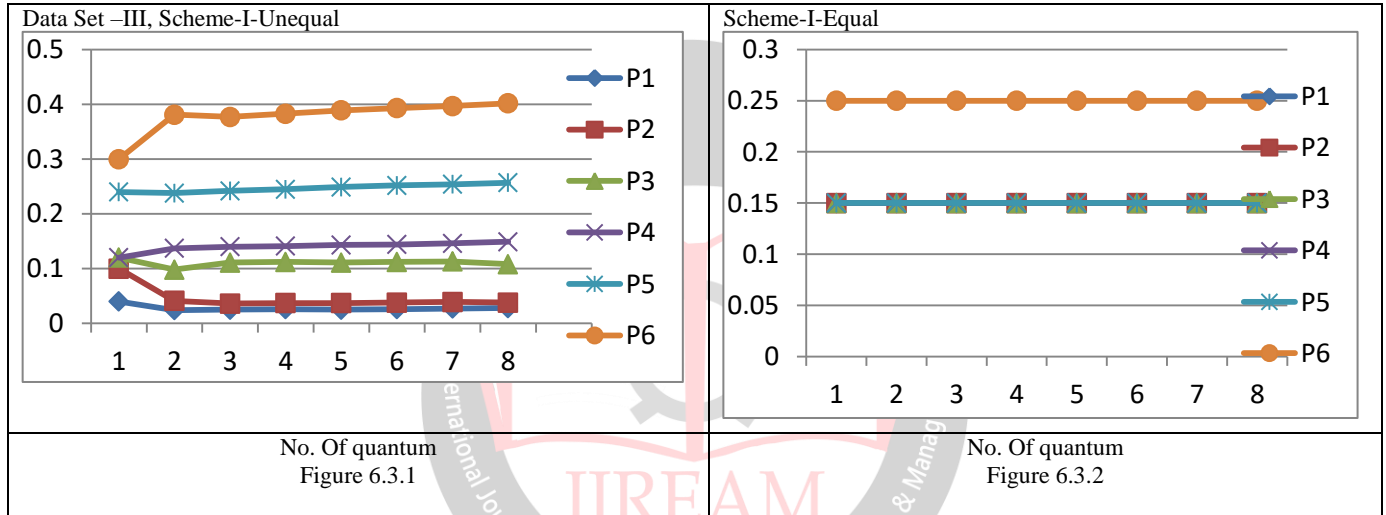
6.3.3 Data Set –III

**Scheme-1:** let initial probability are  $pr_1=.21, pr_2=.36, pr_3=.41, pr_4=0.02, p_{r5}=.04,$

Unequal and equal probability matrix are follow

Unequal								Equal							
$X^{(n)}$								$X^{(n)}$							
		$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$			$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
	$P_1$	.04	.1	.12	.12	.24	.30		$P_1$	.15	.15	.15	.15	.15	.25
	$P_2$	.02	.10	.13	.15	.20	.4		$P_2$	.15	.15	.15	.15	.15	.25
	$P_3$	.04	.05	.06	.10	.25	.5	$X^{(n-1)}$	$P_3$	.15	.15	.15	.15	.15	.25
	$P_4$	.03	.03	.20	.20	.3	.24		$P_4$	.15	.15	.15	.15	.15	.25
	$P_5$	.01	.02	.04	.15	.28	.5		$P_5$	.15	.15	.15	.15	.15	.25
	$P_6$	.03	.04	.13	.15	.25	.4		$P_6$	.15	.15	.15	.15	.15	.25

**Table 6.3.2** The Transition probabilities for Unequal and Equal probabilities  
Graphical pattern For Unequal and Equal



**Unequal:-**The state probability  $P_1, P_2, P_3, P_4, P_5,$  and  $P_6$  are showing independent behavior over the quantum variation and since the pattern of state probability of wait state  $P_6$  is high over the all remain states so this is not good for lottery scheduling.

**Equal:-** In equal transition, the probability for waiting state is becoming very high over all the remaining states

therefore showing inefficiency of Scheme-I with equal transition element.

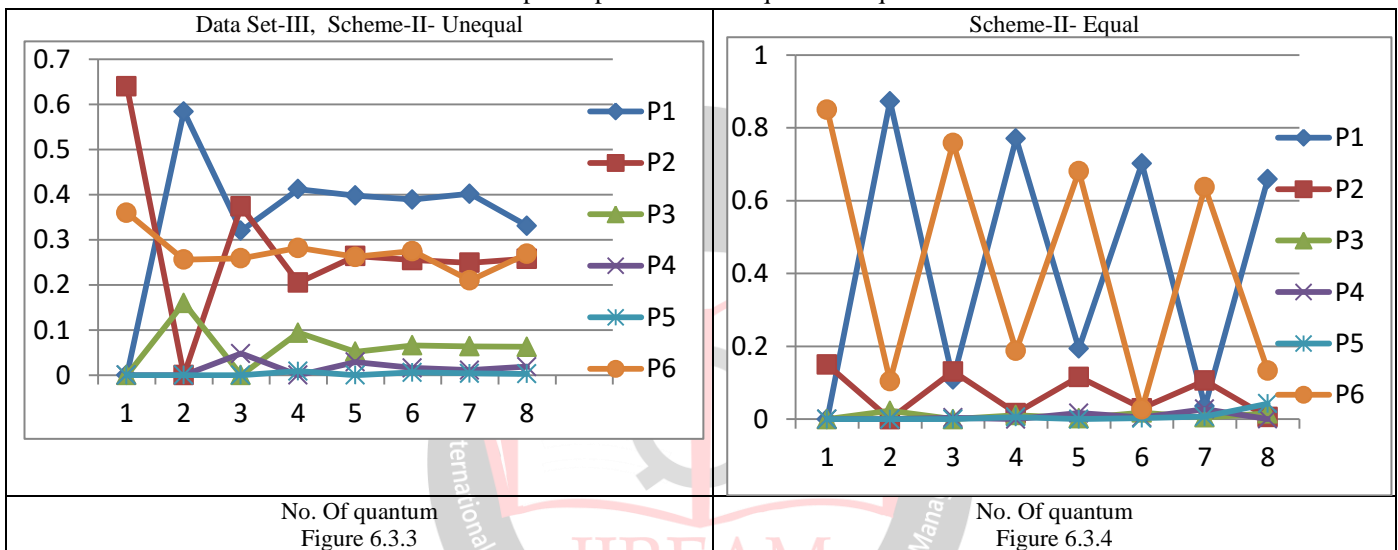
**Scheme-II:-** Let initial probability are  $Pr_1 = 1.0, Pr_2 = 0.00, Pr_3 = 0.00, Pr_4 = 0.00, Pr_5 = 0.00,$  Unequal and equal Probability Matrix are follow

Unequal								Equal							
$X^{(n)}$								$X^{(n)}$							
		$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$			$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
	$P_1$	0	.36	0	0	0	.64		$P_1$	0	.15	0	0	0	.85
	$P_2$	.20	0	.25	0	0	.55		$P_2$	.15	0	.15	0	0	.30
	$P_3$	.1	0	0	.2	0	.7	$X^{(n-1)}$	$P_3$	.15	0	0	.15	0	.70
	$P_4$	.15	0	0	0	.20	.65		$P_4$	.15	0	0	0	.15	.70
	$P_5$	.37	0	0	0	0	.63		$P_5$	.15	0	0	0	0	.85
	$P_6$	1	0	0	0	0	0		$P_6$	.15	0	0	0	0	0

Unequal							Equal						
	P1	P2	P3	P4	P5	P6		P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>
n=1	0	.64	0	0	0	.36	n=1	0	.15	0	0	0	.85
n=2	0.584	0	0.16	0	0	0.256	n=2	.873	0	.023	0	0	.105
n=3	0.32	0.374	0	0.048	0	0.259	n=3	.109	.131	0	.004	0	.759
n=4	0.412	0.205	0.094	0	0.009	0.282	n=4	.771	.017	.011	0	.006	.188
n=5	0.398	0.264	0.052	0.029	0	0.262	n=5	.194	.116	.003	.017	0	.681
n=6	0.389	0.255	0.066	0.016	0.006	0.275	n=6	.702	.030	.018	.005	.003	.027
n=7	0.402	0.249	0.064	0.011	0.004	0.210	n=7	0.036	0.106	0.005	0.028	.007	.637
n=8	0.331	0.258	0.063	0.019	0.003	.269	n=8	.659	.006	.016	.0008	.043	.134

Table 6.3.2 The Transition probabilities for Unequal and Equal probabilities

Graphical pattern for Unequal and Equal



No. Of quantum  
Figure 6.3.3

No. Of quantum  
Figure 6.3.4

**Unequal:-** In this graphical pattern (figure 6.2.3), we observed that state probability  $P_1$  showing the better performance as compared to other states. The special remark is that state probability  $P_2$  also performance high as compare to other processes ( $P_3, P_4, P_5$  and  $P_6$ ). Thus the schedulers execute more jobs as compared to previous one. This shows better performance efficiency under this data set scheduler probability for the state  $P_2, P_3, P_4$  and  $P_5$  is very low as compared to state  $P_1$  and  $P_2$  in Unequal data set and

**Equal:-** The state probability are moved independent of the quantum variation because the pattern of distribution of state probabilities is almost similar in this figure 6.3.4 this equal data set provides chance for job processing with waiting state.

**VII. CONCLUSION**

This paper describes the performance analysis and comparison between the distinct two schemes of the lottery scheduling under a Markov chain model by using equal and unequal transition probability matrix with different data sets having functions of restrictions probabilities in term of some state transition. Scheme-I suffers with high chance

for system reaching to waiting state. although for Unequal transition probabilities system representing better behaviors than more equal probabilities. Thus unequal transition probabilities seem to be more efficient and performing best job execution in data set-I and II. Scheme-II, the chance for waiting state is lower than other states over the unequal transition probabilities and provides better result for process  $P_1$  and  $P_2$ . Hence it is concluded that the state probabilities of lottery based system over these schemes are very useful that leads to improved performance in term of throughput and response time.

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